Please note:
Till October 2016, the COS1501 exam was a written exam, where you were supplied with an examination book in which to complete your exam. As from May 2017 the COS1501 exam will be a fill-in exam paper. In other words you will write all your answers in the fill-in exam paper that you will receive from the invigilator.

We supply you with a practice fill-in exam below. Please try the questions before you look at the solutions. Your e-tutor is available for any questions you may have regarding the solutions for specific questions.

Please note that questions will differ in the exam, and sections may not count exactly the same, but the exam will consist of questions similar to the questions in the old exam papers available on myUnisa. Please contact your e-tutor if you have difficulty with questions in old exam papers. If you post your effort for a question on the discussion forum of the e-tutor site, the e-tutor will assist you and give you comments. Please do not email the lecturers for solutions to old exam papers. It is to your advantage that you work out the solutions for yourself, and then discuss the questions that you had difficulty with, with your e-tutor.

Afrikaanse studente: U mag die vraestel in Afrikaans beantwoord.

This paper is a fill-in paper and consists of 16 pages plus an additional 4 pages for rough work (pp.17-20).

Instructions:

1. Answer all the questions in all 6 sections on the fill-in paper.
2. **Please do all rough work on the last four pages marked 'ROUGH WORK'.**
3. The mark for each question appears in brackets next to the question.

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SECTION 1
SETS AND RELATIONS (Multiple-Choice Questions)

Each question comprises 2 marks.
Circle the alternative that you think is the correct alternative to select.
<u>There is ONLY one correct alternative per question. If you circle more than one alternative, a zero mark will be awarded for that question.</u>
There is space at the end of the paper for rough work.

[16 marks]

Suppose U = {a, b, c, d, {b, c}, {a, b, c}} is a universal set with the following subsets:

A = {a, b}, B = {b, c, {b, c}} and C = {b, {a, b, c}}.

Answer questions 1.1 to 1.8 using the given sets.

Question 1.1
Which one of the following sets represents $A \cup B \cup C$?

1. {b}
2. {a, b, c}
3. {a, b, c, {b, c}, {a, b, c}}
4. {a, {b, c}, {a, b, c}}

Question 1.2
Which one of the following sets represents $A \cap B \cap C$?

1. $\emptyset$
2. {b}
3. {a, b, c}
4. {b, c, {b, c}}

Question 1.3
Which one of the following sets represents A – C?

1. $\emptyset$
2. {b}
3. {a}
4. {a, {a, b, c}}

[TURN OVER]

Question 1.4
Which one of the following sets represents U + B?

1. {b, c, {b, c}}
2. {a, {a, b, c}}
3. {a, d, {a, b, c}}
4. {a, b, c, d}

Question 1.5
Which one of the following sets represents C′?

1. {a, c, d, {b, c}}
2. {a, d, {b, c}}
3. {d, {b, c}}
4. {a, b, c, d}

Question 1.6
Which one of the following sets is a subset of $\mathcal{P}$(C)?

1. {b}
2. {{a, b, c}}
3. {{{a, b, c}}}
4. {b, {a, b, c}}

Question 1.7
Let P = {(b, c), (b, a), (c, a), (a, a)} be a relation on U. Which one of the following statements is **true**?

1. P is antisymmetric, but not transitive.
2. P is transitive, but not antisymmetric.
3. P is neither antisymmetric nor transitive.
4. P is antisymmetric and transitive.

Question 1.8

Let T = {(a, a), (b, b), (c, c), (a, c), (c, b)} be a relation on A ∪ B. Which one of the following alternatives provides (an) ordered pair(s) that must be added to T to make it an equivalence relation?

1. only (a, b)
2. only (a, b) & (c, a)
3. only (a, b), (c, a) & (b, c)
4. Not one of the above alternatives provides all the ordered pairs that should be added.

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SECTION 2
SET THEORY

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. [19 marks]

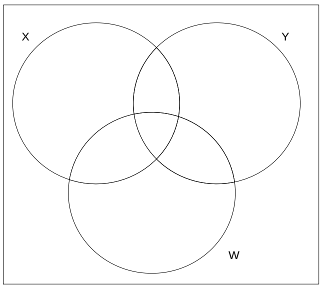
Question 2.1

Complete the following Venn diagrams to show whether or not, for all subsets X, Y and Z of a universal set U

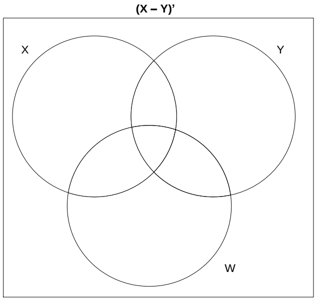
$$(X - Y)' \cap (X \cup W) = X' + (Y \cap W). \quad \textbf{(7)}$$

Left hand side: $(X - Y)' \cap (X \cup W)$

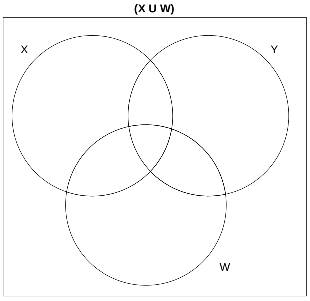
X - Y



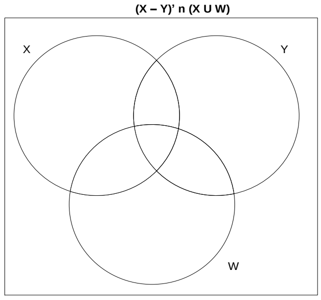
(X – Y)'



(X U W)

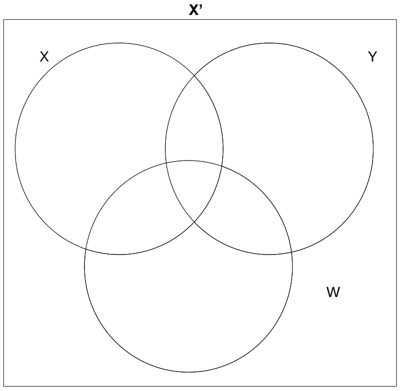


(X – Y)' n (X U W)

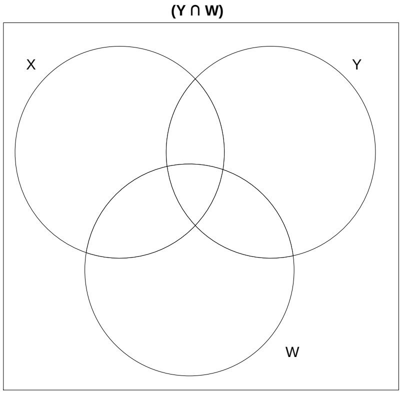


Right hand side: $X' + (Y \cap W)$.

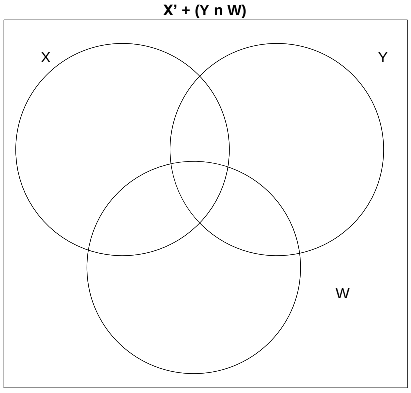
X'



(Y ∩ W)



X' + (Y n W)



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Question 2.2

Prove without using Venn diagrams, that

$X \cap (Y' - W) = X \cap (Y' \cap W')$ for all subsets X, Y and W of a universal set U. **(4)**

$x \in X \cap (Y' - W)$
iff
iff
iff
iff
iff

Question 2.3

Let U = {1, 2, 3, 4} be a universal set with subsets X = {1, 2} and Y = {3, 4} and W = {2, 4}.
Use the sets X, Y and W to show that:

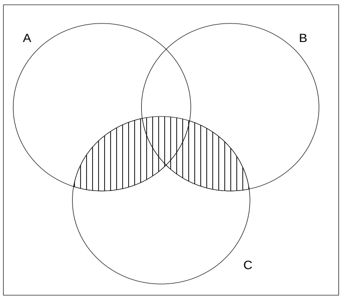
$$[X \cup (Y \cap (W' + X'))] = [(X \cup (Y \cap W)].$$ **(4)**

***Note**: Do not use Venn diagrams in your answer.*

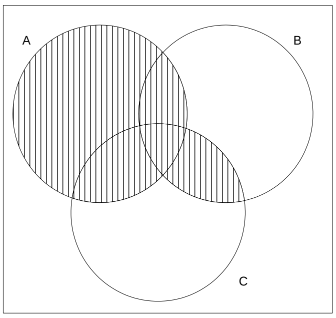
Question 2.4

From the given Venn diagram representations of sets A, B and C, write down the corresponding expressions:

a) Shaded area in the Venn diagram below represents: **(2)**



b) The shaded area in the Venn diagram below represents: **(2)**



SECTION 3
RELATIONS AND FUNCTIONS

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. [26 marks]

Question 3.1

a) Which two properties does a bijective function have?
Note: Do not make use of specific examples in your answer. **(2)**

b)(i) Provide the definition of an injective function. **(2)**

b)(ii) Provide an injective function from {1, 2, 3} to {a, b, c, d, e}. **(2)**

c) Give an example of a relation T on set A = {3, 4, 5} that is a strict partial order. **(2)**

d) Give an example of a partition of set B = {0, 1}. Give justification why your example set is a partition. **(4)**

Question 3.2

Let R be a relation on $\mathbb{Z}$ defined by

$(x, y) \in R$ iff $y - x = 7m$, $m \in \mathbb{Z}$.

a) Prove that R is transitive.
(Note: Do not make use of specific examples in your proof.) **(3)**

b) Give one equivalence class of R. (Show elements in the set.) **(2)**

Question 3.3

Let f and g be functions on $\mathbb{Z}$ defined by

$(x, y) \in f$ iff $y = x^2 + 2x$

and

$(x, y) \in g$ iff $y = -x + 3$.

[TURN OVER]

a) (i) Use the definition for ran(g) to determine the range of g. **(3)**

(ii) Is g surjective? Justify your answer. **(2)**

b)(i) Determine $f \circ g(x)$. Show all the steps. **(3)**

b)(ii) Show that ordered pair (-3, 0) is not an ordered pair in function g. **(1)**

SECTION 4
OPERATIONS AND MATRICES

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. **[10 marks]**

Question 4.1

Consider the following matrices:

$$A = \begin{bmatrix} 5 & 2 & -4 \\ 3 & 6 & 1 \\ 2 & 0 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}.$$

Determine the matrix A · B. Show your calculations. **(3)**

Question 4.2

*	**a**	**b**	**c**
a	a		
b		b	
c			b

a) Complete the table by providing an example of a binary operation * defined on {a, b, c}, with element **a** as the identity element of *. **(2)**

[TURN OVER]

b) Determine (a*b)*c and a*(b*c). **(2)**

c) Considering your answer in (b) is it possible to decide whether * is associative? Justify your answer. **(1)**

Question 4.3

Use the table below to determine whether a*(b*(b*c)) = (c*a)*(a*b). Show all the steps and state as part of your and whether a*(b*(b*c)) = (c*a)*(a*b) is an identity.

*	**a**	**b**	**c**
a	a	c	a
b	b	b	c
c	c	c	b

(2)

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC

Write your answers in the space provided. There is space for rough work at the end of the fill-in paper. [18 marks]

Question 5.1

a) For each of the following statements, if you think the statement is true, circle T (for true), else circle F (for false):

(i)	$(p \vee \neg(\neg(\neg q)) \equiv (\neg q \wedge p)$	T	F
(ii)	$(\neg(\neg(\neg p)) \rightarrow q) \equiv (p \vee q)$	T	F
(iii)	$\neg (q \rightarrow p) \equiv (\neg p \rightarrow q)$	T	F

(3)

b)

(i) Complete the truth table below. Do NOT add any rows. (*Hint:* Complete the highlighted column last). **(2)**

p	**q**	**r**	**¬q**	**¬r**	**(¬r ∨ p)**	**q → (¬r ∨ p)**	**q → (¬r ∨ p) ↔ (p ∨ ¬q) ∧ r**	**(p ∨ ¬q)**	**(p ∨ ¬q) ∧ r**
T	T	F							
T	F	T							

(ii) Complete the truth table for the following statement. (*Hint:* Complete the highlighted column last). **(5)**

p	**r**	**¬p**	**¬r**	**(p ∨ r)**	**(¬r ∧ p)**	**(p ∨ r) ∧ (¬r ∧ p)**	**[(p ∨ r) ∧ (¬r ∧ p)] ↔(¬p → r)**	**(¬p → r)**
T	T							
T	F							
F	T							
F	F							

(ii) Is the expression in (ii) above a contradiction, tautology or neither? **(1)**

Question 5.2

Consider the statement: $\exists\ x \in Z^+, [(x - 1 \geq 0) \wedge (x + 6 < 8)]$.

a) Is the given statement true? Justify your answer. **(2)**

b) Simplify the negation statement given below so that the *not*-symbol (¬) does not occur to the left of any quantifier. The *not*-symbol may also not occur outside of any parentheses. **Show all the steps**. **(5)**

Negation: $\neg[\forall x \in \mathbb{Z}, [(x - 5 \leq -1) \vee (x^2 + 2x \neq 15)]]$.
≡
≡
≡

SECTION 6
MATHEMATICAL PROOFS

Write your answers in the space provided. There is space for rough work at the end of the paper. **[11 marks]**

Question 6.1

Provide a direct proof to show that, for all n $\in \mathbb{Z}$,

if $n^2 - 5n + 4 < 0$, then $n > 0$.

Note: Do not make use of specific examples in your proof. Show all the steps. **(4)**

Question 6.2

Write down the converse of the statement:

if $7x^3 - 4x + 8$ is even, then x is odd.

(Only write down the converse statement – do not proof anything). **(2)**

Question 6.3

Provide a proof by contrapositive to show that for all x $\in \mathbb{Z}$,
if $x^2 + 2x$ is odd, then $x - 1$ is even.
Note: Do not make use of specific examples in your proof. **(5)**

ROUGH WORK

ROUGH WORK

ROUGH WORK

ROUGH WORK