UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS1501

May/June 2015

THEORETICAL COMPUTER SCIENCE I

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	MRS HW DU PLESSIS
SECOND	MRS D BECKER

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

Afrikaanse studente: U mag die vraestel in Afrikaans beantwoord.

This paper consists of 8 pages.

Instructions:

1 Answer all the questions

2 Any rough notes must be done in your answer book

3 The mark for each question appears in brackets next to the question

4 Please answer the questions in the given order If you want to do a question later, leave enough space

EVERYTHING OF THE BEST!

[TURN OVER]

SECTION 1
SETS AND RELATIONS (Multiple-Choice Questions)

Each question comprises 2 marks.
Choose only one alternative per question and then write the question number and the alternative that you regard as the correct answer in the answer book. **[16 marks]**

Suppose U = {1, {1}, 2, a, b} is a universal set with the following subsets

A = {1, 2}, B = {{1}, a} and C = {1, a, b}

Answer questions 1 1 to 1.8 by using the given sets

Question 1.1
Which one of the following sets represents $A \cup B$?

1. {1, 2, a}
2. {{1}, 2, a}
3. {1, {1}}
4. {1, {1}, 2, a}

Question 1.2
Which one of the following sets represents $C - B$?

1. {1, b}
2. {a}
3. {b}
4. {1, {1}, b}

Question 1.3
Which one of the following sets represents $B \cap C$?

1. {1, a}
2. {a}
3. {{1}, a, b}
4. {{1}, a}

[TURN OVER]

Question 1.4

Which one of the following sets represents B′ ?

1 U + B
2 {1, 2, b}
3 {2, b}
4 {1, b}

Question 1.5

Which one of the following sets represents B + C?

1 {1, {1}, b}
2 {b}
3 {1, {1}, a, b}
4 {1, a}

Question 1.6

What is the cardinality of $\mathcal{P}$(A)?

1 32
2 8
3. 4
4. 2

Question 1.7

Suppose the set T is defined as T = {{1}, {{1}, a}, {2, b}} Which one of the following statements regarding the set T is **true**?

1 U is a subset of T
2 T is a subset of U
3 T is the power set of U
4 T is a partition of U

Question 1.8

Let S = {(1, a), (b, 1), (b, b), (b, a)} be a relation on C Which ordered pair must be removed from S to make S a strict partial order?

1 (b, 1)
2 (b, a)
3 (1, a)
4 (b, b)

[TURN OVER]

SECTION 2
SET THEORY

Write your answer to each question out in full in your answer book. **[20 marks]**

Question 2.1

a) Draw Venn diagrams to show that $A + (B \cap C) = (A + B) \cap (A + C)$, with $A, B, C \subseteq U$, is not an identity

(Hint Draw the Venn diagrams step-by-step) **(6)**

b) Provide a counterexample, and then use it to show that $A + (B \cap C) \neq (A + B) \cap (A + C)$ **(5)**

Question 2.2 **(9)**

Prove that, for all $X, Y, W \subseteq U$,

$(Y \cup (X \cap W')) = (Y \cup X) \cap (Y \cup W')$ is an identity.

(Note Do not make use of specific examples in your proof Do not draw Venn diagrams)

SECTION 3
RELATIONS AND FUNCTIONS

Write your answer to each question out in full in your answer book. **[21 marks]**

Question 3.1

a) Let $A = \{1, 2, 3\}$, and let B be a relation on A defined by
$B = \{(1, 3), (2, 1), (3, 2)\}$

(i) Determine the composition relation $B \circ B$ **(3)**

(ii) B is an irreflexive relation Write down one ordered pair that can be added to B such that B will then be neither reflexive nor irreflexive **(1)**

b) Let $A = \{1, a\}$ and $B = \{1, b\}$ Give an example of an injective function from A to B **(2)**

c) Let $A = \{2, 3, b\}$, $B = \{1, b, c, d\}$ and $C = \{1, 2, a\}$ For each of the following functions, write down whether it is injective, surjective or both

[TURN OVER]

(i) Function M $A \to B$, defined by $M = \{(2, 1), (b, b), (3, c)\}$

(ii) Function N $A \to C$, defined by $N = \{(2, 2), (3, 1), (b, a)\}$

(iii) Function S $B \to C$, defined by $S = \{(1, 1), (b, 2), (c, 2), (d, a)\}$ **(3)**

Question 3.2

a) Let R be a relation on $\mathbb{Z}$ defined by $(x, y) \in R$ iff the difference between x and y is multiple of 3

Prove that R is symmetric

(Note Do not make use of specific examples in your proof) **(3)**

b) Let f and g be functions on $\mathbb{Z}^+$ defined by

$(x, y) \in f$ iff $y = 2x - 3$

and

$(x, y) \in g$ iff $y = x^3 - 1$

(i) Prove that g is an injective function **(4)**

(ii) Is $(-1, -2) \in g$? Give a reason for your answer **(1)**

(iii) Determine $f \circ g(x)$ (Show all the steps) **(4)**

SECTION 4
OPERATIONS AND MATRICES

Write your answer to each question out in full in your answer book. **[16 marks]**

Question 4.1

Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 2 \\ 3 & 1 & 0 \end{bmatrix}$$

a) Is $A \cdot B$ equal to $B \cdot A$? Determine both matrices and then draw a conclusion (Show all your calculations) **(7)**

[TURN OVER]

b) Provide a matrix D such that **(3)**

$$D - B = \begin{bmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{bmatrix}$$

Question 4.2

Consider the table for the binary operation $\Diamond$ on $\{a,b\}$ and answer the questions below

$\Diamond$	**a**	**b**
a	b	b
b	a	a

(i) Give a counterexample to prove that $\Diamond$ is not **commutative** **(1)**

(ii) Give a counterexample to prove that $\Diamond$ is not **associative** **(2)**

(iii) Does the binary operation $\Diamond$ have an **identity** element? **(1)**

(iv) Write the binary operation $\Diamond$ in list notation **(2)**

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC

Write your answer to each question out in full in your answer book. [18 marks]

Question 5.1

a) For each of the following statements, write down whether the statement is **true** or **false** **(4)**
NO truth tables are needed for this question.

(i) $p \wedge (p \rightarrow q) \equiv p \wedge (\neg p \vee q)$

(ii) $\neg p \vee r \equiv p \vee \neg r$

(iii) $\neg(\neg q \wedge r) \equiv q \vee \neg r$

(iv) $(p \wedge q) \wedge r \equiv p \vee (q \vee r)$

b) (i) **Draw the following truth table in your answer book** and then complete the table for the following compound statement **(5)**

$$[\neg p \rightarrow (q \vee r)] \leftrightarrow (p \vee \neg r)$$

p	q	r	$\neg p$	$\neg r$	$q \vee r$	$\neg p \rightarrow (q \vee r)$	$\leftrightarrow$	$p \vee \neg r$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

(ii) Is the given statement a tautology, a contradiction or neither? **(1)**

Question 5.2

(i) Consider the statement $\exists\ x \in Z, [((x - 2) < -3) \wedge (x \leq 0)]$

Write down the negation of the given expression Simplify the expression so that the *not*-symbol ($\neg$) does not occur to the left of any quantifier The *not*-symbol may also not occur outside of any parentheses Show all the steps **(7)**

(ii) Is the original statement, the negation statement, or are both the original and the negation statements **true**? **(1)**

SECTION 6
MATHEMATICAL PROOFS

Write your answer to each question out in full in your answer book. **[9 marks]**

Question 6.1 **(4)**

Provide a direct proof to show that, for all $n \in Z$,

if n is a multiple of 5, then $n^2 + 3n + 5$ is a multiple of 5

(Note: Do not make use of specific examples in your proof.)

Question 6.2

a) Provide the converse of the statement

if x is odd then $x^2 + 3x + 4$ is even **(1)**

b) Provide a proof by contrapositive to show that for all $x \in Z$,

if $x^3 + 4x + 2$ is odd, then x is odd **(4)**

(Note: Do not make use of specific examples in your proof.)