Tutorial letter 202/1/2017

Theoretical Computer Science 1

COS1501

Semester 1

School of Computing

Discussion of Assignment 02

Dear Student,

The solutions to the second assignment MCQ questions are discussed in this tutorial letter. A discussion of the self-assessment questions is provided in tutorial letter 102. In the exam it will be expected of you to write out the answers to all the questions and provide proofs where required, thus it is very important that you also do all the self-assessment questions. Take note of the hints provided in tutorial letter 101 since these hints will help you to avoid making common errors in the exam. Please work through the practice exam under Additional resources on MyUnisa, to get used to the format of the fill-in paper that you will be writing in the exam. (This has been explained in the tutorial letter containing the discussion of assignment 1).

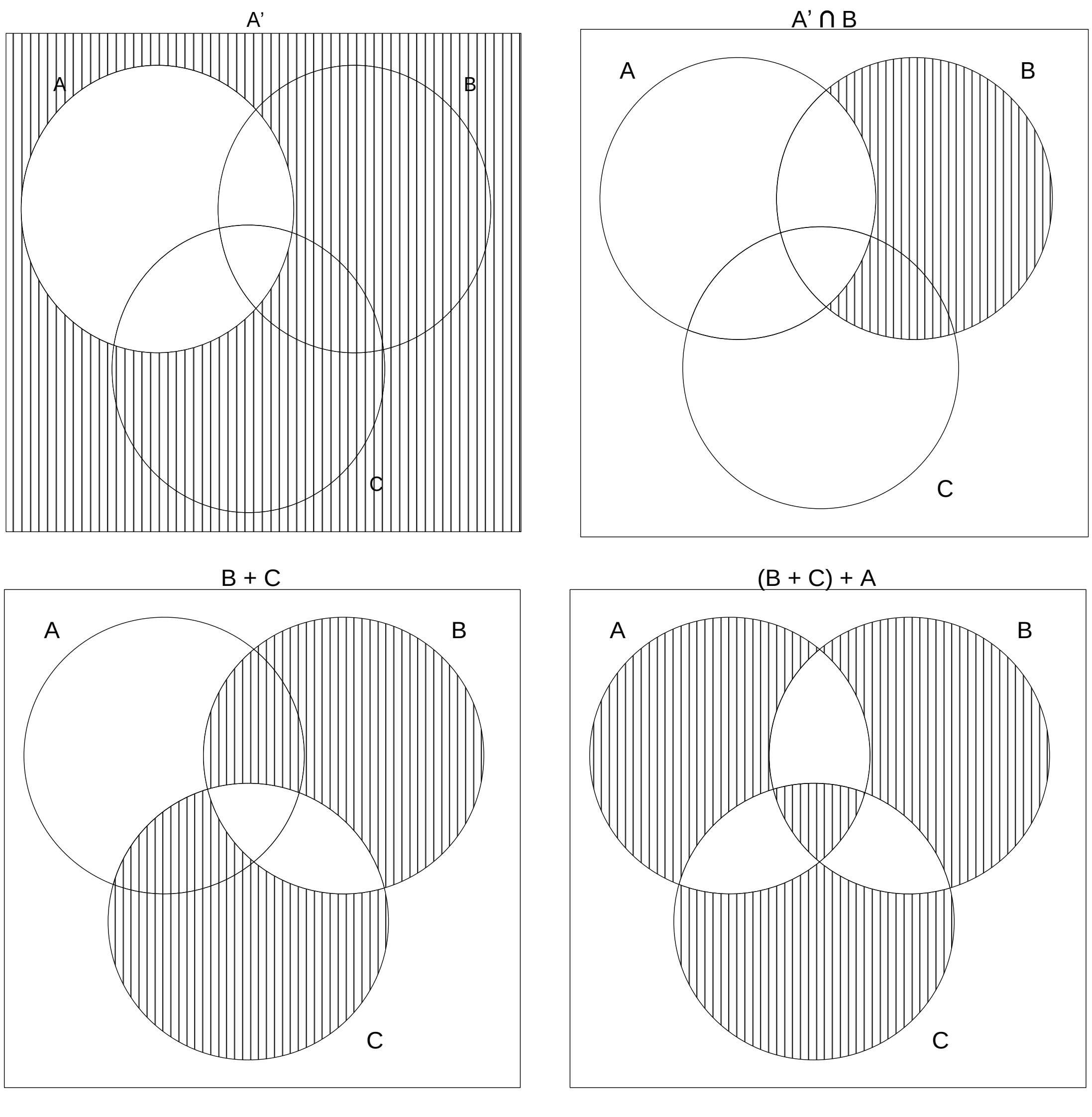
Regards,
COS1501 Team

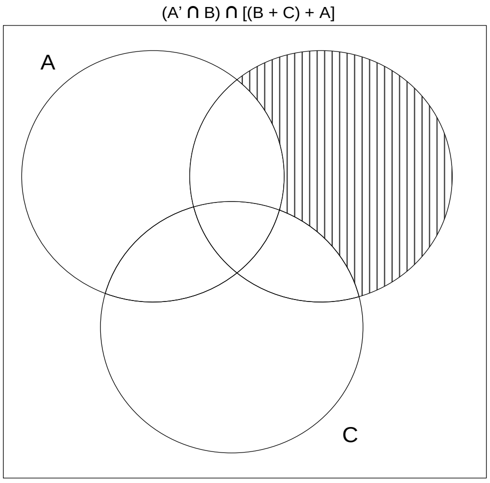
Discussion of assignment 02, semester 1

Question 1 **Alternative 2**

We determine the Venn diagram for the set $(A' \cap B) \cap [(B + C) + A]$ step by step:

Answer:





Refer to study guide, pp 50, 51.

Question 2 **Alternative 3**

Let A, B and C be subsets of a universal set U = {1, 2, 3}.

The statement A – (B ∩ C) = (A – B) ∪ C is NOT an identity. Which of the following sets A, B and C can be used in a *counterexample* to prove that the given statement is not an identity?

1. A = {1}, B = {3} & C = { }
2. A = {1, 2}, B = { } & C = {2}
3. A = {1}, B = {1, 2} & C = {2, 3}
4. A = {1}, B = {2, 3} & C = {1}

Discussion

Given: A, B, C ⊆ U with U = {1, 2, 3}, and A – (B ∩ C) = (A – B) ∪ C is not an identity.

*We do **not** start our counterexample solution with A – (B ∩ C) ≠ (A – B) ∪ C.*

***First** we **determine A – (B ∩ C)**, **then** we **determine (A – B) ∪ C** by using the sets provided in the different alternatives, then we **compare the answers** and get to a conclusion.*

We consider the different alternatives:

1. We use the sets A = {1}, B = {3} & C = { } to determine A – (B ∩ C) and (A – B) ∪ C and then we compare the answers.

A – (B ∩ C)	= {1} – ({3} ∩ { })
	= {1} – { }
	= {1}
(A – B) ∪ C	= ({1} – {3}) ∪ { }

$= \{1\} \cup \{\ \}$
$= \{1\}$

Thus $(A - (B \cap C) = (A - B) \cup C$.

2. We use the sets $A = \{1, 2\}$, $B = \{\ \}$ & $C = \{2\}$ to determine $A - (B \cap C)$ and $(A - B) \cup C$ and then we compare the answers.

$A - (B \cap C) = \{1, 2\} - (\{\ \} \cap \{2\})$
$= \{1, 2\} - \{\ \}$
$= \{1, 2\}$
$(A - B) \cup C = (\{1, 2\} - \{\ \}) \cup \{2\}$
$= \{1, 2\} \cup \{2\}$
$= \{1, 2\}$

Thus $A - (B \cap C) = (A - B) \cup C$.

3. We use the sets $A = \{1\}$, $B = \{1, 2\}$ & $C = \{2, 3\}$ to determine $A - (B \cap C)$ and $(A - B) \cup C$ and then we compare the answers.

$A - (B \cap C) = \{1\} - (\{1, 2\} \cap \{2, 3\})$
$= \{1\} - \{2\}$
$= \{1\}$
$(A - B) \cup C = (\{1\} - \{1, 2\}) \cup \{2, 3\}$
$= \{\ \} \cup \{2, 3\}$
$= \{2, 3\}$

Clearly $\{1\} \neq \{2, 3\}$ thus $A - (B \cap C) \neq (A - B) \cup C$.

4. We use the sets $A = \{1\}$, $B = \{2, 3\}$ & $C = \{1\}$ to determine $A - (B \cap C)$ and $(A - B) \cup C$ and then we compare the answers.

$A - (B \cap C) = \{1\} - (\{2, 3\} \cap \{1\})$
$= \{1\} - \{\ \}$
$= \{1\}$
$(A - B) \cup C = (\{1\} - \{2, 3\}) \cup \{1\}$
$= \{1\} \cup \{1\}$
$= \{1\}$

Thus $A - (B \cap C) = (A - B) \cup C$.

Alternatives 1, 2 and 4 do not provide counterexamples, but a counterexample is provided in alternative 3, thus this alternative should be selected.

Refer to study guide, pp 41-44, 60,61.

Question 3 **Alternative 3**

We want to prove that for all A, B, C $\subseteq$ U,

$(A \cup C) - (C \cap B) = (A - C) \cup [(A - B) \cup (C - B)]$ is an identity.

Discussion:
In the proof we apply the definitions for union, intersection, difference and complement of sets. The notation should be correct and all the necessary steps should appear in the proof.
We complete the proof below. You will see that step 4 and step 6 of the proof agrees with alternative 3.

$z \in (A \cup C) - (C \cap B)$

iff $(z \in A$ or $z \in C)$ and $(z \notin (C \cap B))$

iff $(z \in A$ or $z \in C)$ and $(z \notin C$ or $z \notin B)$

iff $(z \in A$ or $z \in C)$ and $(z \in C'$ or $z \in B')$ Step 4

iff $[(z \in A$ or $z \in C)$ and $(z \in C')]$ or $[(z \in A$ or $z \in C)$ and $(z \in B')]$

iff $[(z \in A$ and $z \in C')$ or $(z \in C$ and $z \in C')]$ or
$[(z \in A$ and $z \in B')$ or $(z \in C$ and $z \in B')]$ Step 6

iff $[(z \in A$ and $z \in C')]$ or $[(z \in A$ and $z \in B')$ or $(z \in C$ and $z \in B')]$

(note that $(z \in C$ and $z \in C')$ is impossible, so we removed it in step 7)

iff $[(z \in A - C)]$ or $[(z \in (A - B)$ or $(z \in C - B)]$

iff $z \in (A - C) \cup [(A - B) \cup (C - B)]$

Refer to study guide, pp 41-43, 55-57.

Question 4 **Alternative 1**

There are 35 grade 7 pupils in a class.

Of these pupils,

26 own cell phones,
10 own laptops,
7 own iPads.
(Pupils do not necessarily own only one device.)
Furthermore,
4 own cell phones and iPads,
4 own laptops and iPads,
3 own cell phones and laptops.

How many pupils own cell phones, iPads and laptops?

1. 3
2. 7
3. 11
4. 35

Solution:

$|U| = 35$, $|C| = 26$, $|L| = 10$, $|P| = 7$, (U = universal set; C = cell phone; L = laptop; P = iPad)

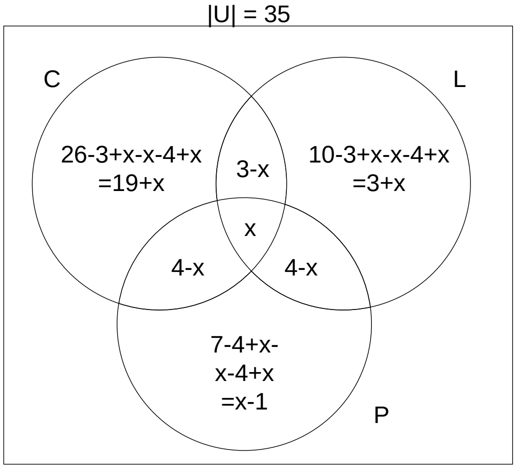
$|C \cap P| = 4$,

$|L \cap P| = 4$,

$|C \cap L| = 3$,

Let ***x*** pupils own cell phones, iPads and laptops, ie $|R \cap L \cap P| = x$.

Now we can fill in the various regions in the following Venn diagram.
We initially fill in x for $|R \cap L \cap P|$



$|U| = 35 = 19 + x + (3 – x) + x + (4 – x) + (4 – x) + 3 + x + x - 1$

ie $35 = 32 + x$

ie $x = 3$, ie 3 pupils own cell phones, laptops and iPads.

From the argument provided we can deduce that alternative 1 should be selected.

Refer to study guide, pp 63 – 66.

Let T be a relation from A = {0, 1, 2, 3, 4} to B = {0, 2, 4, 6, 8} such that

(a, b) $\in$T iff 2a < b. (A, B $\subseteq$ U = $\mathbb{Z}$.)

(Hint: Write down all the elements of T. For example, if 1 $\in$ A and 4 $\in$ B then 2 $\cdot$ a = 2 $\cdot$ 1 = 2, and 2 < 4, thus (1, 4) $\in$ T.)

Question 5 **Alternative 4**

Which one of the following alternatives provides only elements belonging to T?

1. (0, 0), (0, 4), (0, 8)
2. (1, 2), (1, 4), (1, 6)
3. (2, 6), (3, 8), (4, 16)
4. (0, 2), (1, 6), (3, 8)

We first write down all the ordered pairs in T:
T = {(0, 2), (0, 4), (0, 6), (0, 8), (1, 4), (1, 6), (1, 8), (2, 6), (2, 8), (3, 8)}

We consider ordered pairs provided in the different alternatives:

1. Is (0, 0) $\in$ T? No, **2(0) $\nless$ 0**. Thus (0, 0) $\notin$ T. The ordered pairs in this alternative does not provide only elements belonging to T.

2. Is (1, 2) $\in$ T? No, **2(1) $\nless$ 2.** Thus (1, 2) $\notin$ T. The ordered pairs in this alternative does not provide only elements belonging to T.

3. Is (2, 6) $\in$ T? Yes, **2(2) < 6** thus (2, 6) $\in$ T.
   Is (3, 8) $\in$ T? Yes, **2(3) < 8** thus (3, 8) $\in$ T.
   Is (4, 16) $\in$ T? No, ordered pair (4, 16) is not in T, because 16 $\notin$ B = {0, 2, 4, 6, 8}.
   Since (4, 16) $\notin$ T, the ordered pairs in this alternative does not provide only elements belonging to T.

4. Is (0, 2) $\in$ T? Yes, **2(0) < 2** thus (0, 2) $\in$ T.
   Is (1, 6) $\in$ T? Yes, **2(1) < 6** thus (1, 6) $\in$ T.
   Is (3, 8) $\in$ T? Yes, **2(3) < 8** thus (3, 8) $\in$ T.

Thus all the given ordered pairs in this alternative are elements of T.

From the arguments provided we can deduce that alternative 4 should be selected.

Refer to study guide, p 73.

Question 6 **Alternative 2**

Which one of the following statements regarding the relation T is false?

1. T is transitive.
2. T satisfies trichotomy.
3. T is antisymmetric.
4. T is irreflexive.

Discussion

We first provide definitions using some relation R on A:

Irreflexive:

We ask the question: Is it true that for all x $\in$ A we have that (x, x) $\notin$ R?

*(For **no** element x $\in$ A we have that (x, x) $\in$ R.)*

Antisymmetric: We ask the question: Is it true that for all x, y $\in$ A,
if x ≠ y and (x, y) $\in$ R then (y, x) $\notin$ R?

Transitive: R is transitive iff it has the property that for all x, y, z $\in$ A,
whenever (x, y) $\in$ R and (y, z) $\in$ R, then (x, z) $\in$ R.
We ask the question: Is it true that for all x, y, z $\in$ A, if (x, y) $\in$ R and (y, z) $\in$ R, then (x, z) $\in$ R?

Trichotomy: A relation R on A satisfies the requirement for trichotomy iff, for every x and y chosen from A such that x ≠ y, we have that x and y are comparable,
i.e. for all x, y $\in$ A such that x ≠ y, x R y or y R x (i.e. (x, y) $\in$ R or (y, x) $\in$ R).

T is a relation from A = {0, 1, 2, 3, 4} to B = {0, 2, 4, 6, 8} such that

(a, b) $\in$T iff 2a < b. (A, B $\subseteq$ U = $\mathbb{Z}$.)

We provide the set T:

T = {(0, 2), (0, 4), (0, 6), (0, 8), (1, 4), (1, 6), (1, 8), (2, 6), (2, 8), (3, 8)}

We consider the different alternatives:

1. For transitivity:
   (0, **2**), (**2**, 6) $\in$ T implies that (0, 6) must be in T, which is indeed true: (0, 6) $\in$ T;
   (0, **2**), (**2**, 8) $\in$ T implies that (0, 8) must be in T, which is indeed true: (0, 8) $\in$ T.
   There are no other pairs to consider. From the above we deduce that T is a transitive relation.

2. Trichotomy:
   We provide a counterexample to prove that T *does not satisfy trichotomy*:
   (2, 4) $\notin$ T and (4, 2) $\notin$ T.

3. Antisymmetry:

From the ordered pairs given in relation T above it is clear that T is antisymmetric. Whenever (x, y) ∈ T it is true that (y, x) ∉ T.

4. Irreflexive:

From the ordered pairs given in relation T above it is clear that T is irreflexive. For all x ∈ A it is true that (x, x) ∉ T.

From the arguments provided it is clear that alternative 2 should be selected.

Refer to study guide, pp 75-78.

Consider the following relation on set B = {1, {2}, a, {b}, c}:

P = {(1, a), ({2}, {b}), ({b}, 1), (c, {2})}

R = {(a, {b}), ({b}, {2}), (1, {2})}

Question 7 **Alternative 2**

Which one of the following alternatives represents the domain of P (dom(P))?

1. {1, 2, b, c}
2. {1, {2}, {b}, c}
3. {1, 2, a, b}
4. {1, {2}, a, {b}}

Let's first look at the definition for the domain of a relation:
Given a relation T from X to Y, the domain of T is defined by:
dom(T) = {x | for some y ∈ Y, (x, y) ∈ R} (i.e. the set of first co-ordinates)

We look at relation P = {(**1**, a), (**{2},** {b}), (**{b},** 1), (**c**, {2})} with all the first coordinates highlighted.

Dom(P) = {1, {2}, {b}, c}. It is clear that alternative 2 should be selected.

Question 8 **Alternative 1**

Which one of the following relations represents the composition relation R ○ R (ie R; R)?

1. {(a, {2})}
2. {(a, {b}), ({b}, 2)}
3. {(a, {b}), ({b}, {2}), (a, {2})}
4. {({b}, 1)}

Discussion
We first look at the definition of a composition relation:
*Given relation **R** from B to B and R from B to B, the composition of **R** followed by R*

*(R ○**R** or **R**; R) is the relation from B to B defined by*

*R ○**R** = **R**; R = {(m, o) | there is some n ∈ B such that (m, n) ∈ **R** and (n, o) ∈ R}.*

*(R and **R** is exactly the same relation, but for the purpose of our explanations we make the subtle distinction.)*

R = {(a, {b}), ({b}, {2}), (1, {2})} is defined on **B = {1, {2}, a, {b}, c}:**

To determine **R**; *R* we start with the pair (a, **{b}**) of **R**, and then we look for a pair in *R* that has as first co-ordinate a **{b},** and then see where it takes us.
Link (a, **{b}**) of **R** with (**{b}**, {2}) of *R*, then (a, {2}) ∈ **R**; *R*.
No other pairs can be linked, so
R ○ **R** = {(a, {2})}.

From the above one can conclude that alternative 1 should be selected. Clearly no other alternative is suitable.

Refer to study guide, pp 79, 108, 109.

Question 9 **Alternative 4**

Which one of the following relations represents the composition relation R ○ P (ie P; R)?

1. {(a, {b}), ({b}, 1), ({b}, {2}), ({2}, {b}), (1, {2})}
2. {(1, a), (a, {b}), ({b}, 1), ({b}, {2}), ({2}, {b}), (1, {2})}
3. {(a, 1), ({b}, {b}), (1, {b})}
4. {(1, {b}), ({2}, {2}), ({b}, {2})}

P = {(1, a), ({2}, {b}), ({b}, 1), (c, {2})} and R = {(a, {b}), ({b}, {2}), (1, {2})} is defined on set

B = {1, {2}, a, {b}, c}.

To determine P; R we start with the pair (1, **a**) of P, and then we look for a pair in R that has as first co-ordinate an **a**, and then see where it takes us.

Link (1, **a**) of P with (**a**, {b}) of R, then (1, {b}) ∈ P; R,
link ({2}, **{b}**) of P with (**{b}**, {2}) of R, then ({2}, {2}) ∈ P; R and
link ({b}, **1**) of P with (**1**, {2}) of R, then ({b}, {2}) ∈ P; R,

No other pairs can be linked, so R ○ P = {(1, {b}), ({2}, {2}), ({b}, {2})}. Alternative 4 is therefore the correct alternative.

Refer to study guide, pp 108, 109

Question 10 **Alternative 3**

The relation R is irreflexive and not transitive. Which ordered pair(s) should be included in R for it to remain irreflexive, but become transitive?

1. (1, {b})
2. ({2}, {b}), ({2}, {2}), (a, {2})
3. (a, {2})
4. (a, {2}), ({b}, {b})

Discussion

A relation R $\subseteq$ A $\times$ A is transitive *iff R has the property that for all x, y, z $\in$ A, whenever (x, y) $\in$ R and (y, z) $\in$ R, then (x, z) $\in$ R. Let's say the y in the ordered pairs (x, y) and (y, z) plays the role of an "intermediary".*

R = {(a, {b}), ({b}, {2}), (1, {2})} is defined on B = {1, {2}, a, {b}, c}. We identify the ordered pairs that have an "intermediary". We name the new transitive relation 'R_1'.

For transitivity:

(a, {b}), ({b}, {2}) $\in$ R_1 thus (a, {2}) in R should also be an element of R_1.

There are no more ordered pairs in R with an 'intermediary'.

We now have R_1 = {(a, {b}, **(a, {2})**, ({b}, {2}), (1, {2})}.

The ordered pair in bold is therefore the only ordered pair that has to be added to relation R in order to make it a transitive relation. Relation R_1 = {(a, {b}, **(a, {2})**, ({b}, {2}), (1, {2})} is also still an irreflexive relation. Alternative 3 is therefore the correct alternative.

Refer to study guide p 77.

Suppose U = {1, {1}, 2, {2}, {1, {2}}, {{1}, 2}} is a universal set with the subset A = {1, {1}, 2, {2}}.

Let R = {(1, {1}), (1, {2}), (1, 2), (2, {1}), (2, {2}), ({1}, {2})} be a relation on A.

Answer questions 11 & 12 by using the given sets A, U and the relation R.

Question 11 **Alternative 3**

Which one of the following statements regarding relation R is true?

1. R is a weak partial order.
2. R does not satisfy trichotomy.
3. R is a strict total (linear) order.
4. R is an equivalence relation.

Discussion:
The relations mentioned in alternatives 1-4 have different properties:
A weak partial order is reflexive, antisymmetric and transitive.
A relation R on A satisfies the requirement for trichotomy iff, for every x and y chosen from A such that x ≠ y, we have that x and y are comparable, i.e. for all x, y ∈ A such that x ≠ y, x R y or y R x (ie (x, y) ∈ R or (y, x) ∈ R).
A strict total (linear) order is irreflexive, antisymmetric and transitive, and satisfies trichotomy.
An equivalence relation is reflexive, symmetric and transitive.

We will write down what we can determine about relation R, by investigating the properties stated above:

- R is not reflexive. We give a counterexample: (1, 1) ∉ R.
- R is antisymmetric, ie whenever (x, y) ∈ R it is true that (y, x) ∉ R.
- R is transitive. See the discussion in Question 10. Whenever (x, y) ∈ R and (y, z) ∈ R, then it is true that (x, z) ∈ R.
- R does indeed satisfy trichotomy.

We now look at the different alternatives:

1. A weak partial order is reflexive, antisymmetric and transitive: R is not reflexive. R is therefore not a weak partial order.

2. R does indeed satisfy trichotomy. If you do not agree, read the definition of trichotomy again. This is therefore not the correct alternative to select.

3. A strict total order is irreflexive, antisymmetric and transitive, and satisfies trichotomy: R has all these properties, and is therefore a strict total order. This alternative should therefore be selected.

4. An equivalence relation is reflexive, symmetric and transitive: R is not reflexive, so it is not an equivalence relation.

Refer to study guide, pp 84 - 92.

12 **Alternative 3**

Which one of the following sets is a partition S of U?

1. {**{**1, {1}, 2, {2}**}**, **{**2, {1, {2}}**}**, **{**{1}, 2**}**}
2. {**{**1**}**, **{**2**}**, **{**1, {2}**}**, **{**{1}, 2**}**}
3. {**{**1, {1, {2}}**}**, **{**{1}, 2**}**, **{**{2}, {{1}, 2}**}**}
4. {**{**1, {1}, 2, {2}**}**, **{**1, 2**}**}

(A partition of the given set U can be defined as a set S = {S_1, S_2, S_3, …}. The members of S are subsets of U (each set S_i is called a part of S) such that

a. for all i, $S_i \neq \emptyset$ (that is, each part is nonempty),

b. for all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$ (that is, different parts have nothing in common), and

c. $S_1 \cup S_2 \cup S_3 \cup \ldots = U$ (that is, every element in U is in some part S_i).

It is possible to form different partitions of U depending on which subsets of U are used to be elements of S.

*Test whether the sets given in the different alternatives meet all the criteria given in the above definition. Note that the **elements** of a **partition** of U must be **subsets** of **U**. Subsets of U are formed when you keep the outside brackets of U and then throw away all, some or no element of U. For example, keep the outside brackets of U, then throw away the element 1, then the subset {{1}, 2, {2}, {1, {2}}, {{1}, 2}} of U is formed. Refer to study guide, pp 94, 95.)*

We consider the sets provided in the different alternatives:

1. Let P = {**{**1, {1}, 2, {2}**}**, **{**2, {1, {2}}**}**, **{**{1}, 2**}**} (say).
We test whether P is a partition of U = {1, {1}, 2, {2}, {1, {2}}, {{1}, 2}}.

{1, {1}, 2, {2}**}**, **{**2, {1, {2}}**}** and **{**{1}, 2**}** are three **non-empty** subsets of U,
{1, {1}, 2, {2}**}** ∩ **{**2, {1, {2}}**}** ∩ **{**2, {1, {2}}**}** = {2} ≠ ∅. P is therefore not a partition of U, as the intersection between the different subsets of P should be the empty set.

2. Let P = {**{**1**}**, **{**2**}**, **{**1, {2}**}**, **{**{1}, 2**}**} (say).
We test whether P is a partition of U = {1, {1}, 2, {2}, {1, {2}}, {{1}, 2}}.

{1**}**, **{**2**}**, **{**1, {2}**}** and **{**{1}, 2**}** are all **non-empty** subsets of U;
{1**}** ∩ **{**2**}** ∩ **{**1, {2}**}** ∩ **{**{1}, 2**}** = ∅,
but **{**1**}** ∪ **{**2**}** ∪ **{**1, {2}**}** ∪ **{**{1}, 2**}** = {1, 2, {1}, {2}} ≠U. Therefore P is not a partition of U.

3. Let P = {**{**1, {1, {2}}**}**, **{**{1}, 2**}**, **{**{2}, {{1}, 2}**}**} (say).
We test whether P is a partition of U = {1, {1}, 2, {2}, {1, {2}}, {{1}, 2}}.

{1, {1, {2}}**}**, **{**{1}, 2**}** and **{**{2}, {{1}, 2}**}** are three **non-empty** subsets of U,

{1, {1, {2}}**}** ∩ **{**{1}, 2**}** ∩ **{**{2}, {{1}, 2}**}** = ∅ and

{1, {1, {2}}**}** ∪ **{**{1}, 2**}** ∪ **{**{2}, {{1}, 2}**}** = U.

Because P has the above properties, it is a partition of U.

4. Let P = **{{**1, {1}, 2, {2}**}**, **{**1**,** 2**}}** (say).
We test whether P is a partition of U = {1, {1}, 2, {2}, {1, {2}}, {{1}, 2}}.

{1, {1}, 2, {2}**}** and **{**1**,** 2**}** are two **non-empty** subsets of U,

but **{{**1, {1}, 2, {2}**}** ∩ **{**1**,** 2**}}** = {1, 2} ≠ ∅. Therefore P is not a partition of U.

From the arguments provided, it is clear that alternative 3 should be selected.

Refer to study guide, pp 94, 95.

Let R be the relation on $\mathbb{Z}^+$ (the set of positive integers) defined by

(x, y) ∈ R iff 3x + y > 6.

Answer questions 13 to 14 by using the given relation R.

Question 13 **Alternative 4**

Which one of the following is an ordered pair in R?
1. (-1, 10)
2. (1, 1)
3. (1, 3)
4. (3, 1)

Relation R on $\mathbb{Z}^+$ is defined by (x, y) ∈ R iff 3x + y > 6.
We consider the ordered pairs provided in the different alternatives:

1. R is a relation on $\mathbb{Z}^+$ but x = -1 ∉ $\mathbb{Z}^+$ so (-1, 10) is not an ordered pair in R.

2. Let x = 1 and y = 1 then

   3x + y = 3(1) + 1 ≯ 6.

   thus (1, 1) ∉ R.

3. Let x = 1 and y = 3 then

   3x + y = 3(1) + 3 = 6 ≯ 6.

thus $(1, 3) \notin R$.

4. Let $x = 3$ and $y = 1$ then
   $3(3) + 1 = 10 > 6$.
   thus $(3, 1) \in R$. This is the correct alternative to choose.

From the arguments provided we can deduce that alternative 4 should be selected.

Refer to study guide, pp 71-73.

Question 14 **Alternative 1**

Which of the following sets of ordered pairs can be used together in a counterexample to prove that R is not symmetric.

1. (3, 1) & (1, 3)
2. (-2, 6) & (6, -2)
3. (1, 7) & (7, 1)
4. (0, 7) & (7, 0)

Discussion
Refer to the definition of symmetry provided previously in this tutorial letter.

Alternative 1:

Let $x = 3$ and $y = 1$, then $3x + y = 3(3) + 1 = 10 > 6$, ie $(3, 1) \in R$;

Let $x = 1$ and $y = 3$, then $3(1) + 3 = 6 \not> 6$ ie $(1, 3) \notin R$. This proves that R is not symmetric.

Alternative 2:

$(-2, 6) \notin R$ and $(6, -2) \notin R$ because R is defined on $\mathbb{Z}^+$ and $-2 \notin \mathbb{Z}^+$.

Alternative 3:

Let $x = 7$ and $y = 1$, then $3x + y = 3(7) + 1 = 22 > 6$, ie $(7, 1) \in R$;

Let $x = 1$ and $y = 7$, then $3(1) + 7 = 10 > 6$ ie $(1, 7) \in R$. These ordered pairs are both in R. Thus this does not prove that R is not symmetric. Note, one set of ordered pairs does not prove that a relation is symmetric.

Alternative 4:

$(0, 7) \notin R$ and $(7, 0) \notin R$ because R is defined on $\mathbb{Z}^+$ and $0 \notin \mathbb{Z}^+$.

From the above discussion it can be derived that alternative 1 should be selected.

Refer to study guide, p 76.

Question 15 **Alternative 2**

Which of the following sets is equal to the set:

$\{v \mid v \in \mathbb{Z}, v^2 + 2v - 35 < 0\}$

1. $\{v \mid v \in Z, -7 > v < 5\}$
2. $\{w \mid w \in Z, -7 < w < 5\}$
3. $\{v \mid v \in Z, -7 < v > 5\}$
4. $\{w \mid w \in Z, -7 > w > 5\}$

We simplify the given set:

$\{v \mid v \in \mathbb{Z}, v^2 + 2v - 35 < 0\}$
$= \{v \mid v \in \mathbb{Z}, (v - 5)(v + 7) < 0\}$
$= \{v \mid v \in \mathbb{Z}$, [$v - 5 < 0$ and $v + 7 > 0$] or [$v - 5 > 0$ and $v + 7 < 0$]$\}$
$= \{v \mid v \in \mathbb{Z}$, [$v < 5$ and $v > -7$] or [$v > 5$ and $v < -7$ (which is impossible)]$\}$
$= \{v \mid v \in \mathbb{Z}$, $v < 5$ and $v > -7\}$
$= \{v \mid v \in \mathbb{Z}, -7 < v < 5\}$
$= \{w \mid w \in \mathbb{Z}, -7 < w < 5\}$ we can substitute a variable by any letter. This proves that alternative 2 should be selected.

Note: A discussion of the self-assessment questions is provided in tutorial letter 102.