Tutorial Letter 203/2/2017

Special Relativity and Riemannian Geometry

APM3713

Semester 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains the solutions to Assignment 03.

BAR CODE

Memo for Assignment 3 S2 2017

Four vectors and tensors (§ 2.2.4 - 2.3.5 (excluding 2.3.1 - 2.3.4))

Question 1: Transformation of energy

An electron (mass $m_e = 0.511\,\mathrm{MeV/c^2}$) is moving along the x-axis of an inertial reference frame S with speed $v = 0.8c$, momentum $0.682\,\mathrm{MeV/c}$ and total energy $0.852\,\mathrm{MeV}$. What is its total energy in an inertial frame S' that is moving in the standard configuration with speed $0.6c$ relative to S?

- $0.852\,\mathrm{MeV}$
- $0.738\,\mathrm{MeV}$
- **$0.554\,\mathrm{MeV}$***
- $0.511\,\mathrm{MeV}$
- $0.443\,\mathrm{MeV}$

The Lorentz factor for the two frames is

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\\ &= \frac{1}{\sqrt{1-0.6^2}}\\ &= \frac{5}{4}\end{aligned}$$

Using the transformation equation for the total energy in S' gives

$$\begin{aligned}E' &= \gamma\left(E - Vp_x\right)\\ &= \frac{5}{4}\left(0.852\,\mathrm{MeV} - (0.6c)\,(0.682\,\mathrm{MeV/c})\right)\\ &= \frac{5}{4}\left(0.852\,\mathrm{MeV} - 0.409\,\mathrm{MeV}\right)\\ &= 0.554\,\mathrm{MeV}\end{aligned}$$

Question 2: Transformation of momentum

For the electron described in the previous question, what is the measured value of the momentum in the S' frame?

- $0.682\,\mathrm{MeV}/c$
- $0.593\,\mathrm{MeV}/c$
- $0.285\,\mathrm{MeV}/c$
- **$0.214\,\mathrm{MeV/c}$*$**
- $0.046\,\mathrm{MeV}/c$

Since the electron is moving in the x-direction, p_x is the only non-zero component of the momentum vector. The appropriate transformation equation is

$$\begin{aligned} p'_x &= \gamma\left(p_x - VE/c^2\right) \\ &= \frac{5}{4}\left[0.682\,\mathrm{MeV}/c - (0.6c)\,(0.852\,\mathrm{MeV})\,/c^2\right] \\ &= \frac{5}{4}\left[0.682\,\mathrm{MeV}/c - 0.511\,\mathrm{MeV}/c^2\right] \\ &= 0.214\,\mathrm{MeV}/c \end{aligned}$$

You can also use the energy-momentum relation in the S' frame:

$$\begin{aligned} E'^2 &= p'^2c^2 + m^2c^4 \\ (0.554\,\mathrm{MeV})^2 &= p^2c^2 + \left(0.511\,\mathrm{MeV/c^2}\right)^2 c^4 \\ 0.307\ (\mathrm{MeV})^2 &= p^2c^2 + 0.261\ (\mathrm{MeV})^2 \\ p^2c^2 &= 0.046\ (\mathrm{MeV})^2 \\ p &= 0.214\,\mathrm{MeV}/c \end{aligned}$$

Question 3: Kinetic energy

A proton (mass $m_p = 938.3\,\mathrm{MeV}/c^2$) is moving with speed $0.4c$ along the x-axis relative to the laboratory frame. What is its kinetic energy?

- $7.6 \times 10^{18}\,\mathrm{MeV}$
- $1023\,\mathrm{MeV}$
- $178.7\,\mathrm{MeV}$
- $273\,\mathrm{MeV}$
- **$84.45\,\mathrm{MeV}$***

Take the laboratory frame to be S and let the proton be stationary in the S' frame. The Lorentz factor for the two frames is

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-0.4^2}} \\ &= 1.09\end{aligned}$$

The kinetic energy is then

$$\begin{aligned}E_K &= (\gamma - 1)\,mc^2 \\ &= (1.09 - 1)\,(938.3\,\mathrm{MeV}) \\ &= 84.45\,\mathrm{MeV}\end{aligned}$$

Question 4: Four momentum

What is the value of the first element of the four-momentum P^0 for the proton described in the previous question?

- 1210 MeV/c
- 1117 MeV/c
- **1023 MeV/c***
- 409.1 MeV/c
- 84.45 MeV/c

The value of $P^0$is given by E/c, where E is the total energy of the proton. For the total energy, we get

$$\begin{aligned} E &= \gamma mc^2 \\ &= (1.09)\left(938.3\,\mathrm{MeV}/c^2\right)c^2 \\ &= 1023\,\mathrm{MeV} \end{aligned}$$

Therefore, we have $P^0 = 1023\,\mathrm{MeV/c}$

Question 5: Transformation of tensors

Using equation (2.110) in the study guide, how would a contravariant tensor of rank 1 A^ν transform in general?

- $A'^\mu = \sum_{\nu=0}^3 \frac{\partial x'^\nu}{\partial x^\mu} A^\mu$
- $A'^\mu = \sum_{\nu=0}^3 \frac{\partial x^\mu}{\partial x'^\nu} A_\nu$
- $A'^\mu = \sum_{\nu=0}^3 \frac{\partial x^\mu}{\partial x'^\nu} A^\nu$
- $\mathbf{A}'^\mu = \sum_{\nu=\mathbf{0}}^{\mathbf{3}} \frac{\partial \mathbf{x}'^\mu}{\partial \mathbf{x}^\nu} \mathbf{A}^\nu$*
- $A'^\mu = \sum_{\nu=0}^3 \frac{\partial x^\nu}{\partial x'^\mu} A^\mu$