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UNISA university of south africa

COS1501

MAY/JUNE 2018

THEORETICAL COMPUTER SCIENCE I

STUDENT NUMBER									

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Question No	Marks Examiners					
	1		2		3	
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Subject

Number of paper

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Examination centre

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UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS1501

MAY/JUNE 2018

THEORETICAL COMPUTER SCIENCE I

Duration 2 Hours

100 Marks

EXAMINERS

FIRST MRS HW DU PLESSIS

SECOND MR CL PILKINGTON

Closed book examination.

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Afrikaanse studente: U mag die vraestel in Afrikaans beantwoord.

This paper is a fill-in paper and consists of 18 pages plus an additional 2 pages for rough work (pp 19-20).

Instructions:

1 Answer all the questions in all 6 sections on the fill-in paper

2. **Please do all rough work in the areas marked 'ROUGH WORK'. There are additional rough work pages at the end of the paper.**

3 The mark for each question appears in brackets next to the question

EVERYTHING OF THE BEST!

[TURN OVER]

SECTION 1
SETS AND RELATIONS (Multiple-Choice Questions)

Each question comprises 2 marks.
Circle the alternative that you think is the correct alternative to select.
There is ONLY one correct alternative per question. If you circle more than one alternative, a zero mark will be awarded for that question.
There is additional space at the end of the paper for rough work. **[16 marks]**

Suppose **U = {a, {b, c}, c, d, {d, e}, e}** is a universal set with the following subsets.

A = {{b, c}, c, {d, e}}, **B = {a, {b, c}, d, e}** and **C = {a, c, d, e}.**

Answer questions 1.1 to 1 8 using the given sets, by **circling** the alternative number that you select.

Question 1.1
Which one of the following sets represents $A \cup B$?

1. {a, b, c, d, e}
2. {a, {b, c}, c, d, e}
3 {a, {b, c}, c, {d, e}}
4 {a, {b, c}, c, {d, e}, d, e}

Question 1.2
Which one of the following sets represents $B \cap C$?

1 {a, c, d, e}
2 {a, d, e}
3 {d, e}
4 {c, {b, c}}

Question 1.3
Which one of the following sets represents $C - A$?

1 {a, {b, c}, d, e, {d, e}}
2. {c, d, e}
3 { }
4 {a, d, e}

Question 1.4

Which one of the following sets represents U + B?

1. U
2 {c, {d, e}}
3. {a, {b, c}, d, e}
4 (U – A) – C

Question 1.5

Which one of the following sets represents $C \cap B'$?

1 {c}
2. {a, d, e}
3. {a, c, d, e}
4 {a, c, {d, e}}

ROUGH WORK

Question 1.6

Which one of the following statements regarding $\mathcal{P}(A)$ is true?

1. $\mathcal{P}(A) = \{ \{ \}, \{\{b, c\}\}, \{c\}, \{\{d, e\}\}, \{\{b, c\}, c\}, \{\{b, c\}, \{d, e\}\}, \{c, \{d, e\}\} \}$
2. The cardinality of $\mathcal{P}(A)$ is 16
3. $c \in \mathcal{P}(A)$
4. $\{\{ \}\} \subset \mathcal{P}(A)$

Question 1.7

Let T = {(a, a), (c, d), (e, d), (a, d), (e, a), (e, c)} be a relation on the set C. Which one of the following statements is **true**?

1. T satisfies trichotomy
2. T is reflexive.
3. T is transitive.
4. T is symmetric.

Question 1.8

Let S = {(a, c), (a, d), (a, e), (d, e), (c, d)} be a relation on set C. Which of the following statements regarding S is true?

1. S is a strict total order.
2. S is a weak partial order
3. If (e, c) is added to S, S would satisfy trichotomy
4. If (e, c) is added to S, it would make S transitive

ROUGH WORK

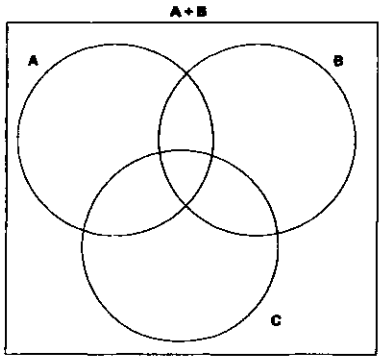
SECTION 2
SET THEORY

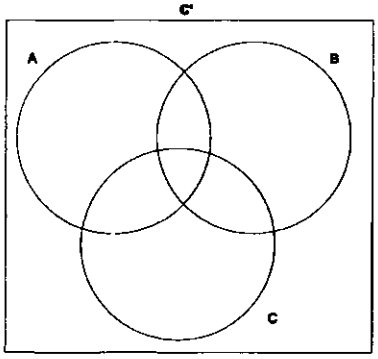
Write your answers in the space provided. There is additional space for rough work at the end of the fill-in paper. [19 marks]

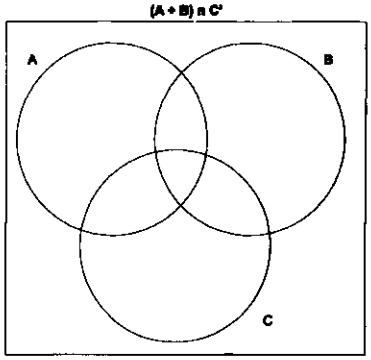
Question 2.1

a) Complete the Venn diagrams to show that $(A + B) \cap C' = (A \cup B) - (A \cap B \cap C)$, with $A, B, C \subseteq U$, is not an identity. **(6)**

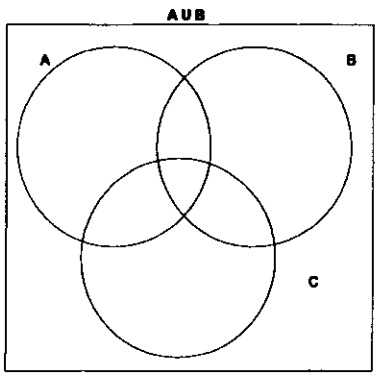
Left hand side:

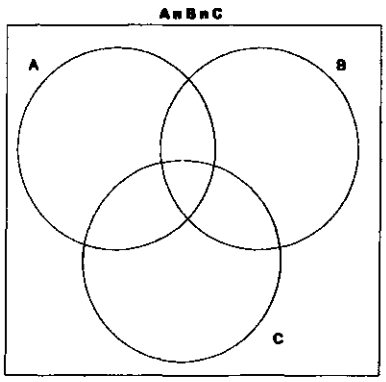


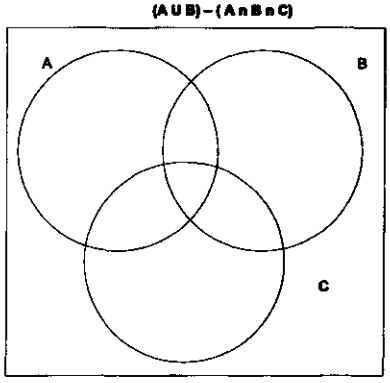




Right hand side:



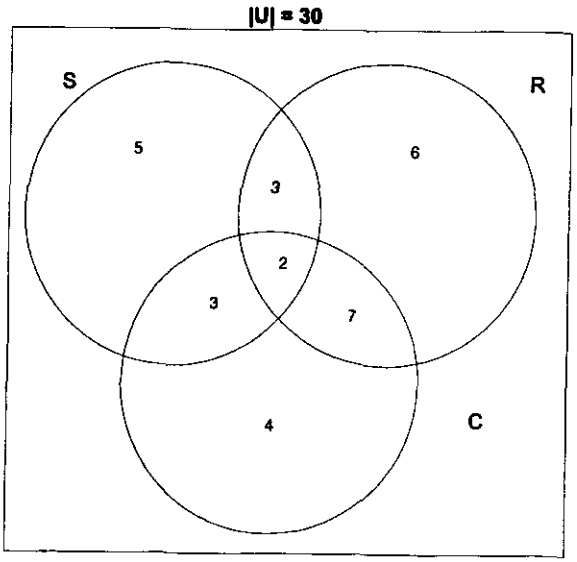




b) Let A = {1, 2, 3}, B = {1, 2, 4} and C = {2, 3, 4}. Without using Venn diagrams, use the sets A, B and C to prove that $(A \cap C) \cup ((B - A) - C) \neq B - ((A \cap B) \cap C)$ **(4)**

c) Consider the following Venn diagram, and then answer the questions that follow:

The Venn diagram represents the pupils playing soccer, rugby and cricket in Mr Smith's class of 30 pupils



(i) How many pupils play both rugby and cricket, but not soccer? **(1)**

(ii) How many pupils play soccer only? **(1)**

(iii) How many pupils play rugby? **(1)**

Question 2.2

Prove without using Venn diagrams, that

$X \cup (Y - W) = (X \cup Y) \cap (X \cup W')$ for all subsets X, Y and W of a universal set U. **(6)**

$x \in X \cup (Y - W)$
iff
iff
iff
iff
iff
iff

ROUGH WORK

SECTION 3
RELATIONS AND FUNCTIONS

Write your answers in the space provided. There is additional space for rough work at the end of the fill-in paper. **[20 marks]**

Question 3.1

a) Provide an example of a relation T on set A = {a, b, c} that is irreflexive, antisymmetric and satisfies trichotomy **(3)**

T =

b) Let S = {(1, 2), (4, 3), (3, 3), (2, 4)} be a relation on set B = {1, 2, 3, 4}

(i) Which ordered pair must be removed from S to make S an irreflexive relation? **(1)**

(ii) Which ordered pairs must be added to S to make S a transitive relation? **(2)**

c) Let A = {1, 2, 3}, B = {2, 3, 4} and C = {1, 2, 3, 4}. For each of the following statements, circle T if you think the statement is true, and F if you think the statement is false. **(3)**

S = {(2, 2), (3, 1), (4, 3)} is an injective function from B to C	T	F
G = {(4, 2), (1, 1), (3, 3)} is a surjective function from C to A	T	F
H = {(2, 1), (3, 3), (4, 2)} is a bijective function from B to A	T	F

Question 3.2

Let A = {a, b, c} and B = {c, d, a}.

Let L = {(a, a), (c, d), (b, c), (c, a)} and M = {(c, a), (a, c), (a, d), (c, d)} be two relations from A to B

(a) Determine M ∘ L (ie L; M). **(3)**

M ∘ L =

(b) Determine $L \circ L$ (ie $L; L$). **(2)**

$L \circ L =$

ROUGH WORK

Question 3.3

Let f and g be functions on Z^+ defined by:

$(x, y) \in f$ iff $y = 5x^2 - 2$ and $(x, y) \in g$ iff $y = 3x + 2$

a) Is g an injective function? If your answer is yes, give a proof. If your answer is no, give a counterexample. **(3)**

b(i) $g \circ g(x)$ is also a function on Z^+. Determine $g \circ g(x)$. **(2)**

$g \circ g(x) =$

b(ii) Determine if the ordered pair (-2, 2) is in function f. Show your calculations. **(1)**

SECTION 4
OPERATIONS AND MATRICES

Write your answers in the space provided. There is additional space for rough work at the end of the fill-in paper. **[13 marks]**

Question 4.1

(a) Consider the following matrices:

$$Let\ A = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 2 & 2 \end{bmatrix} \quad and\ C = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}.$$

Calculate (A • B) • C. Show your calculations. **(4)**

(***Hint***: Calculate (A • B) first, then multiply the result with C)

(b) Consider the following matrix.

$$D = \begin{bmatrix} -2 & 0 & 1 \\ 2 & 4 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Provide an identity matrix I such that D I = I D = D **(3)**

Question 4.2

Given the table below:

*	**a**	**b**	**c**
a	c	a	b
b	a	b	c
c	c	c	b

a) Provide a counterexample to prove that the binary operation * is not commutative. **(1)**

b) Provide a counterexample to prove that the binary operation * is not associative. **(2)**

c) Is ((b * b) * c) * c = [(a * c) * (b * c)] * (c * a)? Justify your answer by applying the binary operation * to the left hand side and the right hand side of the equation separately, and then draw a conclusion. **(2)**

d) Does the binary operation * have an identity element? Motivate your answer. **(1)**

ROUGH WORK

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC

Write your answers in the space provided. There is additional space for rough work at the end of the fill-in paper. **[20 marks]**

Question 5.1

a) For each of the following statements, if you think the statement is true, circle T (for true), else circle F (for false)

(i)	$(\neg p \wedge q) \rightarrow p \equiv p \vee \neg q$	T	F
(ii)	$\neg(\neg(p \rightarrow q)) \vee q) \equiv \neg p \vee q$	T	F
(iii)	$p \vee \neg(\neg p \wedge q) \vee (p \vee \neg q) \equiv p \vee \neg q$	T	F

(3)

b) Complete the following truth table for all the possible values of p and q. **(2)**

p	q	$\neg p$	$\neg p \leftrightarrow q$

c) **(i)** Complete the truth table for the following expression Make sure that you complete all the empty blocks.

$$[p \wedge (q \rightarrow r)] \leftrightarrow [(\neg p \vee q) \rightarrow (p \wedge r)]$$ **(5)**

p	q	r	$\neg q$	$\neg r$	$p \wedge \neg q$	$(p \wedge \neg q) \vee r$	$\leftrightarrow$	$p \vee (\neg r \rightarrow \neg q)$	$\neg r \rightarrow \neg q$
T	T	T	F	F			T	T	
T	T	F	F	T		F		T	
T	F	T	T	F	T	T	T	T	T
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	F		T		
F	T	F	F	T	F	F		F	
F	F	T	T	F			T		T
F	F	F	T	T		F		T	T

(ii) Looking at your answers in the shaded column, is the expression a tautology, contradiction or neither? Give a reason for your answer. **(2)**

Question 5.2

Consider the statement: $\exists x \in Z^+, [((x - x^2 \geq 0) \wedge ((x - 1) \geq 0)]$

a) Is the given statement true? Justify your answer. **(2)**

b) (i) Simplify the negation statement given below so that the *not*-symbol ($\neg$) does not occur to the left of any quantifier The *not*-symbol may also not occur outside of any parentheses. **Show all the steps.** **(4)**

Negation: $\neg[\exists x \in Z^+, [((x - x^2 \geq 0) \wedge ((x - 1) \geq 0)]$.
$\equiv$
$\equiv$
$\equiv$

b) (ii) Is the resulting simplified statement in b(i) true? Give a reason for your answer **(2)**

ROUGH WORK

SECTION 6
MATHEMATICAL PROOFS

Write your answers in the space provided. There is additional space for rough work at the end of the fill-in paper. **[12 marks]**

Question 6.1

Provide a direct proof to show that, for all $n \in Z$, if n is odd, then $3n^2 + 3n - 11$ is odd.

Note: Do not make use of specific examples in your proof. **(4)**

Suppose
then
i.e.
i e.
i.e.
i.e.

Question 6.2

Show by means of a counter-example that the statement $\forall x \in Z, -x^3 - 5x - 7 > 0$ is not true. **(2)**

Let x =
then
i.e.
i.e.

Question 6.3

Given the following statement If $x + 1$ is a multiple of 3, then $x^2 + 3x + 3$ is odd

a) Provide the **converse** statement **(1)**

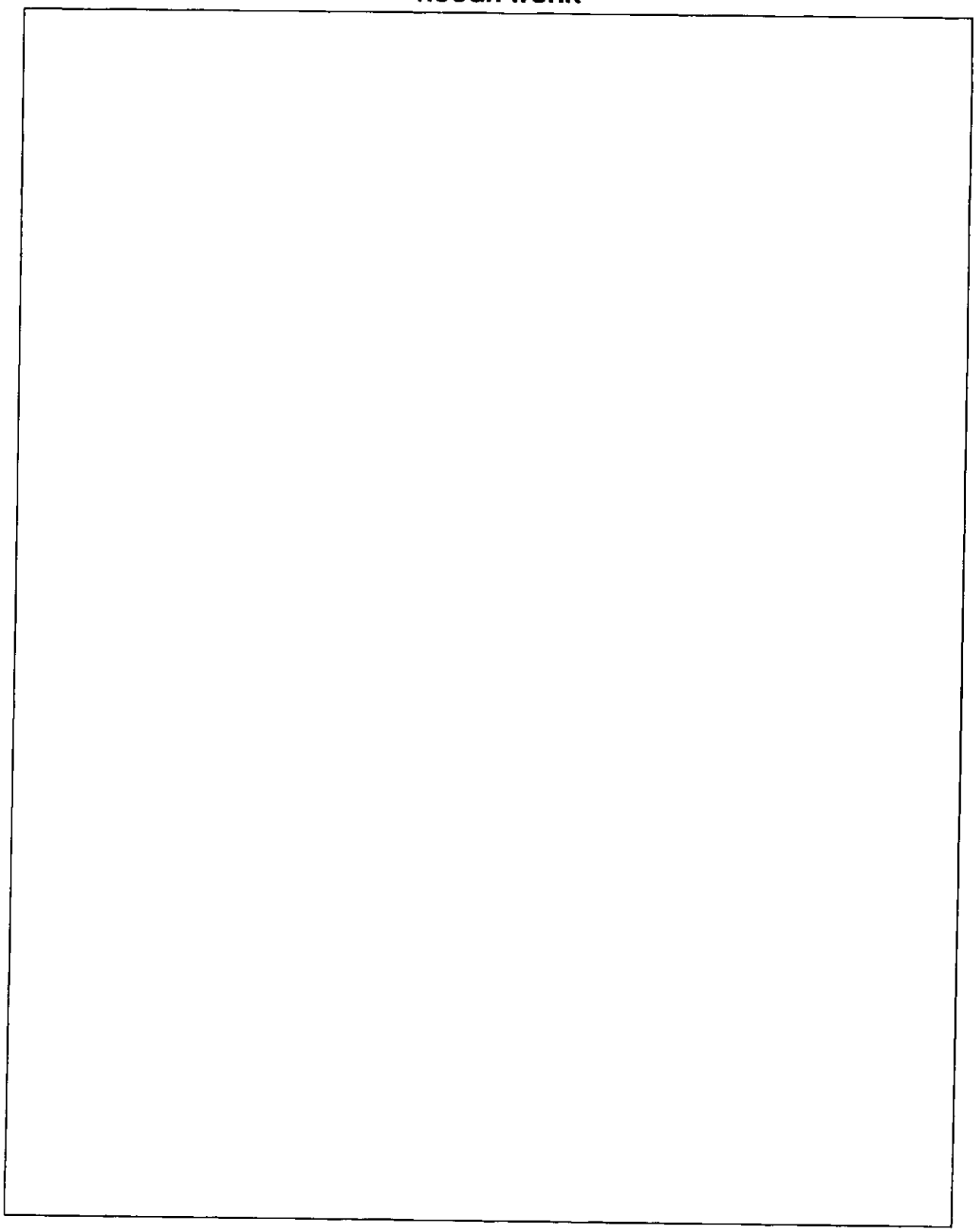
b) Provide the **contrapositive** statement **(1)**

Question 6.4

Provide a contrapositive proof to prove that for all $x \in Z$, if $3x^2 - 5x + 7$ is even, then x is odd.
Note. Do not make use of specific examples in your proof **(4)**

Suppose
then
assume

ROUGH WORK



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