UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM2616

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COMPUTER ALGEBRA

Duration . 2 Hours

100 Marks

EXAMINERS :
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Closed book examination.

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This paper consists of 3 pages
Answer **all** the questions

QUESTION 1

Given that the following have been defined in a MuPAD session
n positive integer
x array of n identifiers
g $n \times n$ matrix
write a MuPAD procedure called mygam that takes the above as input and outputs $n \times n \times n$ array defined by

$$C_{abc} = \frac{1}{2}\sum_{i=1}^{n} h_{ci}\left(\frac{\partial g_{ai}}{\partial x_b} + \frac{\partial g_{bi}}{\partial x_a} - \frac{\partial g_{ab}}{\partial x_i}\right)$$

where h is the matrix inverse of g **[20 Marks]**

QUESTION 2

Write a MuPAD procedure, called mydiff(fx), that does not use the operator D or diff The input parameters are f which is a polynomial in x, and x which is an identifier. The output should be the first derivative of f with respect to x **[20Marks]**

[TURN OVER]

QUESTION 3

Write LaTeX code, in the form of a complete document, for the following

1. In what follows, Ω is a bounded domain of $\mathbb{R}^3$ with boundary Γ We define the following.

$$\mathbf{X} = \left\{ \varphi \in \mathbf{H}^1(\Omega) \quad \varphi_{/\Gamma} = 0 \right\}$$

Poincaré inequality

$$\|\varphi\| \leq C_\Omega \|\nabla \varphi\|, \tag{1}$$

holds for $\varphi \in \mathbf{X}$

2 Let

$$\phi(x) = \left[\sqrt{\sum_{n=1}^{\infty} \frac{\partial^n \epsilon}{\partial x_n^n} \frac{1}{\sqrt{n}} \phi^{(n)}(x)} \right]^{\frac{1}{n}} \tag{2}$$

Show that ϵ and $\varphi$are well defined for $x > 0$, in (1) and (2). **[20Marks]**

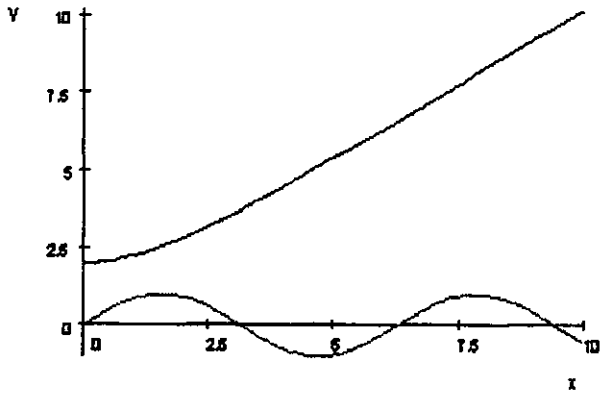
QUESTION 4

Write a MuPad code that produces the graph shown below, containing plots of

(a) $y = \sin x$

(b) The solution of the differential equation

$$\frac{dy}{dx} = \frac{x}{y}, \quad \text{with } y(0) = 2$$



[20Marks]

QUESTION 5

Let $f = (1+a)^3 \sin x \cos x - e^{3x} \cos^2(2x)$ Use MuPad to find a representation of f in the form $f = a + bx + cx^2$ (a, b, c numerical constant) valid for $x << 1$ You may assume that $e^x = 1 + x + \frac{x^2}{2}$, $\cos x = 1 - \frac{x^2}{2}$, and $\sin x = x$

[20Marks]

TOTAL: [100Marks]