

Tutorial Letter 202/1/2018

COMPUTER ALGEBRA

APM2616

Semester 1

Department of Mathematical Sciences

This tutorial letter contains solutions
for assignment 02.

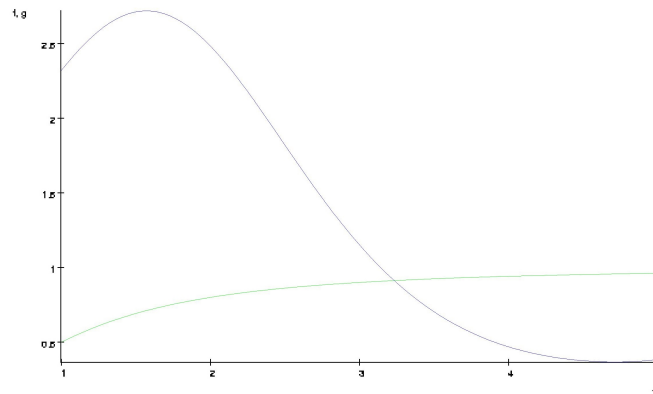
BAR CODE

QUESTION 1

Write MuPAD code to draw a graph that plots the functions $f(t) = e^{\sin t}$ and $g(t) = \frac{t^2}{1+t^2}$ in the range $t = -1$ to 5 . The axes should be appropriately labelled, and horizontal directions should be the same. The graph of f is to be blue and that of g green. **[10 Marks]**

SOLUTION 1

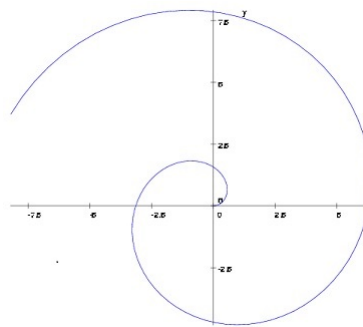
```
f:=plot::Function2d(exp(sin(t)),t= -1..5,Color=RGB::Blue):
g:=plot::Function2d(t^2/(1+t^2),t= -1..5,Color=RGB::Green):
plot(f,g,Scaling=Constrained,Labels=["t", "f, g"]):
```



[10 Marks]

QUESTION 2

Write MuPAD code to plot a spiral $(u \cos u, u \sin u)$ coloured blue so as to produce the result given below. **[10 Marks]**



SOLUTION 2

```
spiral:=[Mode = Curve, [u*cos(u), u*sin(u)], u = [0,9],Color=[Flat,RGB::Blue]]
plot2d(Scaling=Constrained,spiral);
```

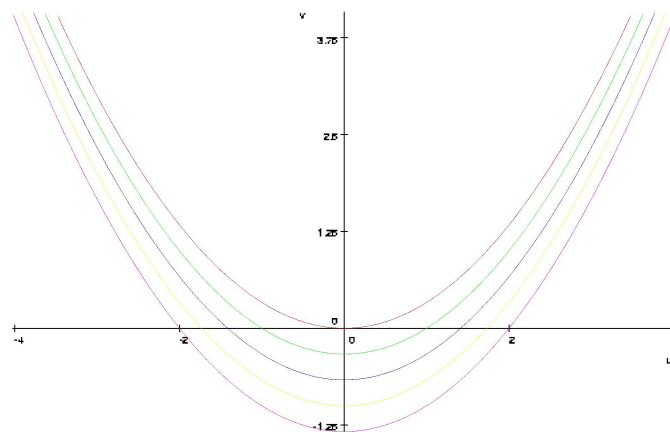
[10 Marks]

QUESTION 3

Plot contours of the function $f(u, v) = u^2 - 3v$ at $f = 0, 1, 2, 3$ and 4 . The axes, contours and graph should be appropriately labelled. **[10 Marks]**

SOLUTION 3

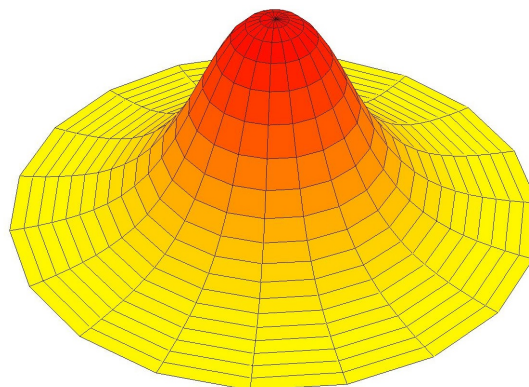
```
f:=u^2-3*v:
s:= plot::implicit(
[f,f-1,f-2,f-3,f-4], u = -4..4, v = -4..4):
plot(s,Labels=["u", "v"]):
```

**[10 Marks]****QUESTION 4**

Using smooth functions, construct a 3D graph that looks like a wide-brimmed hat. **[10 Marks]**

SOLUTION 4

```
s:=plot::cylindrical([u,v,(25-u^2)/(6+u^3)],u=0..5,v=-PI..PI):
plot(s,Scaling=Constrained,Axes=None);
```

**[10 Marks]**

QUESTION 5

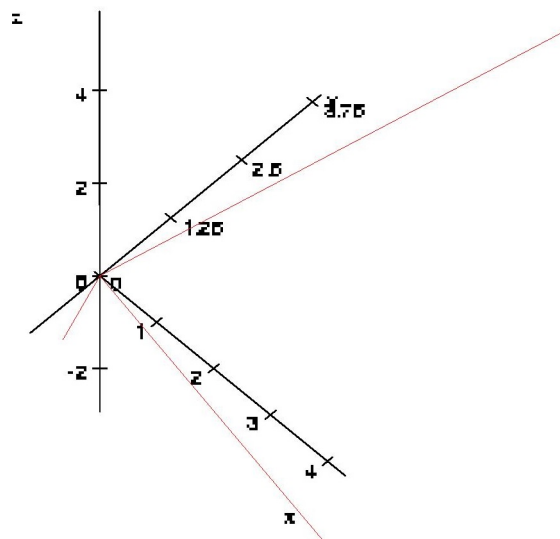
Find the eigenvalues and eigenfunctions of the matrix $\begin{pmatrix} 5 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 6 \end{pmatrix}$. Draw a 3D graph showing the eigenvectors, with each eigenvector having the magnitude of its corresponding eigenvalue. Each axis should have the same scale. **[15 Marks]**

SOLUTION 5

```
m:=matrix([[5,2,1],[2,3,2],[1,2,6]]);
[d, X, res] := numeric::eigenvectors(m):
XX:=matrix(3,3):
for i from 1 to 3 do
for j from 1 to 3 do
XX[i,j]:=X[i,j]*d[j]
end_for
end_for:
for i from 1 to 3 do
ll[i]:=plot::line([0,0,0],[XX[1,i],XX[2,i],XX[3,i]])
end_for:
plot(ll[1],ll[2],ll[3],Axes=Origin,Scaling=Constrained,
CameraPoint=[1000, -1000,2000]);
```

```
+--      +-
|  5, 2, 1  |
|           |
|  2, 3, 2  |
|           |
|  1, 2, 6  |
|           |
+--      +-

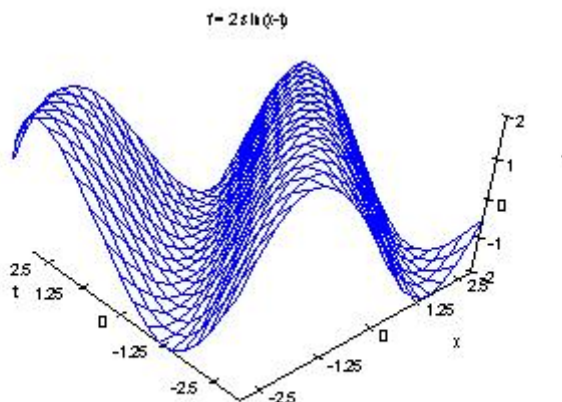
```



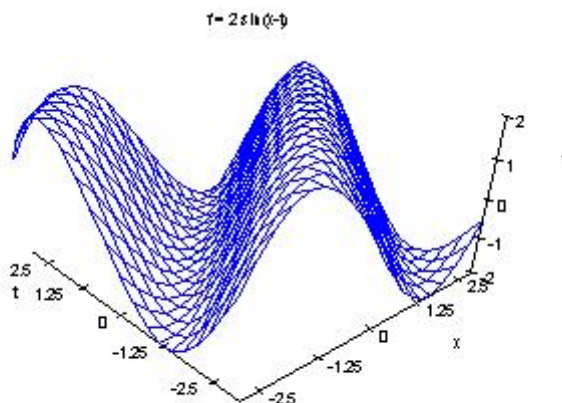
[15 Marks]

QUESTION 6

The following diagram is a plot of $f(x, t) = 2 \sin(x - t)$. Write MuPAD code that reproduces the diagram (the graphical object is uniformly coloured blue). **[15 Marks]**

**SOLUTION 6**

```
plot3d(CameraPoint=[-8,-9,7], Title="f = 2 sin(x-t)",
Labels=["x", "t", "f"], Axes=Corner, [Mode=Surface,
[x,t,2*sin(x-t)],x=[-11/12*PI,11/12*PI],
t=[-PI,PI],Color=[Flat,RGB::Blue],
Style=[WireFrame,Mesh]]);
```

**[15 Marks]****QUESTION 7**

Write \LaTeX code to reproduce the text on Pages 6–8. Use the packages *"amsmaths"* and *"amsmaths"*, the pages must be 126mm wide, 180mm high and the top margin is 0.4cm. Please note that manual numbering/reference of/to equation, bibliography, page, section or any other item will not be accepted. **[30 Marks]**

A Stability results on the Orientation of Red Blood Cells in Large Arteries

J. M .W. Munganga

1 Introduction

Mathematical modeling of blood flow has become a useful tool in supplementing experimental data and hence enhance that understanding of the hemostatic system. A model that could predict the effect of magnetic fields on the orientation of red blood cells could be of great value to the study of clot formation, the effect of foreign objects like pace makers, electrodes and artificial heart valves in the body. Munganga and Al [1, 2, 3] studied the thermodynamic stability, existence, stability and uniqueness of solutions in the absence of body forces.

A closure approximation (linear and quadratic), is used to approximate the tensor \mathcal{A} which features in equations (1), (2) and (5), by a function of the second order tensor \mathbf{A} .

2 Constitutive Equations for Particle Suspensions

In this section we construct constitutive equations for the orientation tensors as well as for the stress tensor.

2.1 The Constitutive equation for the orientation tensors

$$\begin{aligned} \frac{DA_{ij}}{Dt} + (W_{ij}A_{ik} - A_{jk}W_{ki}) - \lambda(D_{ik}A_{kj} + A_{ik}D_{kj} - 2A_{ijkl}D_{kl}) \\ - D_r(\delta_{ij} - nA_{ij}) = 0, \end{aligned} \quad (1)$$

($n = 2$ or 3)

or

$$\frac{D\mathbf{A}}{Dt} + (\mathbf{A}\mathbf{W} - \mathbf{W}\mathbf{A}) - \lambda(\mathbf{A}\mathbf{D} + \mathbf{D}\mathbf{A} - 2\mathbf{A}\mathbf{D}) - D_r(\mathbf{I} - n\mathbf{A}) = 0, \quad (2)$$

Equation (1) and (2) are known as the "evolution equation" for the orientation tensor \mathbf{A} , [1, 2, 3].

2.2 Constitutive Equation for the Stress

The total stress \mathbf{T} is a modification of the constitutive equation for incompressible Newtonian fluids, and is given by

$$\mathbf{T} = -p\mathbf{I} + 2\mu\mathbf{D} + \mathbf{E} \quad (3)$$

where p is the pressure, μ is the solvent viscosity, \mathbf{D} is the deformation tensor. The extra stress \mathbf{E} is found by solving for the stress field around a single massless particle.

Often the contribution of D_r to the stress is not significant, and this term is usually neglected. This will be the case henceforth. Thus, the stress will be expressed in the form;

$$\mathbf{T} = -p\mathbf{I} + 2\mu_I\mathbf{D} + \mathbf{S} \quad (4)$$

where

$$\mathbf{S} = 2\mu_I[N_p\mathbf{A}\mathbf{D} + N_s(\mathbf{A}\mathbf{D} + \mathbf{D}\mathbf{A})] \quad (5)$$

and

$$\mu_I = \mu(1 + hH), \quad N_p = \frac{hK}{1 + hH}, \quad N_s = \frac{hB}{1 + hH}, \quad (6)$$

N_p and N_s are positive constants known as the particle and shear number respectively.

The linear approximation is exact for random distribution of particles, for which

$$\mathbf{A} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \text{ in } \mathbb{R}^3 \quad \text{or} \quad \mathbf{A} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \text{ in } \mathbb{R}^2. \quad (7)$$

3 Conclusion

In this article blood has been modelled on pages 1– 3, as a suspension with particles which represented a rouleaux formation of erythrocytes adhering side by side forming a cylindrical body. Normal human blood was regarded as semi-diluted suspension, in which rouleaux (thereafter referred to as particles) have a low probability of making contact, though the motion of the rouleaux and the fluid are coupled. We assumed that the number of the particles per unit volume is uniform though the orientation of particles may not be.

References

- [1] Munganga J.M.W., Reddy B.D., Diatezwa K.J., Aspect of the thermodynamic stability of fibre suspension flows, *Journal of Non-Newtonian Fluid Mechanics*, **92** (2000) 135-150.
- [2] Munganga J.M.W., Reddy B.D., Local and Global Existence of solution to Equations for flows of fibre suspensions, *Mathematical Models and Methods in Applied Sciences*, **Vol. 12, No 8** (2002) 1177-1203.
- [3] Munganga J.M.W., Existence and Stability of Solutions to the Equations of Fibre Suspension Flows, Doctoral Thesis, University of Cape Town, Septembre 1999.

SOLUTION 7

```

\documentclass[12pt]{article}
\usepackage{amsfonts}
\usepackage{amsmath}
\setlength{\textheight}{180mm} \setlength{\textwidth}{126mm}
\setlength{\oddsidemargin}{0cm} \setlength{\evensidemargin}{0cm}
\setlength{\topmargin}{0.4cm}
\begin{document}
\title{A Stability results on the Orientation of Red Blood Cells in Large Arteries}
\author{J. M .W. Munganga}
\date{}
\maketitle
\section{Introduction}
Mathematical modeling of blood flow has become a useful tool in supplementing experimental data and hence enhance that understanding of the hemostatic system. A model that could predict the effect of magnetic fields on the orientation of red blood cells could be of great value to the study of clot formation, the effect of foreign objects like pace makers, electrodes and artificial heart valves in the body. Munganga and Al \cite{Munganga Reddy Diatezwa 2000, Munganga and Reddy 2002, Munganga 1999} studied the thermodynamic stability, existence, stability and uniqueness of solutions in the absence of body forces.
\newline
A closure approximation (linear and quadratic), is used to approximate the tensor  $\mathcal{A}$  which features in equations (\ref{26}), (\ref{27}) and (\ref{34}), by a function of the second order tensor  $\mathbf{A}$ .
\section{Constitutive Equations for Particle Suspensions}
In this section we construct constitutive equations for the orientation tensor as well as for the stress tensor.
\subsection{The Constitutive equation for the orientation tensors}
\begin{eqnarray}
\frac{DA_{ij}}{Dt} + (W_{ij}A_{ik} - A_{jk}W_{kj}) - \lambda (D_{ik}A_{kj} + A_{ik}D_{kj} - 2\mathcal{A}_{ijkl}D_{kl}) \nonumber \\
& \& - D_r(\delta_{ij} - nA_{ij}) \nonumber \\
& \& = 0, \quad \text{\label{26}} \nonumber \\
& (n=2 \text{ or } 3) \& \quad \text{\nonumber}
\end{eqnarray}
or
\begin{equation}
\frac{D\mathbf{A}}{Dt} + \left( \mathbf{AW} - \mathbf{WA} \right) - \lambda \left( \mathbf{AD} + \mathbf{DA} - 2\mathcal{A}\mathbf{D} \right) - D_r \left( \mathbf{I} - n\mathbf{A} \right) = 0, \quad \text{\label{27}}
\end{equation}
Equation (\ref{26}) and (\ref{27}) are know as the "evolution equation" for the orientation tensor  $\mathbf{A}$ , \cite{Munganga Reddy Diatezwa 2000, Munganga and Reddy 2002, Munganga 1999}.

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\subsection{Constitutive Equation for the Stress}

The total stress \mathbf{T} is a modification of the constitutive equation for incompressible Newtonian fluids, and is given by

$$\mathbf{T} = -p\mathbf{I} + 2\mu \mathbf{D} + \mathbf{E} \quad \text{\label{30}}$$

where p is the pressure, μ is the solvent viscosity, \mathbf{D} is the deformation tensor. The extra stress \mathbf{E} is found by solving for the stress field around a single massless particle. \newline

Often the contribution

of \mathbf{D}_r to the stress is not significant, and this term is usually neglected. This will be the case henceforth. Thus, the stress will be expressed in the form;

$$\mathbf{T} = -p\mathbf{I} + 2\mu \mathbf{D} + \mathbf{S} \quad \text{\label{33}}$$

where

$$\mathbf{S} = 2\mu \mathbf{I} \left[N_p \mathcal{A} \mathbf{D} + N_s \left(\mathbf{A} + \frac{d\mathbf{A}}{dt} \right) \right] \quad \text{\label{34}}$$

and

$$\mu \mathbf{I} = \mu \left(1 + hH \right), \quad N_p = \frac{hK}{1 + hH}, \quad N_s = \frac{hB}{1 + hH}, \quad \text{\label{34'}}$$

N_p and N_s are positive constants known as the particle and shear number respectively.

The linear approximation is exact for random distribution of particles, for which

$$\mathbf{A} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

\text{\label{35}} \mathbb{R}^3 \text{\label{36}} or \mathbf{A} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}

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\end{bmatrix}%
\text{in\ } \mathbb{R}^2 \text{.}
\end{equation}
\section{Conclusion}

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In this article blood has been modelled as a suspension with particles which represented a rouleaux formation of erythrocytes adhering side by side forming a cylindrical body. Normal human blood was regarded as semi-diluted suspension, in which rouleaux (thereafter referred to as particles) have a low probability of making contact, though the motion of the rouleaux and the fluid are coupled. We assumed that the number of the particles per unit volume is uniform though the orientation of particles may not be.

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\begin{thebibliography}{99}
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\bibitem{Munganga Reddy Diatezwa 2000} Munganga J.M.W., Reddy B.D., Diatezwa K.J., Aspect of the thermodynamic stability of fibre suspension flows, \emph{Journal of Non-Newtonian Fluid Mechanics}, \textbf{92} (2000) 135-150.
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\bibitem{Munganga and Reddy 2002} Munganga J.M.W., Reddy B.D., Local and Global Existence of solution to Equations for flows of fibre suspensions, \emph{Mathematical Models and Methods in Applied Sciences}, \textbf{Vol. 12, No 8} (2002) 1177-1203.
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\bibitem{Munganga 1999} Munganga J.M.W., Existence and Stability of Solutions to the Equations of Fibre Suspension Flows, Doctoral Thesis, University of Cape Town, Septembre 1999.
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\end{thebibliography}
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[30 Marks]