Tutorial Letter 202/2/2017

Special Relativity and Riemannian Geometry

APM3713

Semester 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains the solutions to Assignment 02.

BAR CODE

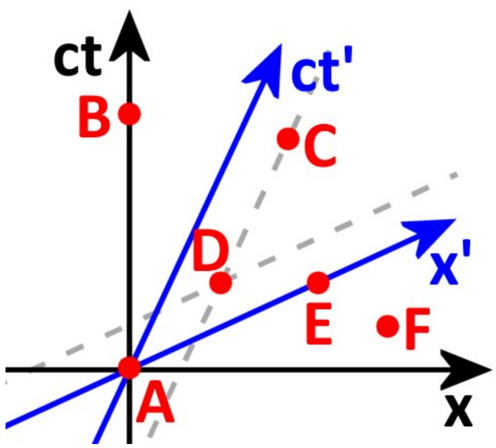
Memo for Assignment 2 S2 2017

Minkowski spacetime (§ 1.4)

Physical laws in relativity (§ 2.1 - 2.2.3)

Questions 1 - 3: Spacetime diagrams

Consider the spacetime diagram below to answer the following questions



1. In the S' frame, the following two events occur at the same position

- A and B
- A and E
- A and D
- **D and C***
- D and E

The S' frame is indicated by the blue coordinate axes. For two events to occur at the same position, they must occur at the same space coordinate. With the help of the dashed line that is parallel to the x' axis, it is clear that events C and D have the same x' coordinate.

2. To which point would observers in both the S and S' frame assign the same spacetime coordinates?

- **A***
- B
- C
- E
- None of the points

The only point to which observers in both S and S' will assign the same spacetime coordinates is the origin, event A where $(x,\, ct) = (x',\, ct') = (0,\, 0)$.

3. Which of the following statements are true *for all frames*?

- **Event A happens before event C***
- Events E and F are causally related
- Events A and B occur at the same position
- Events A and B occur at the same time
- Event C caused event F

Event A will happen at time $ct = ct' = 0$. It is not possible to draw a coordinate axes where Event C will be below the space axis. The spacetime diagram will have to look something like the figure below, which is not a valid Lorentz transformation. The axes of a Lorentz transformation will always be symmetric about the line $ct = x$ (shown as a dashed line in the figure).

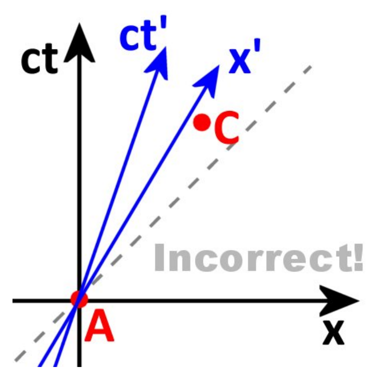
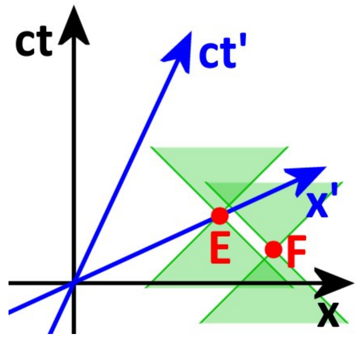


Figure 1: Not a correct Lorentz transformation!

Events E and F will be causally related if they can be related by a signal that travels slower than the speed of light, or equivalently, if the two events are in each others lightcones. From the figure below it is clear that they are not in each others lightcones (or, a signal connecting the two events will have to move faster than the speed of light), so one event cannot cause the other and they are not causally related.



Events A and B occur at the same position in S ($x = 0$), but this is not true for all frames. In S' (and all other inertial frames with a positive V relative to S, A will occur at the origin and B will occur at some negative value of x.

Events A and B do not occur at the same time in S, so they do not occur at the same time in all frames, since you can give a counter example.

Events C and F are causally related, since they can be connected with a signal that moves slower than c, or are in each others light cones. But, event F will occur before C in all frames, so there is no way in which C could cause F, although it would be possible for F to cause C.

Question 4: Mass energy

What is the mass energy of a proton with rest mass $m_p = 1.67 \times 10^{-27}$ kg?

- 5.01×10^{-19} joules
- **1.50×10^{-10} joules***
- 1.50×10^{44} joules
- 1.67×10^{-27} joules
- 9×10^{16} joules

The mass energy for the proton is

$$E = mc^2 = \left(1.67 \times 10^{-27}\,\text{kg}\right)\left(3 \times 10^8\right)^2 = 1.50 \times 10^{-10}\,\text{J}$$

Question 5: Momentum

How fast must a body be travelling so that its correct relativistic momentum is 1 % greater than the classical momentum?

- $0.04c$
- $0.42c$
- **0.14c***
- $1.41c$
- $1.35c$

For the relativistic momentum to be 1 % greater than the classical momentum, we must have

$$\begin{aligned} p_{rel} &= 1.01 p_{clas} \\ \gamma m u &= 1.01 m u \\ \gamma &= 1.01 \\ \sqrt{1 - V^2/c^2} &= 0.990 \\ 1 - V^2/c^2 &= 0.980 \\ V^2/c^2 &= 0.02 \\ V &= 0.141c \end{aligned}$$

Question 6: Collisions

A proton and neutron collide in an elastic collision. Before the collision, the neutron is stationary and the proton has momentum $\mathbf{p}_p = (0.4,\ -0.2,\ 0.8)$ MeV/c and the proton's momentum after the collision is $(-0.2,\ -0.5,\ 0.6)$ MeV/c. What is the neutron's momentum after the collision?

- $(0.2,\ 0.5,\ -0.6)$ MeV/c
- $(0.2,\ -0.7,\ 1.4)$ MeV/c
- **(0.6, 0.3, 0.2) MeV/c***
- $(0,\ 0,\ 0)$ MeV/c
- $(-0.6,\ 0.7,\ 0.2)$ MeV/c

The collision is elastic so that the kinetic energy is conserved. This is not really relevant to solve this problem, but be sure to know what it means. Total energy and momentum is always conserved. This means that the total momentum before the collision should be equal to the total momentum after the collision. A stationary object has no speed, and therefore no momentum. We can write this as

$$\begin{aligned} \mathbf{p}_n^{before} + \mathbf{p}_p^{before} &= \mathbf{p}_n^{after} + \mathbf{p}_p^{after} \\ \mathbf{p}_n^{after} &= \mathbf{p}_n^{before} + \mathbf{p}_p^{before} - \mathbf{p}_p^{after} \\ &= (0,\ 0,\ 0) + (0.4,\ -0.2,\ 0.8) - (-0.2,\ -0.5,\ 0.6) \\ &= (0.6,\ 0.3,\ 0.2)\ \text{MeV}/c \end{aligned}$$

Question 7: Kinetic energy

A proton (mass $m_p = 938.3\,\mathrm{MeV}/c^2$) is moving with speed $0.4c$ along the x-axis relative to the laboratory frame. What is its kinetic energy?

- $7.6 \times 10^{18}\,\mathrm{MeV}$
- $1023\,\mathrm{MeV}$
- $178.7\,\mathrm{MeV}$
- $273\,\mathrm{MeV}$
- **$85.39\,\mathrm{MeV}$***

Take the laboratory frame to be S and let the proton be stationary in the S' frame. The Lorentz factor for the two frames is

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-0.4^2}} \\ &= 1.091\end{aligned}$$

The kinetic energy is then

$$\begin{aligned}E_K &= (\gamma - 1)\,mc^2 \\ &= (1.091 - 1)\,(938.3\,\mathrm{MeV}) \\ &= 85.39\,\mathrm{MeV}\end{aligned}$$