UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS1501

October/November 2015

THEORETICAL COMPUTER SCIENCE I

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS HW DU PLESSIS
SECOND MRS D BECKER

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 8 pages

Instructions:

Afrikaanse studente U mag die vraestel in Afrikaans beantwoord.

1 Answer all the questions

2 Any rough notes must be done in your answer book

3 The mark for each question appears in brackets next to the question

4 Please answer the questions in the given order If you want to do a question later, leave enough space

EVERYTHING OF THE BEST!

SECTION 1
SETS AND RELATIONS (Multiple-Choice Questions)

Each question comprises 2 marks.
Choose only one alternative per question and then write the question number and the alternative that you regard as the correct answer in the answer book.
[16 marks]

Suppose U = {{1}, 2, a, b, {c}, {d}} is a universal set with the following subsets

A = {{1},{c}, {d}}, B = {2, a, b} and C = {{1}, 2}

Answer questions 1 1 to 1 8 by using the given sets

Question 1.1
Which one of the following sets represents A ∪ C?

1 {{1, c, d}, {1, 2}}
2 {1, 2, c, d}
3 {{1}, 2, {c}, {d}}
4 {{1}}

Question 1.2
Which one of the following sets represents A ∩ C?

1 {1}
2 {{1}}
3 {2, {c}, {d}}
4 C

Question 1.3
Which one of the following sets represents B + C?

1 ∅
2 {2}
3 {{1}, a, b}
4 {{1}, 2, a, b}

Question 1.4
Which one of the following sets represents U − B?

1. A ∪ C
2. A + C
3. B′
4. {1, c, d}

Question 1.5
Which one of the following sets is an element of the power set $\mathcal{P}$(C)?

1. {{1},{2}}
2. {{1, 2}}
3. {{2}}
4. {{1}}

Question 1.6
Which one of the following sets represents A′?

1. {1, c, d}
2. B ∪ C
3. {{1}, 2, a, b}
4. {2, a, b}

Question 1.7
Let R = {({1}, 2), (2, 2)} be a relation on C Which one of the following statements is **NOT true**?

1. R is antisymmetric
2. R satisfies trichotomy
3. R is reflexive
4. R is transitive

Question 1.8
Let S = {({1}, {1}), (2, 2), (a, a), (b, b), ({c}, {c}), ({d}, {d}), (a, b), (b, a), (2, {c}), ({c}, 2)} be an equivalence relation on U Which one of the following **does NOT** represent an equivalence class of S?

1. [2] = {2, c}
2. [a] = {a, b}
3. [b] = {a, b}
4. [{c}] = {2, {c}}

[TURN OVER]

SECTION 2

SET THEORY

Write your answer to each question out in full in your answer book.

[23 marks]

Question 2 1 **(4)**

Suppose X = {a}, Y = {b, c}, W = {a, c} and U = {a, b, c} with X, Y, W $\subseteq$ U Can the given sets be used to prove that (X + Y) $\cup$ (X + W) $\neq$ X $\cap$ (Y $\cup$ W) ? Justify your answer

Question 2.2 **(9)**

There are 30 children in Ms Silangwe's class All children partake in at least one of three extramural activities tennis, netball and soccer

12 play tennis,

21 play netball, and

14 play soccer

(It doesn't necessarily mean that these pupils only partake in only one activity)

Furthermore

6 play tennis and netball,

6 play tennis and soccer,

7 play soccer and netball,

2 play tennis, netball and soccer

(a) How many pupils partake in tennis only?

(b) How many pupils partake in soccer only?

(c) How many pupils partake in only netball, or both tennis and soccer?

Note Include a diagram in your solution

Question 2.3 **(10)**

Prove, for all sets X, Y $\subseteq$ U, that

Y − (X $\cap$ Y) = Y − X is an identity

Note Do not include specific examples or Venn diagrams in your proof

SECTION 3
RELATIONS AND FUNCTIONS

Write your answer to each question out in full in your answer book. [23marks]

Question 3.1

a) Let A = {1, 2, 3}, and let B and C be relations on A defined by
B = {(1, 2), (2,1), (3,2)} and C = {(2, 3)}

(i) Is B a strict partial order? If your answer is yes, justify your answer If your answer is no, give a counterexample. **(2)**

(ii) Determine the composition relation C ∘ B (or B; C) **(2)**

(iii) Provide a relation P on A that is neither reflexive nor irreflexive **(1)**

b) Let S = {(a, 2), (b, 1)} be a relation from A = {a, b, c} to B = {1, 2}

(i) Is S functional? Justify your answer **(2)**
Note: Do not make use of specific examples in your answer

(ii) S is not a function Add one ordered pair to S to make S a function **(1)**

Question 3.2

a) (i) Give the definition of a reflexive relation R on set A **(1)**

(ii) Let R be a relation on $\mathbb{Z}$, (the set of integers), defined by
$(x, y) \in R$ iff $y \leq x + 1$
Is R reflexive? If your answer is yes, provide a proof. If your answer is no, provide a counterexample **(2)**

b) Let f and g be functions on Z defined by

$(x, y) \in g$ iff $y = x - 1$

and

$(x, y) \in f$ iff $y = x^2 - 2x + 1$

From the table on the next page, choose an option from column B that best matches each entry in column A For example, option **(i)** in column A best matches option **g** in column B because dom(f) and dom(g) are equal to $\mathbb{Z}$ Write this in your answer book as

(i) g

Now do the same for (ii) to (vii).

Column A	Column B
(i) Dom(f) and Dom(g)	a. $x^2 - 4x + 4$
(ii) $f \circ g(x)$	b. is an ordered pair in function g only
(iii). $g \circ f(x)$	c is neither injective nor surjective
(iv). ordered pair (1, 0)	d. $x^2 - 2x$
(v). ordered pair (0, -1)	e. is bijective
(vi) function f	f is an ordered pair in both functions f and g
(vii). function g	g are equal to $\mathbb{Z}$

(12)

SECTION 4

OPERATIONS AND MATRICES

Write your answer to each question out in full in your answer book.

[10 marks]

Question 4.1

(a) Consider the matrices **(3)**

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 2 & 1 \\ 4 & 1 & -4 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 2 & 4 & 3 \\ 1 & -1 & 0 \\ 5 & 0 & 2 \end{bmatrix}$$

Determine A - B.

(b)

(i)Define an identity matrix I with respect to a matrix A. **(1)**

(ii) Consider the matrix C.

$$C = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

Give an identity matrix with respect to C **(2)**

Question 4.2

a) Let Y = {1, 2} and let ◊ Y × Y → Y be a binary operation Draw the following table in your answer book

◊	1	2
1		
2		

(i) Complete the table by providing an example of a binary operation ◊ is **not** commutative. **(2)**

(ii) Translate your completed table into list notation **(2)**

[TURN OVER]

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC
Write your answer to each question out in full in your answer book. **[18 marks]**

Question 5.1
Let p, q and r be simple declarative statements Answer the following questions concerning these statements

a) For each of the following statements, write down in your answer book whether they are **true** or **false. (You do not have to draw truth tables for this question).**

(i) $\neg(\neg(\neg p)) \equiv \neg p$ **(1)**
(ii) $p \vee (\neg(\neg p)) \equiv p$ **(1)**
(iii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge r$ **(1)**

b) Are the statements $[p \wedge \neg (q \to r)]$ and $[p \wedge (q \wedge r)]$ logically equivalent? Copy the following truth table **in your answer book** and complete the table. Then deduce your answer from your completed table **(6)**

p	q	r	$q \to r$	$\neg(q \to r)$	$(q \wedge r)$	$p \wedge \neg(q \to r)$	$\leftrightarrow$	$p \wedge (q \wedge r)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

Question 5.2

Consider the statement $\exists x \in Z^+, [(2x = x^2) \wedge ((x - 4) < 0)]$

a) Write down the negation of the given expression then simplify the expression so that the *not*-symbol (¬) does not occur to the left of any quantifier The *not*-symbol may also not occur outside of any parentheses Show all the steps **(7)**

b) Is the original statement, the negation statement, or are both the original and the negation statements **true**? Justify your answer **(2)**

SECTION 6
MATHEMATICAL PROOFS

Write your answer to each question out in full in your answer book. [10 marks]

Question 6.1 **(4)**

Provide a direct proof to show that, for all $n \in Z$,

if n is a multiple of 2 then $5n^2 + 3n + 1$ is an odd number

Note Do not make use of specific examples in your proof

Question 6.2

Provide a proof by contrapositive to show that for all $x \in Z$,

if $3x^2 + 2$ is even, then x is even **(6)**

Note Do not make use of specific examples in your proof