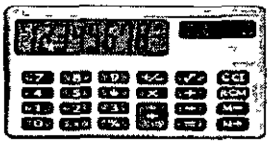
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

APM3711
SECOND PAPER

May/June 2018

Numerical Methods II

Duration 2 Hours

100 Marks

EXAMINERS
FIRST DR GM MOREMEDI
SECOND DR EM RAPOO

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

[TURN OVER]

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This paper consists of 5 pages

INSTRUCTIONS

Answer all the questions

[TURN OVER]

QUESTION 1

(a) Use the second-order Taylor method to find $y\,(1\ 1)$ when y is the solution to the differential equation

$$\frac{dy}{dx} = xy + 1, \quad y\,(1) = 2$$

(6)

(b) Euler's method is used to solve a first-order differential equation

$$\frac{dy}{dx} = f\,(y, x)$$

from $x = 0$ to $x = 4$

(i) If the step length is $h = \Delta x = 0\ 1$, we get an error of 0 12 for $y\,(4)$ What step length should we use to reduce the error for $y\,(4)$ to 0 01?

(ii) How would this change in the step size influence the local error? (8)

(c) Explain briefly **why** the modified Euler method is more accurate than the Euler method (2)

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QUESTION 2

(a) (i) Why is a Runge-Kutta method usually preferred to a Taylor series method of the same order?

(ii) Is there an upper limit to the order of a Runge-Kutta method? Justify your answer! (4)

(b) Explain briefly how the accuracy of a Runge-Kutta method can be determined by

(i) halving the step size at the end of each interval,

(ii) using two Runge-Kutta methods with different orders

Which method is more efficient? Why? (8)

(c) The predictor and corrector formulas of the Adam-Moulton method are

$$y_{n+1} = y_n + \tfrac{h}{24}\left(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}\right) + \tfrac{251}{720}h^5y^5\,(\zeta_1)$$
$$y_{n+1} = y_n + \tfrac{h}{24}\left(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right) - \tfrac{19}{720}h^5y^{(5)}\,(\zeta_2)$$

Apply the Adams-Moulton method to calculate the approximate value of $y\,(0\ 8)$ and $y\,(1\ 0)$ from the differential equation

$$y' = t + y$$

and the starting values

t	y(t)
0 0	0 95
0 2	0 68
0 4	0 55
0 6	0 30

Use 3 decimal digits with rounding at each step (8)

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QUESTION 3

(a) Consider the boundary value problem

$$\begin{aligned} y" = \quad & y + x \\ & y(1) = 1, \; y(3) = 2 \end{aligned}$$

Solve this system over the interval $1 \leq x \leq 3$ by using the shooting method Use the Euler method with $\Delta x = 1$ (8)

(b) Compare briefly the shooting method and the method of finite differences in solving boundary value problems, by stating the main advatanges and the main disadvantages of each method (4)

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QUESTION 4

(a) Explain why the numerical solution of a characteristic value problem of the type

$$u'' + f(x)\,u' + k^2 u = 0, \quad u(0) = 0, \quad u(1) = 0$$

by the method of finite differences, leads to an eigenvalue problem Here, $f(x)$ is an arbitrary function of x (5)

(b) The matrix

$$A = \begin{pmatrix} -2 & 2 \\ 1 & 1 \end{pmatrix}$$

has an eigenvalue near -2 5 and another one near $+1$ 5

(i) Use the power method to find the eigenvalue near -2 5 Do three iterations, starting with the vector $(1, 0)^T$

[TURN OVER]

(ii) Explain how you can use suitable shifting with the power method to find the eigenvalue near 1.5 with fewer iterations, **without** having to find the inverse of any matrix. (9)

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QUESTION 5

(a) (i) Write down the finite difference equation corresponding to the three-dimensional partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + u + xyz = 0$$

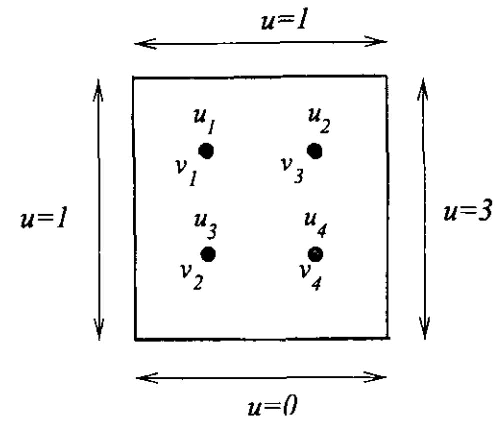
at the grid point (x_i, y_j, z_k) of a rectangular mesh with $\Delta x = \Delta y = \Delta z = h$. Denote $u(x_i, y_j, z_k)$ by $u_{i,j,k}$.

(ii) Re-write the difference equation in (i) into an iteration formula appropriate for Liebmann's method. Use the superscripts u^k, u^{k+1} to show how new values are computed from previous ones. (7)

(b) The equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is to be solved in a rectangular region. The grid (with $\Delta x = \Delta y = 1$), the boundary values and the numbering of the node points are indicated in the sketch below.



(i) Set up the two sets of equations for solving the values of u at the four node points by the ADI method. Use the notation of the sketch, with function values denoted by u_i in rowwise traverse and by v_i in columnwise traverse.

(ii) Explain how you would proceed to solve the problem by using the ADI method. (14)

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QUESTION 6

(a) The function e^x is to be approximated by a fifth-order polynomial over the interval $[-1, +1]$ Why is a Chebyshev series a better choice than a Taylor (or MacLaurin) expansion? (4)

(b) Given the power series

$$f(x) = 1 - x - 2x^3 - 4x^4$$

and the Chebyshev polynomials

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1, \end{aligned}$$

economize the power series $f(x)$ twice (6)

(c) Find the Padé approximation $R_3(x)$, with numerator of degree 2 and denominator of degree 1, to the function $f(x) = x^2 + x^3$ (7)

[17]

TOTAL: [100]

First Examiner.	Dr GM Moremedi
Second Examiner	Dr EM Rapoo
External Examiner	Dr R Narain (UKZN)