Tutorial Letter 201/2/2017

Special Relativity and Riemannian Geometry

APM3713

Semester 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains the solutions to Assignment 01.

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Memo for Assignment 1 S2 2017

Special relativity basics (§ 1.1 - 1.2)

Consequences of Lorentz transformations (§ 1.3)

Question 1: Lorentz transformation

Alice sees an explosion happening and measures the spacetime coordinates of the explosion to be $(t,\ x,\ y,\ z) = (1.5\,\text{ns},\ 2\,\text{m},\ 1\,\text{m},\ 0\,\text{m})$. Bob is riding past in a train at a constant speed of $V = 0.4c$ in the positive x-direction. Use the Lorentz transformation equations to determine what time Bob measures the explosion taking place. (Hint: $1\,\text{ns}$ (nanosecond) $= 10^{-9}$s).

- $1.5\,\text{ns}$
- $1.407 \times 10^{-9}\,\text{s}$
- $-1.17 \times 10^{-9}\,\text{s}$
- $1.5\,\text{s}$
- $\mathbf{-1.41}\,\text{ns}$*

The train is moving at $V = 0.4c$, so the Lorentz factor between Alice's frame (S) and Bob's frame (S') is

$$
\begin{aligned}
\gamma &= \frac{1}{\sqrt{1 - V^2/c^2}} \\
&= \frac{1}{\sqrt{1 - (0.4c)^2/c^2}} \\
&= \frac{1}{\sqrt{1 - 0.16}} \\
&= \frac{1}{\sqrt{0.84}} \\
&= 1.09
\end{aligned}
$$

The Lorentz transformation equation for the time coordinate is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Using this, we transform Alice's measurements into Bob's frame as follows

$$\begin{aligned}
t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\
&= 1.09\left(1.5\,\mathrm{ns} - \frac{(0.4c)\,(2\,\mathrm{m})}{c^2}\right) \\
&= 1.09\left(1.5\,\mathrm{ns} - \frac{(0.4)\,(2\,\mathrm{m})}{c}\right) \\
&= 1.09\left(1.5\,\mathrm{ns} - \frac{(0.4)\,(2\,\mathrm{m})}{3\times 10^8\mathrm{ms}^{-1}}\right) \\
&= 1.09\left(1.5\,\mathrm{ns} - 2.27\times 10^{-9}\mathrm{s}\right) \\
&= 1.09\,(1.5\,\mathrm{ns} - 2.27\,\mathrm{ns}) \\
&= 1.09\,(-1.17\,\mathrm{ns}) \\
&= -1.41\,\mathrm{ns}
\end{aligned}$$

The negative time coordinate just means that Bob sees the explosion happening at a time before the arbitrarily chosen zero time.

Question 2: Time dilation

A group of astronauts take on a mission to travel to a nearby planet. According to the people on Earth, the spaceship takes 100 years to reach its destination, but the astronauts on the spaceship only aged 30 years during the journey. How fast was the spaceship travelling, assuming that it was moving at a constant velocity?

- $0.83c$
- **0.95c***
- $3.48c$
- $0.91c$

- $1.04c$

The astronauts measure their own proper time. Then the time that the people on Earth measure for the journey (Δt_E) will be related to the time that the astronauts measure (Δt_A) by the time dilation formula so that

$$\begin{aligned}
\Delta t_E &= \gamma \Delta t_A \\
100\,\text{years} &= \gamma \times 30\,\text{years} \\
\gamma &= 3.33 \\
\frac{1}{\sqrt{1 - V^2/c^2}} &= 3.33 \\
1 - V^2/c^2 &= 0.09 \\
V^2/c^2 &= 0.91 \\
V &= 0.95c
\end{aligned}$$

Remember, nothing with mass can travel faster than the speed of light c. If you ever get an answer where a speed is greater than c, you made a mistake somewhere.

Question 3: Summation

The sum

$$B_\alpha = \sum_{\beta=0}^{3} \eta_{\beta\alpha} B^\beta$$

written out in full is

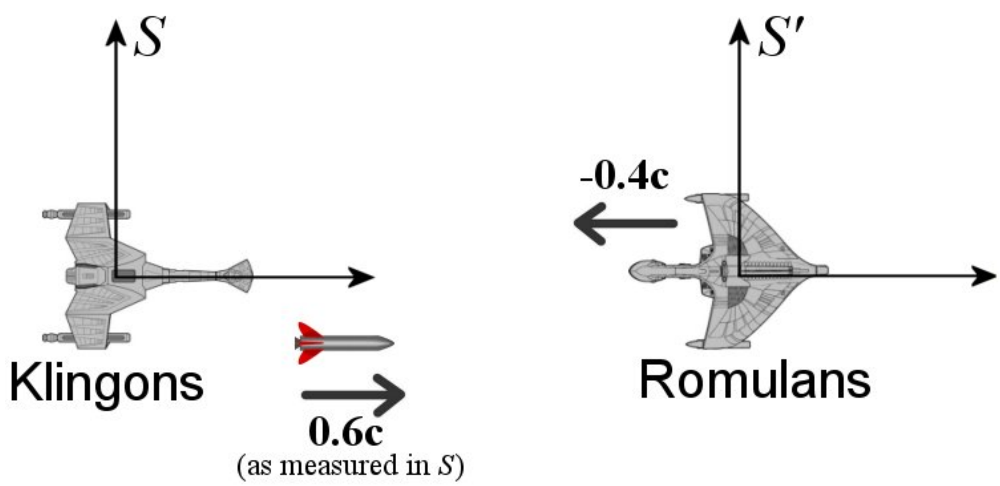
- $B_\alpha = 3\eta_{\beta\alpha} B^\beta$
- $B_\alpha = \eta_{\beta\alpha} B^\beta$
- $B_\alpha = \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta$
- $\mathbf{B}_\alpha = \eta_{\mathbf{0}\alpha} \mathbf{B^0} + \eta_{\mathbf{1}\alpha} \mathbf{B^1} + \eta_{\mathbf{2}\alpha} \mathbf{B^2} + \eta_{\mathbf{3}\alpha} \mathbf{B^3} *$
- $B_\alpha = \eta_{00} B^0 + \eta_{11} B^1 + \eta_{22} B^2 + \eta_{33} B^3$

Question 4: Velocity addition

Two alien races, the Klingons and the Romulans are having a battle in space. During the battle, two of the spaceships approach each other head on at a speed of $0.4c$. The Klingon ship shoots a torpedo in the direction of the Romulan ship. The Klingons measure the torpedo leaving their ship at a speed of $0.6c$. How fast does the Romulans measure the torpedo to be approaching them?

- c
- $0.26c$
- **0.81c***
- $1.32c$
- $0.73c$

When in doubt, draw a picture! It will help you to disentangle the problem and organise your thoughts. It can also show the marker that you have insight into the problem. Below is a rough sketch of the situation.



The two spaceships are approaching each other, moving with a speed $0.4c$ relative to each other. We can interpret this in a few ways mathematically, where the physical situation remains unchanged. For example, we can say that the Klingon ship is moving at $0.2c$ in the positive x-direction, while the Romulan ship is moving at $0.2c$ in the negative x-direction.

Or we can say that the Romulan ship is stationary, with the Klingon ship moving at $0.4c$ in the positive x-direction.

As indicated in the figure above, we will approach this problem by taking the Klingon ship as being stationary, and the Romulan ship travelling towards in at $0.4c$ in the *negative x-direction*. If we call the frame in which the Klingon ship is stationary S (if the figure above were a photograph, the "photographer" would also be stationary in this frame), and the frame in which the Romulan ship is stationary S'. Considering the standard configuration, we can now identify the speed between the frames as $V = -0.4c$.

Now the Klingon ship shoots a torpedo towards the Romulan ship (in the positive x-direction) and they (in the S frame) measures its speed as $v_x = 0.6c$. The question requires you to calculate the speed of the torpedo as measured by the Romulans (in S'). So we use the velocity transformation equation for the x-direction to transform v_x to v'_x.

$$\begin{aligned} v'_x &= \frac{v_x - V}{1 - v_x V/c^2} \\ &= \frac{(0.6c) - (-0.4c)}{1 - (0.6c)(-0.4c)/c^2} \\ &= \frac{c}{1 + 0.24} \\ &= 0.81c \end{aligned}$$

So the Romulans measure the speed of the torpedo to be $0.81c$ in the positive x-direction.