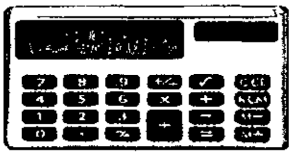
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM3713

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SPECIAL RELATIVITY AND RIEMANNIAN GEOMETRY

Duration 2 Hours | 100 Marks

EXAMINERS

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SECOND PROF DP SMITS

Use of a non-programmable pocket calculator is permissible

Closed book examination

This paper consists of 6 pages.

Some potentially useful formulae can be found on page 5 and 6.

[TURN OVER]

QUESTION 1

Two astronauts, Alice and Bob, leave the Earth and travel to a distant planet 12 lightyears away, as measured from Earth. Assume that the planet and Earth are at rest with respect to each other. The astronauts depart at the same time in different spaceships. Alice travels at a speed of $0.9c$, and Bob travels at $0.5c$. (Hint: A lightyear is the distance traveled by light in one year, which is just c multiplied by one year, or 9.46×10^{12} km. In many problems it is simpler to write it as $1c\cdot$year, since c often cancels out.)

a) What is the distance of the journey according to Alice? (4)

b) What is the duration of the journey according to Bob? (6)

c) What is the speed of Alice's spaceship, as measured by Bob? (4)

d) Bob sends back information to Earth at a frequency of 400 Hz. At what frequency should the people on Earth tune their receivers to receive Bob's message? (3)

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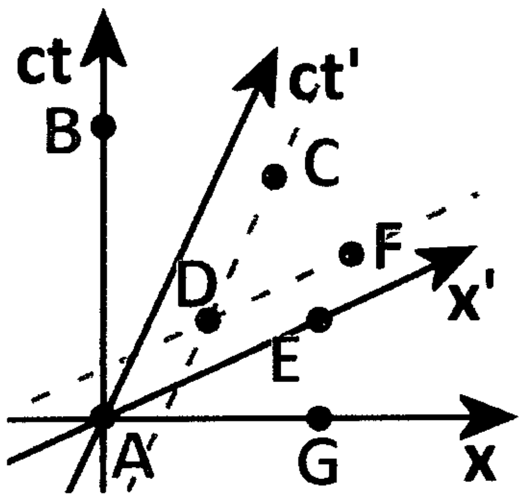
QUESTION 2

a) The Minkowski diagram below shows two reference frames, S and S' that are in the standard configuration. The following questions refer to events A to G as indicated on the diagram. (5)



[TURN OVER]

i. Which event occurs at the same time as Event G according to an observer in the S frame?

ii. Which event occurs at the same location as Event D according to an observer in the S' frame?

iii. To which event(s) would observers in the S and S' frames assign the same spacetime coordinates?

iv. Which event(s) could possibly have caused Event F? Explain your answer.

b) Using the Lorentz transformation equations for intervals, show that the spacetime separation is invariant. That is, show that $(\Delta s)^2 = (\Delta s')^2$ (8)

[13]

QUESTION 3

An electron e^- with kinetic energy 0.45 MeV makes a head-on collision with a positron e^+ that is at rest. (A positron is an antimatter particle that has the same mass as an electron, but opposite charge.) In the collision the two particles annihilate each other and are replaced by two photons γ of equal energy. The reaction can be written as

$$e^- + e^+ \to 2\gamma$$

a) Determine the speed of the electron just before it collided with the positron. (6)

b) What is the momentum of the electron? (3)

c) What is the total energy of the electron? (3)

d) What is the total energy of the positron? (2)

e) What is the energy of each photon? (5)

f) What is the momentum each photon? (2)

g) What is the speed of each photon? Give a reason for your answer. (2)

h) Take the direction of velocity of the electron to be in the positive x-direction. Write down the four-momentum $[P^\mu]$ for the electron. (2)

i) Assume Minkowski spacetime. What is the value of the covariant four-momentum $[P_\mu]$? (3)

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[TURN OVER]

QUESTION 4

The circular cone can be parametrized as

$$\begin{aligned} x(u, v) &= au\cos v \\ y(u, v) &= au\sin v \\ z(u, v) &= u \end{aligned}$$

where a is a constant

a) Find the line element for the surface (9)

b) Let $x^1 = u$ and $x^2 = v$ What is the metric tensor and the dual metric tensor? Explain how you determine these (5)

c) The only non-zero Christoffel coefficients for this surface are Γ^1_{22}, Γ^2_{21} and Γ^2_{21} Calculate these coefficients (10)

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QUESTION 5

a) Suppose that $R_{iklm} = a\left(g_{il}g_{km} - g_{im}g_{kl}\right)$ on some four dimensional Riemannian space and a is a constant Show that for the curvature scalar we have $R = -12a$ (12)

b) The line element of the Euclidean plane in polar coordinates is

$$dl^2 = dr^2 + r^2 d\theta^2$$

The contravariant vector components of a second order tensor $[A^i]$, $i = 1, 2$ is given in polar coordinates as

$$A^1 = r \qquad A^2 = \cos\theta$$

Determine the components of $[A_i]$ if $x^1 = r$ and $x^2 = \theta$ (6)

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Total: [100]

[TURN OVER]

FORMULA SHEET

Speed of light in a vacuum: $c = 3 \times 10^8\,\mathrm{ms}^{-1}$
Mass of electron: $m_e = 0.511\,\mathrm{MeV}/c^2 = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton: $m_p = 938.3\,\mathrm{MeV}/c^2 = 1.67 \times 10^{-27}\,\mathrm{kg}$
$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{MeV} = 10^6\,\mathrm{eV}$

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} E' &= \gamma\,(E - Vp_x) \\ p'_x &= \gamma\left(p_x - \frac{VE}{c^2}\right) \\ p'_y &= p_y \\ p'_z &= p_z \end{aligned}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$\begin{aligned} t' &= \gamma\left(t - \left(V/c^2\right)x\right) \\ x' &= \gamma\,(x - Vt) \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} F'^0 &= \gamma\left(F^0 - VF^1/c\right) \\ F'^1 &= \gamma\left(F^1 - VF^0/c\right) \\ F'^2 &= F^2 \\ F'^3 &= F^3 \end{aligned}$$

$$f_{rec} = f_{em}\sqrt{\frac{c - V}{c + V}}$$

$$L_C\,(\mathrm{P},\,\mathrm{Q}) = \int_P^Q \left[\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2\right]^{1/2} du$$

$$\begin{aligned} v'_x &= \frac{v_x - V}{1 - v_xV/c^2} \\ v'_y &= \frac{v_y}{\gamma\,(1 - v_xV/c^2)} \\ v'_z &= \frac{v_z}{\gamma\,(1 - v_xV/c^2)} \end{aligned}$$

$$\Gamma^i{}_{jk} = \frac{1}{2}\sum_m g^{im}\left(\frac{\partial g_{mk}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m}\right)$$

$$\frac{d^2x^i}{d\lambda^2} + \sum_{j,k} \Gamma^i{}_{jk} \frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0$$

$$k = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$

$$A'^\alpha = \sum_\beta \frac{\partial x'^\alpha}{\partial x^\beta} A^\beta$$

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$$R^l{}_{ijk} = \frac{\partial \Gamma^l{}_{ik}}{\partial x^j} - \frac{\partial \Gamma^l{}_{ij}}{\partial x^k} + \sum_m \Gamma^m_{ik}\Gamma^l{}_{mj} - \sum_m \Gamma^m_{ij}\Gamma^l{}_{mk}$$

$$T^{\mu\nu} = \left(\rho + p/c^2\right) U^\mu U^\nu - p g^{\mu\nu}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

$$\nabla_\beta v_\alpha = \frac{\partial v_\alpha}{\partial x^\beta} - \sum_\lambda \Gamma^\lambda_{\alpha\beta} v_\lambda$$

$$\nabla_\beta v^\alpha = \frac{\partial v^\alpha}{\partial x^\beta} + \sum_\lambda \Gamma^\alpha_{\lambda\beta} v^\lambda$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu}$$

Examiners	Names
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