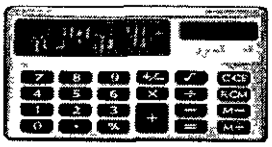
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM3711

May/June 2014

NUMERICAL METHODS II

Duration 2 Hours

100 Marks

EXAMINERS
FIRST DR GM MOREMEDI
SECOND DR EM RAPOO

Use of a non-programmable pocket calculator is permissible

Closed book examination

This paper consists of 5 pages

ANSWER ALL THE QUESTIONS.

QUESTION 1

(a) Use the modified Euler method with two corrections per step and $\Delta t = 1$ to solve the equation

$$\frac{dy}{dt} = ty^2, \quad y(1) = 1$$

at $t = 2$ (5)

(b) Use the second-order Taylor method with $\Delta x = 0.1$ to solve the differential equation

$$\frac{dy}{dx} = y, \quad y(0) = 1$$

at $x = 0.1$ and $x = 0.2$ (6)

(c) Explain briefly the difference between the local and the global error of a numerical method for solving a differential equation (2)

(d) Give the orders of the local as well as the global errors of the following methods

(i) The Euler method

(ii) The modified Euler method

(iii) Second order Taylor method

(iv) 6th order Taylor method (4)

[17]

[TURN OVER]

QUESTION 2

(a) The second-order Runge-Kutta method to solve the equation

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

at the equidistant points x_1, x_2, is given by the equations

$$\begin{aligned} y_{n+1} &= y_n + ak_1 + bk_2, \\ k_1 &= hf(x_n, y_n), \\ k_2 &= hf(x_n + \alpha h,\ y_n + \beta k_1), \end{aligned}$$

where $h = x_{i+1} - x_i$

(i) The four parameters a, b, α, β should be chosen such that $a + b = 1$, $\alpha b = 1/2$, $\beta b = 1/2$ Describe briefly how these conditions are arrived at (3)

(ii) Apply the second-order Runge-Kutta method with $a = 2/3$, $b = 1/3$, $\alpha = \beta = 3/2$ and $\Delta t = 1$ to find $y(1)$ and $y(2)$, if $y(t)$, $t \geq 0$ satisfies the differential equation

$$y' = y + t, \quad y(0) = 2 \tag{6}$$

(b) Briefly compare the fifth-order Runge-Kutta method and the Runge-Kutta-Fehlberg method by stating the main advantages and disadvantages of each method (4)

(c) The iteration formulas of many multistep methods are derived by integrating suitable interpolating and extrapolating polynomials Explain briefly how this is done to obtain the Adams-Moulton method Do not give formulas, but explain at each stage which points polynomials are fitted through, and over which intervals integration is done (4)

[17]

QUESTION 3

(a) Set up the system of equations to solve the boundary value problem

$$\begin{aligned} y'' + y' + y &= x, \quad 0 \leq x \leq 3 \\ y(0) &= 2, \quad y'(3) = 5 \end{aligned}$$

by the method of finite differences, $\Delta x = 1$ (Do NOT solve the equations) (8)

(b) Explain briefly how you would use the 'shooting method" to solve a problem of the type

$$y''' = f(y'', y', y, x), \quad 0 \le x \le 10$$
$$y(0) = A, \quad y(10) = B, \quad y'(0) = C$$

where f is a function and A, B and C are constants (Do NOT solve the problem) (9)

[17]

QUESTION 4

(a) Consider the following characteristic value problem

$$y'' - 2y' + wy = 0, \quad 0 \le x \le 4$$
$$y(0) = y(4) = 0$$

Taking $\Delta x = 1$, use central differences to construct an eigenvalue problem for determining the values of w for which the differential equation has non-zero solutions Identify the matrix, the eigenvalues and the eigenvectors of the problem Do NOT find the eigenvectors and eigenvalues, and do NOT solve the differential equation (8)

(b) Explain how the power method can be used to calculate

(i) the eigenvalue of largest magnitude,

(ii) the eigenvalue of smallest magnitude,

(iii) the eigenvalue closest to a given value s,

of a given non-singular matrix A (7)

[15]

QUESTION 5

(a) (i) Write down the finite difference equation corresponding to the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + xy = 0$$

at the grid point (x_i, y_j) of a rectangular mesh with $\Delta x = \Delta y = h$ Denote $u(x_i, y_j)$ by $u_{i,j}$ (4)

(ii) Re-write the difference equation in (i) into the iteration formula appropriate for Liebmann's method Use the superscripts u^k, u^{k+1} to show how new values are computed from previous ones (2)

(iii) Derive from the difference equation in (ii) the iteration formula used in the S O R (successive overrelaxation) method, with the overrelaxation factor denoted by ω (2)

(iv) Re-write the difference equation in (i) into the two iteration formulas of the A D I (alternating-direction-implicit) method Use the superscripts u^k, u^{k+1} and u^{k+2} to indicate the order of calculations Indicate also which equation corresponds to column wise and which to rowwise traverse, assuming that x-axis is horizontal and y-axis is vertical (4)

(b) Laplace's equation

$$\nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is to be solved in the domain $0 \le x \le 2$, $0 \le y \le 2$, subject to the boundary conditions

$$\begin{aligned} x &= 0 \quad u = 1 \\ x &= 2 \quad \frac{\partial u}{\partial x} = 2 \\ y &= 0 \quad u = 0 \\ y &= 2 \quad u = 1 \end{aligned}$$

(i) Using $\Delta x = \Delta y = 1$, sketch the domain and grid points Give the number of unknown values u_i to be solved, and indicate these u_i in the sketch (3)

(ii) Set up the system of equations to solve the partial-differential equation by the method of finite differences (Do NOT solve the equations) (4)

[19]

QUESTION 6

(a) Given the truncated power series

$$p(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5}$$

and the Chebyshev polynomial

$$T_5(x) = 16x^5 - 20x^3 + 5x,$$

(i) What is the purpose of economizing a power series? (1)

(ii) Economize the power series $p(x)$ (3)

[TURN OVER]

(b) We want to find a Padé approximation $R_6(x)$, with denominator of degree 3, to the function

$$e^{x^2} \approx 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$$

(i) Why is it often preferred to approximate a function by a rational function rather than a polynomial? (1)

(ii) Write down R_6 with coefficients a_i and b_i (4)

(iii) Set up the equations to solve the coefficients (Do NOT solve the equations!) (6)

[15]

First Examiner:	Dr GM Moremedi
Second Examiner:	Dr EM Rapoo
External Examiner:	Dr S Neossi

TOTAL MARKS: [100]