

Tutorial letter 201/1/2017

Theoretical Computer Science 1 COS1501

Semester 1

School of Computing

This tutorial letter contains
a discussion of assignment 01, and
examination information.

Dear Student,

In this tutorial letter the solutions to the first assignment questions are discussed and examination information is provided.

PLEASE ALSO NOTE that the solutions for the activities in the Study guide are available in the learning units on MyUnisa as well as in Additional Resources.

Regards,
COS1501 Team

SOLUTIONS ASSIGNMENT 01 (SEMESTER 1)

Question 1

Alternative 4

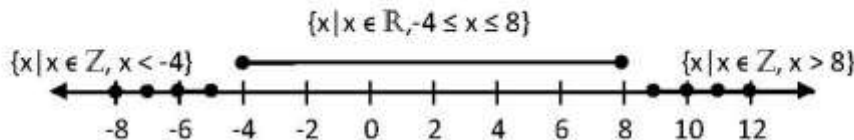
Let us look at the set:

$$\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\} \cap [\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$$

Remember that the union of two sets is the set with elements belonging to the one set or the other set, or to both sets. The intersection of two sets is the set with elements belonging to both sets.

Discussion

We first have to determine the union of the two sets $\{x \mid x \in \mathbb{Z}, x < -4\}$ and $\{x \mid x \in \mathbb{Z}, x > 8\}$ (we will refer to this set as the union set), and then determine the intersection of $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\}$ and the union set. Graphically we can depict the sets as follows:



We can determine the union $\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}$

For $\{x \mid x \in \mathbb{Z}, x < -4\}$, $x \in \{\dots -7, -6, -5\}$. Remember that $x \in \mathbb{Z}$, so we only indicate the set with dots on the graph, and similarly for $\{x \mid x \in \mathbb{Z}, x > 8\}$, $x \in \{9, 10, 11, 12\dots\}$.

For the set $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\}$, we draw a line because $x \in \mathbb{R}$ so all the real numbers between and including -4 and 8 are part of the set.

Thus the set $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\} \cap [\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$ does not contain any elements since no elements belong to set $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\}$ and to the union set.

Alternative 1:

$\{-4, 8\}$ is not the intersection between $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\}$ and the union set since these sets have no element in common.

Alternative 2:

$\{x \mid x \in \mathbb{R}, -4 < x < 8\}$ is not the intersection of the two given sets since they have no elements in common. It should also be clear from the graph that none of the integers between the real numbers -4 and 8, ie the integers -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 are elements of $[\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$, so they cannot possibly be elements of $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\} \cap [\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$.

Alternative 3:

The set $\{x \mid x \in \mathbb{Z}, -4 \leq x \leq 8\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$ is also not the intersection of the two given sets which have no elements in common. It should also be clear from the graph that none of the integers in this set are elements of the set $[\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$, so they cannot possibly be elements of $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\} \cap [\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$.

Alternative 4:

$\{ \}$ must be selected as the intersection between the sets $\{x \mid x \in \mathbb{R}, -4 \leq x \leq 8\}$ and $[\{x \mid x \in \mathbb{Z}, x < -4\} \cup \{x \mid x \in \mathbb{Z}, x > 8\}]$ as we determined in the discussion. By now it should also be clear from the graph that these two sets have no elements in common.

Refer to study guide, pp 11-13.

Question 2

Alternative 3

The set $\{3, 6, 9, 12, 15, \dots\}$ is the set:

1. $\{x \mid x \in \mathbb{Z}, x = 3k \text{ for some } k \in \mathbb{Z}^{\geq}\}$
2. $\{x \mid x \in \mathbb{Z}, x \geq 0, x = 3k \text{ for some } k \in \mathbb{Z}\}$
3. $\{x \mid x \in \mathbb{Z}^+, x = 3k \text{ for some } k \in \mathbb{Z}^+\}$
4. $\{x \mid x \in \mathbb{Z}^{\geq}, x = 3k \text{ for some } k \in \mathbb{Z}\}$

Discussion

When we consider the given sets in the different alternatives, it should be investigated whether the conditions given for a set completely describe the set $\{3, 6, 9, 12, 15, \dots\}$.

Alternative 1:

Let $T = \{x \mid x \in \mathbb{Z}, x = 3k \text{ for some } k \in \mathbb{Z}^{\geq}\}$. Let $x = 0$. It is true that $x \in \mathbb{Z}$. It is also true that 0 is a multiple of 3, because it can be written in the form $x = 3k$, ie $0 = 3(0)$ for $k = 0$. But $0 \notin \{3, 6, 9, 12, 15, \dots\}$. The set in this alternative therefore does not describe the given set since $0 \in T$ but 0 is not an element of the given set.

Alternative 2:

By similar reasoning as provided in alternative 1,

$\{x \mid x \in \mathbb{Z}, x \geq 0, x = 3k \text{ for some } k \in \mathbb{Z}\} \neq \{3, 6, 9, 12, 15, \dots\}$ since $0 \in \{x \mid x \in \mathbb{Z}, x \geq 0, x = 3k \text{ for some } k \in \mathbb{Z}\}$ but $0 \notin \{3, 6, 9, 12, 15, \dots\}$.

Alternative 3:

We can determine the x-values for the set $\{x \mid x \in \mathbb{Z}^+, x = 3k \text{ for some } k \in \mathbb{Z}^+\}$ by using the formula $x = 3k$ for some $k \in \mathbb{Z}^+$. These values of x are thus 3, 6, 9, The condition ' $x \in \mathbb{Z}^+$ ' is also given in the set, but we are sure that 3, 6, 9, ... are all elements of \mathbb{Z}^+ .

Thus $\{x \mid x \in \mathbb{Z}^+, x = 3k \text{ for some } k \in \mathbb{Z}^+\} = \{3, 6, 9, 12, 15, \dots\}$.

Alternative 4:

By similar reasoning as provided in alternative 1,

$\{x \mid x \in \mathbb{Z}^{\geq}, x = 3k \text{ for some } k \in \mathbb{Z}\} \neq \{3, 6, 9, 12, 15, \dots\}$ since $0 \in \{x \mid x \in \mathbb{Z}^{\geq}, x = 3k \text{ for some } k \in \mathbb{Z}\}$ but $0 \notin \{3, 6, 9, 12, 15, \dots\}$.

Refer to study guide, pp 3-13.

Consider the following sets, where U represents a universal set:

$U = \{1, 2, \{3\}, \{1, 3\}\}$ $A = \{1, \{3\}\}$ $B = \{2, \{1, 3\}\}$ $C = \{2, \{3\}\}$.

Questions 3 to 10 are based on the sets defined above.

NOTE: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 3

Alternative 1

$B \cup C$

$$= \{2, \{1, 3\}\} \cup \{2, \{3\}\} = \{2, \{3\}, \{1, 3\}\}$$

Set union: The elements 2, {3} and {1, 3} belong to B or C.

Refer to study guide, p 41.

Question 4

Alternative 3

$A \cap B$

$$= \{1, \{3\}\} \cap \{2, \{1, 3\}\}$$

$$= \{\}$$

Set intersection: There are no elements that belong to both A and B.

Refer to study guide, p 42.

Question 5

Alternative 2

$C - U$ is the set:

$$= \{2, \{3\}\} - \{1, 2, \{3\}, \{1, 3\}\}$$

$$= \{\}$$

Set difference: There are no elements that belong to C that do not belong to U, because $C \subseteq U$.

Refer to study guide, pp 42, 43.

Question 6

Alternative 4

$B + C$

$$= \{2, \{1, 3\}\} + \{2, \{3\}\}$$

$$= \{\{3\}, \{1, 3\}\}$$

Set symmetric difference: The elements $\{3\}$ and $\{1, 3\}$ belong to B or to C, but not both.

It is also the case that $B + C = (B \cup C) - (B \cap C)$, so

$$\begin{aligned} B + C &= (B \cup C) - (B \cap C) && \text{Include elements belonging to } B \cup C \text{ but not to } B \cap C. \\ &= (\{2, \{1, 3\}\} \cup \{2, \{3\}\}) - (\{2, \{1, 3\}\} \cap \{2, \{3\}\}) \\ &= \{2, \{3\}, \{1, 3\}\} - \{2\} \\ &= \{\{3\}, \{1, 3\}\} \end{aligned}$$

Refer to study guide, pp 43, 44.

Question 7

Alternative 2

The elements of $\mathcal{P}(A)$ are all the subsets of A, so we have to determine the subsets of A.

Let's look at the definition of a subset:

*For all sets F and G, F is a subset of G if and only if every element of F is also an element of G. Subsets of G can be formed by **keeping the outside brackets** of G and then throwing away **none, one or more** elements of G.*

The elements of $A = \{1, \{3\}\}$ are 1 and $\{3\}$.

We form the subsets of A:

Throw away no element of set A, then $\{1, \{3\}\} \subseteq A$;

throw away the element 1 of set A, then $\{\{3\}\} \subseteq A$;

throw away the element $\{3\}$ of set A, then $\{1\} \subseteq A$; and

throw away the elements 1 and $\{3\}$ of set A, then $\{\} \subseteq A$;

and that gives all the possible subsets of A.

If $A = \{1, \{3\}\}$ then the subsets of A, namely $\emptyset, \{1\}, \{\{3\}\}$, and $\{1, \{3\}\}$ are all the elements of $\mathcal{P}(A)$, thus

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{\{3\}\}, \{1, \{3\}\}\}.$$

The set provided in alternative 2 is the set $\mathcal{P}(A)$.

The set provided in alternative 1 is a **subset** of $\mathcal{P}(A)$ but **not equal** to $\mathcal{P}(A)$.

The set in alternative 3 is not a subset of $\mathcal{P}(A)$ since the elements 1 and {3} are elements of the set $\{\emptyset, 1, \{3\}, \{1, \{3\}\}$ given in the alternative but $1 \notin \mathcal{P}(A)$ and $\{3\} \notin \mathcal{P}(A)$.

The set in alternative 4 is not a subset of $\mathcal{P}(A)$ since the elements {3} and {1, 3} are elements of the set $\{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$ given in the alternative but $\{3\} \notin \mathcal{P}(A)$ and $\{1, 3\} \notin \mathcal{P}(A)$.

Refer to study guide, pp 40, 45.

Question 8

Alternative 4

*Let's first discuss **proper subsets**:*

*If we throw away **one** or **more** elements of some set G , then the resulting set (let's call it F) is a proper subset of G . So every element of F is also an element of G , but set F has less elements than set G . We may write $F \subset G$.*

$$A \cup C = \{1, \{3\}\} \cup \{2, \{3\}\} = \{1, 2, \{3\}\}.$$

The subsets of $A \cup C$ are: $\{1\}$, $\{2\}$, $\{\{3\}\}$, $\{1, 2\}$, $\{1, \{3\}\}$, $\{2, \{3\}\}$ and $\{1, 2, \{3\}\}$.

By the definition of proper subsets, the proper subsets of $A \cup C$ are:

$\{1\}$, $\{2\}$, $\{\{3\}\}$, $\{1, 2\}$, $\{1, \{3\}\}$ and $\{2, \{3\}\}$.

We consider the sets in the different alternatives:

Alternative 1: $\{\{1, 2, \{3\}\}\}$ is not a proper subset of $A \cup C$, but it is a set containing $A \cup C$ as an element. $\{\{1, 2, \{3\}\}\} \not\subset \{1, 2, \{3\}\}$

Alternative 2: It is true that $\{1, 2, \{3\}\} \subseteq \{1, 2, \{3\}\}$ but $\{1, 2, \{3\}\} \not\subset \{1, 2, \{3\}\}$ because a proper subset of a set has at least one element less than the set.

Alternative 3: $\{\{1, 2\}, \{3\}\} \not\subset \{1, 2, \{3\}\}$ because $\{1, 2\} \notin \{1, 2, \{3\}\}$.

Alternative 4: $\{\{3\}\} \subset \{1, 2, \{3\}\}$ because the only element of $\{\{3\}\}$ namely $\{3\}$, is also an element of $\{1, 2, \{3\}\}$, ie $\{3\} \in \{1, 2, \{3\}\}$ and the number of elements in $\{\{3\}\}$ is less than the number of elements in $A \cup C = \{1, 2, \{3\}\}$. This alternative should be selected.

Refer to study guide, pp 40-41, 44, 45; Tutorial Letter 102, p 9.

Question 9**Alternative 3**

If A is a set, A' denotes the complement of A (the set that contains those elements of some universal set U that are not in A , i.e. $U - A = A'$).

$$A' = U - A = \{1, 2, \{3\}, \{1, 3\}\} - \{1, \{3\}\} = \{2, \{1, 3\}\}.$$

We look at each alternative below:

Alternative 1:

$$B \cup C = \{2, \{1, 3\}\} \cup \{2, \{3\}\} = \{2, \{3\}, \{1, 3\}\} \neq \{2, \{1, 3\}\}.$$

Alternative 2:

$$B + C = \{2, \{1, 3\}\} + \{2, \{3\}\} = \{\{3\}, \{1, 3\}\} \neq \{2, \{1, 3\}\}.$$

Alternative 3:

$$B = \{2, \{1, 3\}\} = A' = \{2, \{1, 3\}\}. \text{ This alternative should therefore be selected.}$$

Alternative 4:

Refer to the definition of set difference in question 5.

$$(U - B) - C = (\{1, 2, \{3\}, \{1, 3\}\} - \{2, \{1, 3\}\}) - \{2, \{3\}\} = \{1\} \neq \{2, \{1, 3\}\}.$$

Refer to study guide, pp 42 - 44.

Question 10**Alternative 1**

The cardinality of any set, is the number of elements in that set.

$U + C = \{1, 2, \{3\}, \{1, 3\}\} + \{2, \{3\}\} = \{1, \{1, 3\}\}$, which has the elements 1 and $\{1, 3\}$ belonging to it. The cardinality of $U + C = \{1, \{1, 3\}\}$ therefore is 2. Alternative 1 should therefore be selected. (Refer to the definition of set symmetric difference in question 6).

Refer to study guide, pp 43-44.

EXAMINATION INFORMATION – PLEASE READ CAREFULLY.

The examination paper will test you on **study units 3 – 10** of the study guide, and the material in **all** the tutorial matter. **Study units 1 – 2** of the study guide provide background material.

Bear in mind that the assignment questions do not cover all the work that we test in the examination paper. You have to prepare **all the work** prescribed above.

It will be expected of you to write down the answers to all the examination questions and provide proofs where required.

How to prepare:

Cover all the study units in the study guide thoroughly and **test yourself** by doing the activities before you look at the solutions in tutorial letter 102.

Work through the 2009 examination paper along with the solutions provided in tutorial letter 103. It is very important that you also do all the self-assessment questions provided in assignments 02, 03 and 04, and take note of the hints provided since these hints will help you to avoid making common errors in the examination. Make sure that you understand the model solutions to **all assignment questions** provided in tutorial letters.

PLEASE NOTE: As from 2017 the exam paper will be a fill-in paper. We will add a practice fill-in paper with solutions to *Additional Resources* on myUnisa and notify you via your myLife email address as soon as it has been added. Please try and do the paper before you look at the solutions. The exam papers from here onwards will have a similar format to the practice paper. Please note that no questions will be repeated from the practice paper or previous papers in the examination, but questions will be similar to the 2009 paper, the practice paper and previous papers on myUnisa. The marks allocated to sections may differ, but sections will follow in the same order. Also please note that we DO NOT supply answers to old exam papers on myUnisa. It is to your advantage to do the questions yourself, and discuss questions that you have problems with, with your e-tutor on the e-tutor site.

Take note of the **structure and notation of solutions** provided in all tutorial matter. For example, when a proof is required and connectives such as “**iff**” or “**if...then**” are left out, the proof is not convincing. Also, **symbols** (e.g. “ \cap ”) should be used as **connectives** for **sets** (e.g. $Y \cap W$), and **words** (e.g. “and”) should be used as **connectives** in **sentences** (e.g. $x \in Y$ **and** $x \in W$). Some proofs should start with the word “**assume**” and then logic reasoning should follow. These and other notation issues are mentioned in the hints provided in the assignments.

Additional advice:

Venn diagrams: Draw your diagrams in stages as described on page 51 of the study guide. Also remember to draw the sets within the context of a universal set, name the sets and provide subscripts for the diagrams.

Relations and functions: You need to **apply** the definitions, not merely give them. For example, when you want to prove that a relation is functional and you only write the statement “for every x there is only one y ”, you will receive no marks since it is neither a properly formulated definition nor a proof. **Relate each answer to the actual definition of the specific relation or function** given in the question.

When solving problems, don't forget the useful **shortcut notations** that help you to express an English sentence in precise mathematical notation. For example, an even number can be expressed as $2k$, an odd number as $2k + 1$, a multiple of three as $3k$ and so on, and two consecutive numbers could be k and $k + 1$, with $k \in \mathbb{Z}$.

Truth tables: Use the notations T and F for *true* and *false*, and when three declarative statements (e.g. p , q and r) are involved, use the same order for the combinations of T and F that is used in the table on page 144 of the study guide.

Mathematical proofs and counterexamples: If you are required to provide a mathematical proof and you give an example instead of a general proof, you will receive no marks for that answer. In other words, **never** attempt to prove that something is true by using an example. However, you **should use a counterexample** to prove that something is not true. If you are required to give a **counterexample** and you attempt to give some mathematical proof instead of a **counterexample**, you will receive no marks for that answer.

Note: Read all the hints provided in tutorial letter 101. These hints will help you to avoid making general mistakes in the examination.