UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM3713

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SPECIAL RELATIVITY AND RIEMANNIAN GEOMETRY

Duration 2 Hours

100 Marks

EXAMINERS

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SECOND PROF DP SMITS

Use of a non-programmable pocket calculator is permissible

Closed book examination

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This paper consists of 6 pages

Some potentially useful formulae can be found on page 5 and 6.

QUESTION 1

A μ-meson with an average lifetime of 2×10^{-6} s is created in the upper atmosphere at an altitude of 6000 m. When it is created it has a velocity of $0.998c$ in a direction towards the Earth.

a) What is the average distance that it will travel before decaying, as determined by an observer on Earth? (8)

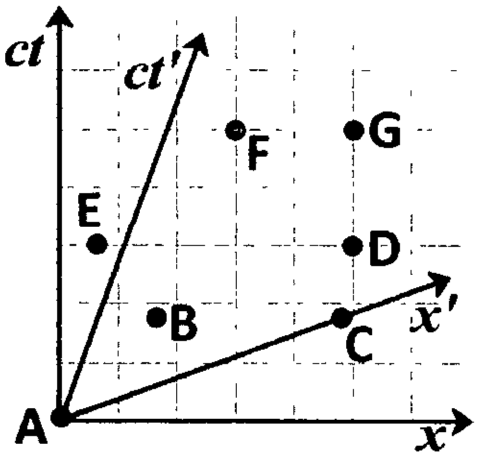
b) Consider an observer at rest with respect to the μ-meson. What is the distance he/she measures from the point of creation of the μ-meson to the Earth? (8)

c) Comment on the physical significance of these results. (4)

[20]

QUESTION 2

The Minkowski diagram below shows two reference frames, S and S', that are in the standard configuration. Events A - G are also indicated.



a) Which event occurs at the same time as Event B in the S' frame? (1)

b) Explain why it is not possible for any particle to be at Event C and, at some later time, at Event B. (2)

c) Event D corresponds to the emission of a light pulse. Which of the indicated events can possibly correspond to the detection of the light pulse? Explain your answer. (3)

[TURN OVER]

d) An observer in S' measures the spacetime coordinates for Event C to be $(x', t') = (45\,\text{km},\ 0\,\text{s})$ and the relative speed of S' to be $V = 0\ 36c$ What would the spacetime coordinates of Event C be as measured in S? (6)

[12]

QUESTION 3

At the Stanford Linear Accelerator, electrons are accelerated to energies of 50 GeV(where 1 GeV = 10^9 eV) Take the electrons' rest mass as 0 511 MeV/c2

a) If this energy were classical kinetic energy, what would be the electrons' speed? (6)

b) Is the above value in (a) of the speed physical? and if not, why? (2)

c) What is the electrons' actual (relativistic) speed? (10)

d) Is the above value in (c) of the speed physical? and if not, why? (2)

[20]

QUESTION 4

Consider a two dimensional space-time with the line element

$$dl^2 = \frac{1}{t^2}\left(c^2dt^2 - dx^2\right),$$

where $t > 0$ and $-\infty < x < \infty$

a) Setting $x^1 = t$ and $x^2 = x$ What is the metric tensor and the dual metric tensor? Explain how you determine these (4)

b) The only non-zero Christoffel coefficients for this surface are $\Gamma^2{}_{12}$, $\Gamma^2{}_{21}$, $\Gamma^1{}_{11}$ and $\Gamma^1{}_{22}$ Calculate these coefficients (10)

c) It is given that the Riemann curvature tensor of the surface is

$$R^1{}_{212} = \frac{1}{c^2t^2}$$

The tensor has the following symmetries

$$R^1_{\ 212} = -R^1_{\ 221} = -R^2_{\ 112} = R^2_{\ 121}$$

and all other components are equal to zero Determine the components of the Ricci tensor for the surface (5)

d) Calculate the Ricci curvature scalar R (4)

e) Calculate the Gaussian curvature of the surface (5)

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QUESTION 5

a) A paraboloid can be parametrized as

$$\begin{aligned} x(u,\,v) &= au\cos v, \\ y(u,\,v) &= au\sin v, \\ z(u,\,v) &= u^2 \end{aligned}$$

Find the line element for the surface (10)

b) Show that $R^i_{\ ijk} = 0$ Explain your steps carefully Hint Use the symmetries of the Riemann tensor given in Question 4(c) (4)

c) The transformation equations between two two-dimensional coordinate systems are given by

$$\begin{aligned} x'^1 &= x^1\cos\theta + x^2\sin\theta \\ x'^2 &= -x^1\sin\theta + x^2\cos\theta \end{aligned}$$

where θ is some angle

Write down the transformation equations for the components of a first order covariant tensor $[A_{ij}]$ (6)

[20]

Total: [100]

[TURN OVER]

FORMULA SHEET

Speed of light in a vacuum: $c = 3 \times 10^8\,\mathrm{ms}^{-1}$
Mass of electron: $m_e = 0.511\,\mathrm{MeV}/c^2 = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton: $m_p = 938.3\,\mathrm{MeV}/c^2 = 1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of pion: $m_\pi = 139.6\,\mathrm{MeV}/c^2 = 2.49 \times 10^{-28}\,\mathrm{kg}$
Mass of muon: $m_\mu = 105.7\,\mathrm{MeV}/c^2 = 1.89 \times 10^{-28}\,\mathrm{kg}$
$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{MeV} = 10^6\,\mathrm{eV}$

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} v'_x &= \frac{v_x - V}{1 - v_x V/c^2} \\ v'_y &= \frac{v_y}{\gamma\,(1 - v_x V/c^2)} \\ v'_z &= \frac{v_z}{\gamma\,(1 - v_x V/c^2)} \end{aligned}$$

$$\begin{aligned} t' &= \gamma\left(t - \left(V/c^2\right)x\right) \\ x' &= \gamma\,(x - Vt) \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} E' &= \gamma\,(E - Vp_x) \\ p'_x &= \gamma\left(p_x - \frac{VE}{c^2}\right) \\ p'_y &= p_y \\ p'_z &= p_z \\ E_K &= (\gamma - 1)mc^2 \end{aligned}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$f_{rec} = f_{em}\sqrt{\frac{c - V}{c + V}}$$

$$\lambda_{em} = \lambda_{rec}\sqrt{\frac{c - V}{c + V}}$$

$$\begin{aligned} F'^0 &= \gamma\left(F^0 - VF^1/c\right) \\ F'^1 &= \gamma\left(F^1 - VF^0/c\right) \\ F'^2 &= F^2 \\ F'^3 &= F^3 \end{aligned}$$

[TURN OVER]

$$L_C\,(\mathrm{P},\ \mathrm{Q}) = \int_P^Q \left[\left(\frac{dx}{du} \right)^2 + \left(\frac{dy}{du} \right)^2 \right]^{1/2} du$$

$$\Gamma^i{}_{jk} = \frac{1}{2} \sum_m g^{im} \left(\frac{\partial g_{mk}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m} \right)$$

$$\frac{d^2 x^i}{d\lambda^2} + \sum_{j,k} \Gamma^i{}_{jk} \frac{dx^j}{d\lambda} \frac{dx^k}{d\lambda} = 0$$

$$k = \frac{|xy - yx|}{(x^2 + y^2)^{3/2}}$$

$$R^l{}_{ijk} = \frac{\partial \Gamma^l{}_{ik}}{\partial x^j} - \frac{\partial \Gamma^l{}_{ij}}{\partial x^k} + \sum_m \Gamma^m_{ik} \Gamma^l{}_{mj} - \sum_m \Gamma^m_{ij} \Gamma^l{}_{mk}$$

$$\begin{aligned} \nabla_\beta v_\alpha &= \frac{\partial v_\alpha}{\partial x^\beta} - \sum_\lambda \Gamma^\lambda_{\alpha\beta} v_\lambda \\ \nabla_\beta v^\alpha &= \frac{\partial v^\alpha}{\partial x^\beta} + \sum_\lambda \Gamma^\alpha_{\lambda\beta} v^\lambda \end{aligned}$$

$$\begin{aligned} A'^\alpha &= \sum_\beta \frac{\partial x'^\alpha}{\partial x^\beta} A^\beta \\ A'_\alpha &= \sum_\beta \frac{\partial x^\beta}{\partial x'^\alpha} A_\beta \end{aligned}$$

$$K = \frac{R_{1212}}{g}$$

$$T^{\mu\nu} = \left(\rho + p/c^2 \right) U^\mu U^\nu - p g^{\mu\nu}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu}$$

Examiners	Names
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