FINAL %	

UNISA university of south africa

COS1501

OCT/NOV 2017

THEORETICAL COMPUTER SCIENCE I

STUDENT NUMBER									

IDENTITY NUMBER													

FOR USE BY EXAMINATION INVIGILATOR

Question No	Marks					
	Examiners					
	1		2		3	
Total						

Subject

Number of paper

Date of examination

Examination centre

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UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS1501

October/November 2017

THEORETICAL COMPUTER SCIENCE I

Duration 2 Hours

100 Marks

EXAMINERS

FIRST MRS HW DU PLESSIS

SECOND MR CL PILKINGTON

Closed book examination

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This paper is a fill-in paper and consists of 14 pages plus an additional 4 pages for rough work (pp 15-18)

Afrikaanse studente: U mag die vraestel in Afrikaans beantwoord.

Instructions:

1 Answer all the questions in all 6 sections on the fill-in paper

2. **Please do all rough work on the last four pages marked 'ROUGH WORK'.**

3 The mark for each question appears in brackets next to the question

EVERYTHING OF THE BEST!

[TURN OVER]

SECTION 1
SETS AND RELATIONS (Multiple-Choice Questions)

Each question comprises 2 marks
Circle the alternative that you think is the correct alternative to select.
There is ONLY one correct alternative per question. If you circle more than one alternative, a zero mark will be awarded for that question.
There is space at the end of the paper for rough work. **[16 marks]**

Suppose **U = {{1}, 2, {1, 2}, a, b, c}** is a universal set with the following subsets

A = {{1}, a, c}, B = {{1}, {1, 2}, b, c} and C = {2, c}.

Answer questions 1 1 to 1 8 using the given sets, by **circling** the alternative number that you select

Question 1.1
Which one of the following sets represents A ∪ C?

1 {1, 2, a, c}
2 {{1, 2}, a, c}
3 {{1}, 2, a, c}
4 {1, {1, 2}, a, c}

Question 1.2
Which one of the following sets represents (B ∩ A)?

1 {{1}, c}
2 {{1, 2}, a, b}
3 {1, c}
4 {{1, c}}

Question 1.3
Which one of the following sets represents C – B?

1 Ø
2 {2}
3 {2, {1}, {1, 2}, b}
4 {{1}, {1, 2}, b, 2}

Question 1.4
Which one of the following sets represents A + C?

1 {1, 2, a, c}
2 {c}
3 {{1}, 2, a, c}
4 {2, {1}, a}

Question 1.5
Which one of the following sets represents $(A \cup C)'$?

1 {{1, 2}, b}
2 (U – B) – A
3 {2, {1}, {1, 2}, a, b}
4 B

Question 1.6
Which one of the following sets is NOT a partition of U?

1 {{2, a, b, c}, {{1}}, {{1, 2}}}
2 {{2, {1, 2}, c}, {{1}, a, b}}
3 {{1, 2}, {1}, {b, c}, {a}, {2}}
4 {{a}, {2}, {b, c}, {{1}}, {{1, 2}}}

Question 1.7
Let T = {(a, c), ({1}, a), (c, c), (c, {1})} be a relation on A. Which one of the following statements is **true** regarding T?

1 T is reflexive, but not transitive
2 T is symmetric and satisfies trichotomy
3 T is irreflexive and transitive
4 T is neither reflexive nor irreflexive

Question 1.8
What is the cardinality of $(A + C) \cap B'$?

1 2
2 3
3 4
4 5

SECTION 2
SET THEORY

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. **[23 marks]**

Question 2.1

Suppose $\mathbf{A} = \{1, 2\}$, $\mathbf{B} = \{2\}$, $\mathbf{C} = \{2, 3\}$ and $\mathbf{U} = \{1, 2, 3\}$ with A, B, $C \subseteq U$ Can the given sets be used to prove that $(\mathbf{A} - \mathbf{B}) \cup (\mathbf{C} - \mathbf{B}) \neq (\mathbf{A} \cap \mathbf{C}) - \mathbf{B}$? Justify your answer Do not use Venn diagrams **(4)**

Question 2.2

A survey involving 30 students is done to determine which study aids students prefer. The following is found:

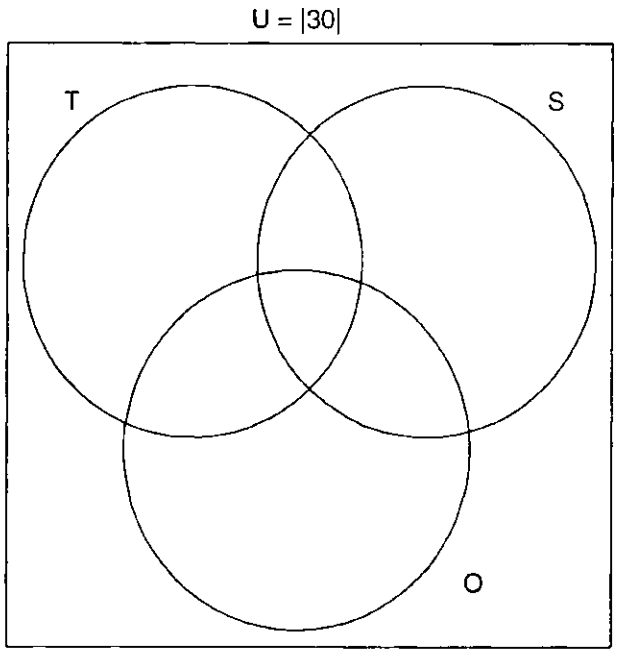
13 students prefer text books,
20 students prefer study guides, and
17 students prefer online lessons

(It doesn't necessarily mean that the students prefer one aid only)

Furthermore

8 students prefer text books and online lessons,
7 students prefer study guides and online lessons, and
10 students prefer text books and study guides

(i) Complete the Venn diagram below with the given information, and then answer the rest of the questions **(5)**



(ii) How many students prefer to have text books, study guides and online lessons? **(2)**

(iii) How many students prefer to have text books only **(1)**

(iv) How many students would like to have study guides and online lessons, but no text books? **(1)**

Question 2.3

Prove without using Venn diagrams, that

$A \cap (B' \cup C) = (A - B) \cup (A \cap C)$ for all subsets A, B and C of a universal set U **(10)**

Show ALL the steps.

$x \in A \cap (B' \cup C)$
iff
iff
iff
iff
iff
iff
iff

SECTION 3
RELATIONS AND FUNCTIONS

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. **[24 marks]**

Question 3.1

Let **A = {1, b, c}** and **B = {2, a, c}** Let **T = {(c, 1), (c, c), (a, b), (2, 1), (2, c)}** be a relation from **B** to **A**

a) Determine T ∘ T **(3)**

T ∘ T =

b) Which ordered pair should be deleted from T to make it an irreflexive relation? **(1)**

Question 3.2

a) Let A = {1, 2} and B = {2, a, b}

Give an example of an injective function from A to B **(2)**

b) Let **A = {1, 2, a, b}**, **B = {2, 3, c}** and **C = {3, a, d}**

For each of the following functions, write down whether it is injective, bijective or surjective You do not have to prove anything

(i) Function M C → B, defined by M = {(3, 3), (d, 2), (a, c)} **(1)**

(ii) Function N $A \rightarrow C$, defined by N = {(b, d), (1, a), (a, 3), (2, d)} **(1)**

(iii) Function F $B \rightarrow A$, defined by F = {(2, 1), (c, b), (3, a)} **(1)**

Question 3.3

a) Let R be a relation on Z defined by $(x, y) \in R$ iff $y = -x + 4$ Prove that R is symmetric **(4)**

b) Let f and g be functions on Z^+ defined by
$(x, y) \in f$ iff $y = 3x + 2$ and $(x, y) \in g$ iff $y = x^3 + 1$

(i) Determine $f \circ g(x)$ **(4)**

$f \circ g(x) =$

(ii) Prove that function f is injective by using the definition of injectivity **(3)**

(iii) Function g is not surjective Give a counterexample to prove it Show how you get to your answer **(2)**

(iv) Is the ordered pair (2, 8) in f? Why / why not? **(2)**

SECTION 4
OPERATIONS AND MATRICES

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. [10 marks]

Question 4.1

Consider the following matrices

$$A = \begin{bmatrix} 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$$

(a) Is it possible to calculate A B? If it is possible, calculate it If it is not possible, give a reason why it is not possible **(2)**

Consider the following matrices

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 3 \\ -2 & 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & -2 \\ -3 & 1 & 0 \\ 2 & 2 & -3 \end{bmatrix}$$

(b) Calculate 2A – B **(3)**

Please turn the page to complete the question in the given space

Question 4.2

Given the incomplete table below for the binary operation *

*	a	b	c
a	b		
b		b	
c			b

a) Complete the table above so that the binary operation * is NOT commutative and that b is the identity element **(3)**

b) Can (a*c)*b and a*(c*b) be used in a counterexample to prove that the operation * is not associative? Motivate your answer **(2)**

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. **[18 marks]**

Question 5.1

a) For each of the following statements, if you think the statement is true, circle T (for true), else circle F (for false)

(i)	$(\neg p \rightarrow q) \equiv (\neg q \rightarrow p)$	**T**	**F**
(ii)	$(p \vee \neg q) \wedge r \equiv p \vee (\neg q \wedge r)$	**T**	**F**
(iii)	$\neg(\neg(p \wedge \neg r)) \equiv \neg(r \vee \neg p)$	**T**	**F**

(3)

b)

(i) Complete the truth table for the following compound statement **(5)**

(Hint Complete the highlighted column last)

$$[(\neg p \rightarrow q) \vee r)] \rightarrow [(\neg r \rightarrow q) \vee p]$$

p	q	r	¬p	¬r	[(¬p→q)	[(¬p→q)∨r]	[(¬p→q)∨r]→[(¬r→q)∨p]	(¬r→q)	(¬r → q) ∨ p
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

(ii) Is the expression a tautology, contradiction or neither? **(1)**

Question 5.2

Consider the statement $\forall x \in Z, [(2x + 3 > 0) \vee (3x - 2 < 0)]$

a) Is the given statement true? Justify your answer **(2)**

b) We give the negation of the statement in (a) below Simplify the negation statement so that the *not*-symbol ($\neg$) does not occur to the left of any quantifier The *not*-symbol may also not occur outside of any parentheses **Show all the steps** **(5)**

Negation: $\neg[\forall x \in Z, [(2x + 3 > 0) \vee (3x - 2 < 0)]]$
$\equiv$
$\equiv$
$\equiv$

c) Is the negation statement true? Justify your answer **(2)**

SECTION 6
MATHEMATICAL PROOFS

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. **[9 marks]**

Question 6.1

(a) Write down the converse of the statement

If n is odd and $n < 0$, then $n^3 + 1 < 0$ **(2)**

(b) Write down the contrapositive of the statement

If $n^3 - 7n^2 + 4$ is even, then n is odd **(2)**

Question 6.2

Provide a direct proof to show that, for all $n \in \mathbb{Z}$, if n is odd, then $4n^2 + 2n + 7$ is odd

Note Do not make use of specific examples in your proof **(5)**

Assume x =
then
ie
ie
ie
ie
ie

ROUGH WORK

ROUGH WORK

ROUGH WORK

ROUGH WORK

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Signature of invigilator

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