Tutorial letter 101/3/2018

NUMERICAL METHODS 1
COS2633

Semesters 1 & 2

Department of Mathematical Sciences

This tutorial letter contains important information about your module
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1 INTRODUCTION

Dear Student

Welcome to the COS2633 module in the Department of Mathematical Sciences at Unisa. We trust that you will find this module both interesting and rewarding.

Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible, but is also available on myUnisa.

1.1 myUnisa

You must be registered on myUnisa (http://my.unisa.ac.za) to be able to submit assignments online, gain access to the library functions and various learning resources, download study material, “chat” to your lecturers and fellow students about your studies and the challenges you encounter, and participate in online discussion forums. myUnisa provides additional opportunities to take part in activities and discussions of relevance to your module topics, assignments, marks and examinations.

1.2 Tutorial matter

A tutorial letter is our way of communicating with you about teaching, learning and assessment. You will receive a number of tutorial letters during the course of the module. This particular tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as the admission requirements for the examination. We urge you to read this and subsequent tutorial letters carefully and to keep it at hand when working through the study material, preparing and submitting the assignments, preparing for the examination and addressing queries that you may have about the course (course content, textbook, worked examples and exercises, theorems and their applications in your assignments, tutorial and textbook problems, etc.) to your COS2633 lecturers.

2 PURPOSE AND OUTCOMES FOR THE MODULE

2.1 Purpose

This module is available as part of a major in Computer Science and Applied Mathematics. The abbreviated syllabus comprises the numerical solution of nonlinear equations and systems of equations, the construction and use of interpolating polynomials, least square approximation, numerical integration and differentiation.

In this module you will learn how to develop and use numerical methods to solve mathematical problems by means of a computer. While the emphasis is on the more practical aspects, a good mathematical background is essential. We therefore advise you to include second year mathematics, in particular MAT2611 and MAT2613, in your curriculum.

The module that follows Numerical Methods 1 is, of course, Numerical Methods 2 (APM3711) which is also available as a subject in Computer Science and Applied Mathematics. Although numerical methods are not dependent on any specific programming language, many software pack-
ages are available as an aid to the study of numerical methods. You are therefore expected to learn one or two programming languages (like Matlab, python, C++ or Maple) on your own and to be able to write relatively simple programs in the language.

2.2 Outcomes

At the end of this module, you should be able to:

1. Draw a rough graph of any given function.
2. Solve various types of nonlinear equations using different kinds of numerical method and interpret the results.
3. Solve sets of equations using a variety of numerical methods.
4. Construct interpolating polynomials and fit curves to a given data.
5. Perform numerical Differentiation and Integration.

3 LECTURER(S) AND CONTACT DETAILS

3.1 Lecturer(s)

The contact details for the lecturer responsible for this module is

Postal address: The COS2633 Lecturers
Department of Mathematical Sciences
Private Bag X6
Florida
1709
South Africa

Additional contact details for the module lecturers will be provided in a subsequent tutorial letter.

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer(s). Tutorial letter 301 will provide additional contact details for your lecturer. Please have your study material with you when you contact your lecturer by telephone. If you are unable to reach us, leave a message with the departmental secretary. Provide your name, the time of the telephone call and contact details. If you have problems with questions that you are unable to solve, please send your own attempts so that the lecturers can determine where the fault lies.

Please note: Letters to lecturers may not be enclosed with or inserted into assignments.

3.2 Department

The contact details for the Department of Mathematical Sciences are:

Departmental Secretary: (011) 670 9147 (SA) +27 11 670 9147 (International)
3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *Study @ Unisa* that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open). Always have your student number at hand when you contact the University.

4 RESOURCES

4.1 Prescribed books

Prescribed books can be obtained from the University's official booksellers. If you have difficulty locating your book(s) at these booksellers, please contact the Prescribed Books Section at (012) 429 4152 or e-mail vospresc@unisa.ac.za.

Your prescribed textbook for this module is:

<table>
<thead>
<tr>
<th>Title:</th>
<th>Numerical Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s):</td>
<td>R.L. Burden, D.J. Faires, A.M. Burden</td>
</tr>
<tr>
<td>Edition:</td>
<td>10th Edition</td>
</tr>
<tr>
<td>Publisher(s):</td>
<td>Cengage Learning</td>
</tr>
<tr>
<td>Year:</td>
<td>2016</td>
</tr>
<tr>
<td>ISBN:</td>
<td>978-1-305-25366-7</td>
</tr>
</tbody>
</table>

Please buy the textbook as soon as possible since you have to study from it directly – you cannot do this module without the prescribed textbook.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information go to:

http://www.unisa.ac.za/brochures/studies

For more detailed information, go to the Unisa website: http://www.unisa.ac.za/, click on Library. For research support and services of Personal Librarians, go to:

http://www.unisa.ac.za/Default.asp?Cmd=ViewContent&ContentID=7102

The Library has compiled numerous library guides:

- find recommended reading in the print collection and e-reserves
  - http://libguides.unisa.ac.za/request/undergrad
request material
- http://libguides.unisa.ac.za/request/request

postgraduate information services
- http://libguides.unisa.ac.za/request/postgrad

finding, obtaining and using library resources and tools to assist in doing research
- http://libguides.unisa.ac.za/Research_Skills

how to contact the Library/find us on social media/frequently asked questions
- http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

For information on the various student support services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication Study @ Unisa that you received with your study material.

6 STUDY PLAN

The sections of the tenth edition that are prescribed for examination purposes are

- **Chapter 2**: sections 2.1 - 2.6;
- **Chapter 3**: sections 3.1 - 3.7;
- **Chapter 4**: sections 4.1 - 4.9;
- **Chapter 6**: sections 6.1 - 6.5;
- **Chapter 7**: sections 7.1, 7.3 - 7.5;
- **Chapter 8**: section 8.1;
- **Chapter 10**: section 10.2.

<table>
<thead>
<tr>
<th>Period</th>
<th>Assignment (due date)</th>
<th>Textbook (10th ed.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20/01 - 26/02</td>
<td>1 (26/02)</td>
<td><strong>Study</strong> chapters 1 and 2</td>
</tr>
<tr>
<td>26/02 - 23/04</td>
<td>2 (23/04)</td>
<td><strong>Study</strong> chapters 3, 4, 6, 7, 8.1 and 10.2</td>
</tr>
<tr>
<td>20/01 - 26/04</td>
<td>3 (26/04)</td>
<td><strong>Study</strong> all chapters</td>
</tr>
</tbody>
</table>

Table 2: Suggested study programme for Semester 1

In addition to the textbook you should also study the following:

- *Tutorial letter 102*, the use of which we discuss in its preface.
Table 3: Suggested study programme for Semester 2

<table>
<thead>
<tr>
<th>Period</th>
<th>Assignment (due date)</th>
<th>Textbook (10th ed.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15/07 - 04/08</td>
<td>1 (04/08)</td>
<td>Study chapters 1 and 2</td>
</tr>
<tr>
<td>04/08 - 25/09</td>
<td>2 (25/09)</td>
<td>Study chapters 3, 4, 6, 7, 8.1 and 10.2</td>
</tr>
<tr>
<td>15/07 - 05/10</td>
<td>3 (05/10)</td>
<td>Study all chapters</td>
</tr>
</tbody>
</table>

**Tutorial letters**, which include detailed discussions and model solutions of the assignments. The assignments and the corresponding tutorial letters are important since they give you an idea of what we expect of you with respect to the *types of problems* to be solved, and their *solutions*. Please note, however, that you should not rely solely on the tutorial letters for your exam preparation. The examination covers the whole syllabus, theory as well as practice, and you should prepare accordingly. We also give *additional explanations* in these letters. The tutorial letters are dispatched to you in the course of the year as they become available and will also be downloadable from the Internet.

**Inventory for the current academic year** that you received on registration and which lists the items available from the Department of Dispatch in Pretoria or the regional offices at the time of registration. Please check the tutorial matter you have received against this inventory and, if necessary, take appropriate action contacting the department of dispatch.

You should read the entire tutorial letter 102, before working through the textbook. You should work through the sections of the prescribed textbook in the order indicated in table 2 or 3 and submit assignments 1, 2 and 3 before the respective due dates.

See the brochure *Study @ Unisa* for general time management and planning skills.

**7 PRACTICAL WORK AND WORK INTEGRATED LEARNING**

There are no practicals for this module.

**8 ASSESSMENT**

**8.1 Assessment criteria**

**Specific outcome 1:** Be able to draw a rough graph of any given function.

**Assessment criteria**

- Ability to extract relevant information for a function, including existence, discontinuities, singularities, symmetries, boundedness, behaviour at very small and very large values of $x$, behaviour at 0, roots and zeros, turning points, . . .
Specific outcome 2: Be able to solve different nonlinear equations using different numerical methods and interpret the results. Numerical techniques include (but are not limited to) bisection, secant and regula falsi methods, fixed-point iteration, Newton’s method and its extensions, accelerating convergence, zeros of polynomials and Müller’s method.

Assessment criteria

- give a mathematical formulation of a method
- identify and understand the meaning of terms in a formulated method
- perform a few iterations of a numerical method
- study convergence of a numerical method
- estimate error of approximation . . .

Specific outcome 3: Be able to solve sets of equations using different numerical methods. Methods include (but are not limited to) Gaussian elimination, pivoting strategies and matrix factorization, Jacobi, Gauss-Seidel and SOR iterative techniques, Newton’s method for functions of many variables.

Assessment criteria

- give a mathematical formulation of a method
- identify and understand the meaning of terms in a formulated method
- perform a few iterations of a numerical method
- study convergence of a numerical method
- estimate error of approximation . . .

Specific outcome 4: Be able to construct interpolating polynomials and fit curves to a given data. Methods include (but are not limited to) interpolation and Lagrange polynomials, data approximation, Hermite interpolation, divided-difference, cubic splines, parametric curves and discrete least squares approximation.

Assessment criteria

- give a mathematical formulation of a method
- identify and understand the meaning of terms in a formulated method
- estimate error of approximation . . .
Specific outcome 5: Be able to perform numerical differentiation and integration. Methods include (but are not limited to):

- **Differentiation**: forward difference, backward difference, centered difference and their various refinements
- **Integration**: trapezoidal rule, Simpson’s rules, midpoint rule, Gaussian Quadrature . . . (standard, composite, mixed, . . .)

Assessment criteria

- give a mathematical formulation of a method
- identify and understand the meaning of terms in a formulated method
- estimate error of approximation . . .

8.2 Assessment plan

A final mark of at least 50% is required to pass the module. If a student does not pass the module then a final mark of at least 40% is required to permit the student access to the supplementary examination. The final mark is composed as follows:

<table>
<thead>
<tr>
<th>Year mark</th>
<th>Final mark</th>
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<tbody>
<tr>
<td>Assignment 01: 20%</td>
<td>Year mark: 20%</td>
</tr>
<tr>
<td>Assignment 02: 60%</td>
<td>Exam mark: 80%</td>
</tr>
<tr>
<td>Assignment 03: 20%</td>
<td></td>
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Example A student obtains the following marks:

Assignment 01: 55%
Assignment 02: 100%
Assignment 03: 65%
Exam: 58%

The year mark is

$$\frac{55 \times 20 + 100 \times 60 + 65 \times 20}{100} = 84\%.$$  

The final mark is

$$\left(\frac{20}{100} \times 84 + \frac{80}{100} \times 58\right), \text{ i.e. } (16.8 + 46.4), \text{ i.e. } 63.2\%.$$

8.3 Assignment numbers

8.3.1 General assignment numbers

The assignments for this module are Assignment 01, Assignment 02, etc.
8.3.2 Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

8.3.3 Assignment due dates

The dates for the submission of the assignments are:

**Semester 1**

- Assignment 01: Monday, 26 February 2018
- Assignment 02: Monday, 23 April 2018
- Assignment 03: Thursday, 26 April 2018

**Semester 2**

- Assignment 01: Monday, 6 August 2018
- Assignment 02: Tuesday, 25 September 2018
- Assignment 03: Friday, 5 October 2018

8.4 Submission of assignments

You may submit written assignments either by post or electronically via myUnisa. Assignments may **not** be submitted by fax or e-mail.

For detailed information on assignments, please refer to the *Study @ Unisa* brochure which you received with your study package.

**Please make a copy of your assignment before you submit!**

To submit an assignment via myUnisa:

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on “Assignments” in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.
8.5 The assignments

Please make sure that you submit the correct assignments for the 1st semester, 2nd semester or year module for which you have registered. For each assignment there is a fixed closing date, the date at which the assignment must reach the University. When appropriate, solutions for each assignment will be dispatched, as Tutorial Letter 201 (solutions to Assignment 01) and Tutorial Letter 202 (solutions to Assignment 02) etc., a few days after the closing date. They will also be made available on myUnisa. Late assignments will not be marked!

Note that Assignment 01 is the compulsory assignment for admission to the examination and must reach us by the due date.

8.6 Other assessment methods

There are no other assessment methods for this module.

8.7 The examination

During the relevant semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. For general information and requirements as far as examinations are concerned, see the brochure Study @ Unisa.

<table>
<thead>
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<th>Registered for . . .</th>
<th>Examination period</th>
<th>Supplementary examination period</th>
</tr>
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<tbody>
<tr>
<td>1st semester module</td>
<td>May/June 2018</td>
<td>October/November 2018</td>
</tr>
<tr>
<td>2nd semester module</td>
<td>October/November 2018</td>
<td>May/June 2019</td>
</tr>
<tr>
<td>Year module</td>
<td>October/November 2018</td>
<td>January/February 2019</td>
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9 FREQUENTLY ASKED QUESTIONS

The Study @ Unisa brochure contains an A–Z guide of the most relevant study information.

10 IN CLOSING

We hope that you will enjoy COS2633 and we wish you all the best in your studies at Unisa!
ADDENDUM A: ASSIGNMENTS – SEMESTER 1

COMPULSORY ASSIGNMENT FOR THE EXAM

ASSIGNMENT 01
Due date: Monday, 26 February 2018
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: 892173

ONLY FOR SEMESTER 1

This assignment covers Bisection, Secant and Regula Falsi Methods, Fixed-Point Iteration, Newton’s Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomials and Müller’s Method

IMPORTANT

• This is a multiple choice assignment. ALL the questions must be answered on a mark reading sheet which you then post to the university. Before answering this assignment, consult the publication Study @ Unisa on how to use and complete a mark reading sheet. You may, however, choose to complete and submit the assignment online, using myUnisa.

• Keep your rough work so that you can compare your solutions with those that will be sent to you after the closing date.

• 10 marks will be awarded for every correct answer.

Question 1: 40 Marks

Consider the function \( f(x) = xe^x - 2 \); we want to study the properties of \( f(x) \) so that we can apply numerical methods to solve the equation \( f(x) = 0 \).

(1.1) Which option is false? (10)

1. the function, \( f(x) \) is well defined and continuous for all \( x \) in the interval \( (0, 2) \)
2. the function, \( f(x) \) has no discontinuity and no singularities
3. the function, \( f'(x) \) is well defined and continuous for all \( x \) in the interval \( (0, 2) \)
4. the function, \( f'(x) \) has no discontinuity and no singularities
5. all of the above
To have an idea on whether we should apply the regula falsi method to determine the root of \( f(x) = 0 \) in a given interval, we may:

1. study carefully the continuity of \( f(x) \) and \( f'(x) \) then conclude
2. check if \( f(x) \) and \( f'(x) \) are differentiable then conclude
3. draw the graph of \( f(x) \) and observe the graphs then conclude
4. apply the functions, \( f(x) \) to a finite set of sample values then conclude
5. none of the above is true

Applying the regula falsi method, with starting points \( x_0 = 0.5 \) and \( x_1 = 1 \) yields the following result:

1. \( 0.847977577064739 \) after at most two iterations
2. \( 0.852551206774865 \) after exactly four iterations
3. \( 0.852604865962733 \) after at least four iterations
4. \( 0.852599625362406 \) after at least five iterations
5. all of the above

Applying the secant method, with starting points \( x_0 = 0.5 \) and \( x_1 = 1 \) yields the following result:

1. \( 0.810371774952277 \) after at least three iterations
2. \( 0.847977577064739 \) after at most one iteration
3. \( 0.852758254751202 \) after exactly two iterations
4. \( 0.852604956977597 \) after at most four iterations
5. all of the above

Question 2: 60 Marks

The function \( f(x) = x^4 - 2x^3 - 5x^2 + 12x - 5 \) has four distinct roots, we want to determine the roots by means of Müller’s method

Which option is false?

1. \( f(x) = x^4 - 2x^3 - 5x^2 + 12x - 5 \) has no singularities and no obvious symmetries and \( f(0) = -5 \)
2. \( f'(x) = 4x^3 - 6x^2 - 10x + 12 \), \( f''(x) = 12x^2 - 12x - 10 \) and \( f'''(x) = 24x - 12 \)
3. \( f''(x) \) has a local extremum at \( x = 0.5 \)
4. the two zeros for \( f''(x) \) are \(-0.5408\) and \(1.5408\)
5. \( f''(x) \) has a local maximum at \( x = 0.5 \)
Applying M"uller's method to compute the zeros of $f'(x)$ yields the following result:

1. $-1.5$ with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$
2. $-1.5$ with the starting points $x_2 = -0.5417$, $x_0 = 1.0417$ and $x_1 = 1.5417$
3. $1$ with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$
4. $2$ with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$
5. none of the above

Which option is false?

1. $f'(x)$ has a local minimum at $1.5408$
2. $f'(x)$ has a local maximum at $-0.5408$
3. $f(x)$ has a local maximum at $-1.5$ and $2$
4. $f(x)$ has a local minimum at $1$
5. all of the above

Applying Müller’s method to compute the zeros of $f(x)$ yields the following result:

1. $-1.5$ with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = -1.5$
2. $-2.4321$ with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = 2$
3. $1.5208$ with the starting points $x_2 = 1$, $x_0 = 1.5$ and $x_1 = 1$
4. $2.3315$ with the starting points $x_2 = 2$, $x_0 = 3$ and $x_1 = 1$
5. none of the above

Applying Müller’s method to compute the zeros of $f(x)$ yields the following result:

1. $-2.4321$ with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = -1.5$
2. $0.5798$ with the starting points $x_2 = -1.5$, $x_0 = -0.25$ and $x_1 = 1$
3. $1.5208$ with the starting points $x_2 = 1$, $x_0 = 1.5$ and $x_1 = 2$
4. $2.3316$ with the starting points $x_2 = 2$, $x_0 = 3$ and $x_1 = 4$
5. all of the above
Select the appropriate option:

(1) secant method and Müller’s method are similar in the sense that they both start with two initial points

(2) secant method yields a complex root even when initial approximation is a real number

(3) Müller’s method determines the next approximation by considering the intersection of a parabola and the $x$–axis through three given points

(4) all of the above

(5) none of the above
ASSIGNMENT 02
Due date: Monday, 23 April 2018
Total Marks: 120
UNIQUE ASSIGNMENT NUMBER: 730868

ONLY FOR SEMESTER 1

This assignment covers Linear Systems of Equations, Linear Algebra, Pivoting Strategies and Matrix Factorization, Jacobi and Gauss-Seidel Iterative Techniques and Error Bounds, Interpolation and Lagrange Polynomials, Data Approximation, Hermite Interpolation, Divided Difference, Cubic Splines, Parametric Curves and Discrete Least Squares Approximation, Numerical Differentiation and Integration

Question 1: 30 Marks

Consider the linear system

\[
\begin{align*}
2.141x_1 - 2.718x_2 + 1.414x_3 - 1.732x_4 &= 3.316 \\
9.869x_1 + 2.718x_2 - 7.389x_3 + 0.428x_4 &= 0 \\
2.236x_1 - 2.449x_2 + x_3 - 1.414x_4 &= 3.141 \\
31.006x_1 + 7.389x_2 - 2.645x_3 + 0.111x_4 &= 1.414
\end{align*}
\]

(1.1) Write the system in matrix notation. (2)

(1.2) Solve the system using:

(a) Gaussian elimination without pivoting. (2)

(b) Gaussian elimination with scaled partial pivoting. (2)

(c) LU decomposition. (2)

(1.3) Show that the total number of arithmetic (multiplication, divisions and additions) operations in the (1.2(a)) is approximately 58.66 (show all details). (2)

(1.4) Suppose we are to solve the equation

\[Ax = b.\]

We solve this by

\[x = \frac{1}{A} b = \frac{1}{\omega A} \omega b = \frac{1}{1 - r} \omega b\]

where \(\omega \neq 0\) is some real number chosen to weight the problem appropriately, and \(r = 1 - \omega A\). Now suppose that \(\omega\) was chosen such that \(|r| < 1\) and \(A \neq 0\). Then the following geometric expansion holds:

\[
\frac{1}{1 - r} = 1 + r + r^2 + r^3 + ...
\]
This gives approximate solution to our problem as,

\[ x \approx [1 + r + r^2 + r^3 + \cdots + r^k] \omega b \]
\[ = \omega b + [r + r^2 + r^3 + \cdots + r^k] \omega b \]
\[ = \omega b + r [1 + r + r^2 + r^3 + \cdots + r^{k-1}] \omega b \]

This suggests an iterative approach to solving the system \( Ax = b \). Let \( x^{(0)} = \omega b \) be the initial estimate of the solution, then for the \( k \)th iterate we have

\[ x(k) = \omega b + r x^{(k-1)} \]
\[ = \omega b + (I - \omega A) x^{(k-1)} \]

If \( |r| < 1 \) then the iterate \( x^{(k)} \) is guaranteed to converge to the true solution.

For some matrix, \( M \) and some scaling factor \( \omega \), we obtain the following algorithm

\[ Mx^{(k)} = \omega b + (M - \omega A) x^{(k-1)} \]

(a) Let \( A \) be the matrix from (1.1) and \( \omega = 1 \)

(i) Solve the system by choosing \( M \) to be the matrix consisting of the diagonal of \( A \). [4]

(ii) Solve the system using the Jacobi method. [2]

(iii) Compare the two results. [2]

(b) Let \( A \) be the matrix from (1.1) and \( \omega = 1 \)

(i) Solve the system by choosing \( M \) to be the lower triangular part of \( A \) including the diagonal entries. [4]

(ii) Solve the system using the Gauss-Seidel method. [2]

(iii) Compare the two results. [2]

(c) Solve the system using Successive Over-Relaxation technique with \( \omega = 0.5 \) and \( x_0 = (3, 0, 3, 1)^T \) [2]

(d) Which method do you prefer and why?

Note: Use four-decimal arithmetic with truncation (not rounding) and do three iterations, starting at \( x_0 = (3, 0, 3, 1)^T \) where applicable.
Question 2: 6 Marks

Find the inverse of

\[
\begin{bmatrix}
3 & 1 & 1 \\
3 & 1 & 2 \\
1 & 2 & 4
\end{bmatrix}
\]

Use Gaussian elimination and work exactly.

Question 3: 8 Marks

Write a computer program which uses Newton’s method to obtain a solution, accurate to five decimal digits, of the pair of simultaneous equations

\[
x^2 + y^2 = 5 \\
x^3 + y^3 = 2
\]

taking \(x = 1\) and \(y = -1\) as the initial values in the iterative process.

Question 4: 18 Marks

Consider the following data:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>0.5</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

(4.1) Set up a difference table through third differences. (3)

(4.2) What is the minimum degree that an interpolating polynomial, that fits all five data points exactly, can have? Explain. (3)

(4.3) Give the (forward) Newton-Gregory polynomial that fits the data points with \(x\) values 0.5, 1, and 1.5. Then compute \(f(1.25)\). (3)

(4.4) Compute an approximate bound for the error in the approximation to \(f(1.25)\) in (4.3) using Newton’s forward interpolating polynomial. (3)

(4.5) Compute \(f(1.25)\) using the Lagrange interpolating polynomial through the data points with \(x\) values 0.5, 1, and 1.5. (3)

(4.6) Construct the natural cubic spline for the last four data points. (3)
Question 5: 12 Marks

Let \((x_0, y_0) = (0, 0)\) and \((x_1, y_1) = (5, 2)\) be the end points of a curve. Use the following guide points to construct parametric cubic approximations \((x(t), y(t))\) and graph the approximations. Construct and graph the cubic Bezier polynomials:

(5.1) \((1, 1)\) and \((6, 1)\)

(5.2) \((0.5, 0.5)\) and \((5.5, 1.5)\)

(5.3) \((2, 2)\) and \((6, 3)\)

Question 6: 19 Marks

Given the data

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0.2 & 0.050446 \\
0.3 & 0.098426 \\
0.6 & 0.33277 \\
0.9 & 0.72660 \\
1.1 & 1.0972 \\
1.3 & 1.5697 \\
1.4 & 1.8487 \\
1.6 & 2.5015 \\
\hline
\end{array}
\]

(6.1) Construct the least squares approximation polynomial of degree three and compute the error.

(6.2) Construct the least squares approximation of the form \(be^{ax}\) and compute the error.

(6.3) Construct the least squares approximation of the form \(bx^a\) and compute the error.

(6.4) Draw the graph of the data points and the approximations in (6.1), (6.2) and (6.3).

Question 7: 17 Marks

(7.1) Approximate

\[
\int_{0.75}^{1.3} \left( \sin^2 x - 2x \sin x + 1 \right) dx
\]

by means of:

(a) Composite trapezoidal rule with \(n = 8\)

(b) Three-term Gaussian quadrature formula

(c) Simpson \(\frac{3}{8}\) rule
(7.2) Estimate the respective truncation errors in (7.1(a)) and (7.1(c)).

(7.3) Determine the integral analytically and then compute the actual errors in (7.1(a)), (7.1(b)) and (7.1(c)) respectively. [Hint: Use Taylor series expansion of $\sin x$.]

Question 8: 10 Marks

The area of the surface described by $z = f(x, y)$ for $(x, y) \in \mathcal{R}$ is given by

$$
\int \int_{\mathcal{R}} \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2} \, dA.
$$

Find an approximation to the area of the surface on the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$ that lies above the region in the plane described by $\mathcal{R} = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$ using:

(8.1) Trapezoidal rule in both directions

(8.2) Simpson $\frac{1}{3}$ rule in both directions

(8.3) Three-term Gaussian quadrature formulas in both directions
Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.
ADDENDUM B: ASSIGNMENTS – SEMESTER 2

COMPULSORY ASSIGNMENT FOR THE EXAM

ASSIGNMENT 01
Due date: Monday, 6 August 2018
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: 693664

ONLY FOR SEMESTER 2

This assignment covers Bisection, Secant and Regula Falsi Methods, Fixed-Point Iteration, Newton’s Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomials and Müller’s Method

IMPORTANT

• This is a multiple choice assignment. **ALL** the questions must be answered on a **mark reading sheet** which you then post to the university. Before answering this assignment, consult the publication **Study @ Unisa** on how to use and complete a mark reading sheet. You may, however, choose to complete and submit the assignment online, using **myUnisa**.

• Keep your rough work so that you can compare your solutions with those that will be sent to you after the closing date.

• 10 marks will be awarded for every correct answer.

Question 1: 50 Marks

Suppose we want to develop an iterative method to compute the $k^{th}$ root of a given positive number $y$, i.e., to solve the nonlinear equation $x^k = N$ given the value of $N$. Consider the following functions

- $g_1(x) = N + x - x^k$
- $g_2(x) = 1 + x - \frac{x^k}{N}$

(1.1) Which option is false? (10)

1. The function, $g_1(x)$ gives a fixed-point problem that is equivalent to the equation $f(x) = 0$
2. The function, $g_2(x)$ gives a fixed-point problem that is equivalent to the equation $f(x) = 0$
3. Newton’s algorithm for $f(x) = 0$ is given by: $x_{n+1} = x_n - \frac{N}{x_n^{k-1}}$
4. The algorithm in option (3) would still apply if $N$ was a negative number, with a complex number as the starting point
5. Not all of the above is true
Consider \( g_1(x) = N + x - x^3 \)

1. the fixed-point iteration scheme \( x_{n+1} = g_1(x_n) \) is (locally) convergent to \( \sqrt[3]{N} \) if \( N = 7 \)
2. the (local) convergence of \( g_1(x) \) cannot be guaranteed because \( g_1(x) \) is not differentiable in the interval that contains \( \sqrt[3]{7} \)
3. the interval of convergence where \( |g_1'(x)| < 1 \) does not contain \( \sqrt[3]{7} \)
4. \( g_1'(x) \) is not continuous
5. None of the above is true

Consider \( g_2(x) = 1 + x - \frac{x^3}{N} \)

1. the convergence of \( g_2(x) \) is not guaranteed because \( g_2'(x) \) is not continuous
2. the convergence of \( g_2(x) \) is not guaranteed because the interval of convergence where \( |g_2'(x)| < 1 \) does not contain \( \sqrt[3]{7} \)
3. the convergence of \( g_2(x) \) is guaranteed because \( g_2(x) \) and \( g_2'(x) \) are continuous and the set of points of intersection of the graphs of \( y = g_2(x) \) and \( y = x \) is not empty
4. \( g_2'(x) \) is not continuous
5. none of the above is true

Consider \( g_2(x) = 1 + x - \frac{x^3}{N} \)

1. there is a guarantee on the convergence of \( g_2(x) \)
2. the convergence of \( g_2(x) \) would not have been guaranteed if \( g_2(x) \) were continuous and differentiable only in an interval that excludes \( \sqrt[3]{7} \)
3. the convergence of \( g_2(x) \) is guaranteed because the interval of convergence where \( |g_2'(x)| < 1 \) contains \( \sqrt[3]{7} \)
4. \( g_2'(x) \) is continuous
5. all of the above is true

The error of the fixed-point function, \( g_3(x) \), given by Newton’s method for \( f(x) = x^3 - 7 \) is:

1. \( e_{n+1} = g_1(x_n) \)
2. \( e_{n+1} = \frac{1}{3x_n} \left[ g_1(x_n) \frac{x_n}{x_n} - 1 \right] \)
3. \( e_{n+1} = g_2(x_n) \)
4. \( e_{n+1} = g_2(x_n) - \frac{g_1(x_n)}{3x_n^2} \)
5. none of the above
Question 2: 50 Marks

The difference between two numbers is 3. If the larger number is added to its square root and the smaller number is added to its square, the product of the two differences equals 66.33. We want to determine the two numbers to within $10^{-3}$.

(2.1) the two numbers say $x$ and $y$ can be determined by:

1. simply using a non-programmable calculator
2. solving the equation $(2x + 3) \left( x + \sqrt{x + 3} \right) \left( x^2 + x \right) = 0$ then deduce the value of $y$ from the equation $y = x - 3$
3. solving the equation $(2x + 3) \left( x + \sqrt{x + 3} \right) \left( x^2 + x \right) = 0$ then deduce the value of $y$ from the equation $y = x + 3$
4. solving the equation $x^2 - \sqrt{x + 3} - 25.11 = 0$ then deduce the value of $y$ from the equation $y = x + 3$
5. none of the above

(2.2) With the starting point 1.5 and applying the Newton’s method to the appropriate equation yields the following result:

1. 6.6093246475 after at exactly two iterations
2. 5.4243003271 after exactly four iterations
3. 5.4243003271 after at least seven iterations
4. 5.4243003271 after at least five iterations
5. none of the above

(2.3) Applying the secant method to the same equation used for the Newton’s methods in question (2.2), yields the following result:

1. 5.1715901498 with starting points 1.5, -1 and after at most two iterations
2. 5.1715901498 with starting points 1.5, 7 and after exactly four iterations
3. 5.2904130851 with starting points 1, 3 and after exactly six iterations
4. 5.2904935273 with starting points 3, 5 and after at most four iterations
5. none of the above
Applying the regula falsi method to the same equation used for the Newton’s methods in question (2.2), with the starting $x_0 = 1$ and $x_1 = 2$, yields the following result:

1. $3.9015664048$ after at most two iterations
2. $5.1163828931$ after exactly five iterations
3. $5.2840053817$ after exactly seven iterations
4. $5.2840053817$ after at most five iterations
5. none of the above

Select the appropriate option:

1. the Newton’s method yields a complex root with the following starting points $-1$, $0$ and $0.5$
2. the Secant method yields a complex root with the starting points $x_0 = -3$, and $x_1 = 1$
3. the Falsi method yields a complex root with the starting points $x_0 = -1$, and $x_1 = -2$
4. all of the above
5. none of the above
Question 1: 30 Marks

Consider the linear system

\[
\begin{align*}
0.06x_1 + 0.08x_2 + 0.07x_3 + 0.08x_4 &= 0.29 \\
0.08x_1 + 0.20x_2 + 0.09x_3 + 0.07x_4 &= 0.44 \\
0.07x_1 + 0.09x_2 + 0.20x_3 + 0.10x_4 &= 0.46 \\
0.06x_1 + 0.08x_2 + 0.10x_3 + 0.20x_4 &= 0.44 \\
\end{align*}
\]

(1.1) Write the system in matrix notation. (2)

(1.2) Solve the system using:

(a) Gaussian elimination without pivoting. (2)

(b) Gaussian elimination with scaled partial pivoting. (2)

(c) LU decomposition. (2)

(1.3) Show that the total number of arithmetic (multiplication, divisions and additions) operations in the (1.2(a)) is approximately 58.66 (show all details). (2)

(1.4) Suppose we are to solve the equation

\[ Ax = b. \]

We solve this by

\[ x = \frac{1}{A} b = \frac{1}{\omega A} \omega b = \frac{1}{1 - r} \omega b \]

where \( \omega \neq 0 \) is some real number chosen to weight the problem appropriately, and \( r = 1 - \omega A \). Now suppose that \( \omega \) was chosen such that \( |r| < 1 \) and \( A \neq 0 \). Then the following geometric expansion holds:

\[ \frac{1}{1 - r} = 1 + r + r^2 + r^3 + ... \]
This gives approximate solution to our problem as,

\[ x \approx (1 + r + r^2 + r^3 + \cdots + r^k) \omega b \]

\[ = \omega b + [r + r^2 + r^3 + \cdots + r^k] \omega b \]

This suggests an iterative approach to solving the system \( Ax = b \). Let \( x^{(0)} = \omega b \) be the initial estimate of the solution, then for the \( k \)th iterate we have

\[ x^{(k)} = \omega b + r x^{(k-1)} \]

\[ = \omega b + (I - \omega A) x^{(k-1)} \]

If \( |r| < 1 \) then the iterate \( x^{(k)} \) is guaranteed to converge to the true solution.

For some matrix, \( M \) and some scaling factor \( \omega \), we obtain the following algorithm

\( Mx^{(k)} = \omega b + (M - \omega A) x^{(k-1)} \)

(a) Let \( A \) be the matrix from (1.1) and \( \omega = 1 \)

(i) Solve the system by choosing \( M \) to be the matrix consisting of the diagonal of \( A \). \[ 4 \]

(ii) Solve the system using the Jacobi method. \[ 2 \]

(iii) Compare the two results. \[ 2 \]

(b) Let \( A \) be the matrix from (1.1) and \( \omega = 1 \)

(i) Solve the system by choosing \( M \) to be the lower triangular part of \( A \) including the diagonal entries. \[ 4 \]

(ii) Solve the system using the Gauss-Seidel method. \[ 2 \]

(iii) Compare the two results. \[ 2 \]

(c) Solve the system using Successive Over-Relaxation technique with \( \omega = 0.5 \) and \( x_0 = (0, 0, 0, 0)^T \)

(d) Which method do you prefer and why?

Note: Use four-decimal arithmetic with truncation (not rounding) and do three iterations, starting at \( x_0 = (0, 0, 0, 0)^T \) where applicable.

Question 2: 6 Marks

Find the inverse of

\[
\begin{bmatrix}
3 & 5 & 1 \\
-1 & 3 & 2 \\
1 & -2 & -1
\end{bmatrix}
\]

Use Gaussian elimination and work exactly.

Question 3: 8 Marks

Consider the nonlinear system

\[ x^2 + y^2 = 5 \]

\[ e^x + xy = 2 \]

(3.1) Approximate the solutions graphically.
(3.2) Use the approximations from part (3.1) as initial approximations for Newton’s method to calculate solutions with an accuracy of $10^{-3}$ in the absolute error definition.

**Question 4: 15 Marks**

(4.1) Construct a divided-difference table from:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-1.1518</td>
</tr>
<tr>
<td>-0.4</td>
<td>0.7028</td>
</tr>
<tr>
<td>0.5</td>
<td>-1.4845</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.14943</td>
</tr>
<tr>
<td>0.0</td>
<td>0.13534</td>
</tr>
</tbody>
</table>

(4.2) Use this divided-difference table to estimate $f(0.15)$, using:

(a) A polynomial of degree 3 through the first four points,

(b) A polynomial of degree 4.

**Question 5: 13 Marks**

The following table lists the quarterly growth of Gross Domestic Product for the RSA from 3rd quarter of 2010 to the end of 2011:

<table>
<thead>
<tr>
<th>GDP (in thousands)</th>
<th>Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1843318</td>
<td>10</td>
</tr>
<tr>
<td>1863705</td>
<td>10</td>
</tr>
<tr>
<td>1884627</td>
<td>11</td>
</tr>
<tr>
<td>1889101</td>
<td>11</td>
</tr>
<tr>
<td>1897054</td>
<td>11</td>
</tr>
<tr>
<td>1911890</td>
<td>11</td>
</tr>
</tbody>
</table>

(5.1) Find the Lagrange polynomial of degree five fitting this data, and use this polynomial to estimate the GDP growth in the quarters 10|2 and 12|1.

(5.2) The GDP growth in 10|2 was approximately 1829347. How accurate do you think your 12|1 figures are?
Question 6: 18 Marks

Consider the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>sin x</th>
<th>( \frac{d}{dx} \sin x = \cos x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.29552</td>
<td>0.95534</td>
</tr>
<tr>
<td>0.32</td>
<td>0.31457</td>
<td>0.94924</td>
</tr>
<tr>
<td>0.35</td>
<td>0.34290</td>
<td>0.93937</td>
</tr>
</tbody>
</table>

(6.1) Use the above values and five-digit rounding to construct a cubic spline \( Q \) with boundary conditions

\[ Q'(x_0) = f'(x_0) \quad \text{and} \quad Q'(x_n) = f'(x_n) \]

which force the slopes of the spline to assume certain values (in our case the values \( f'(x_0) \) and \( f'(x_n) \), respectively) at the two boundaries. Use this spline to approximate \( \sin 0.33 \).

(6.2) Determine the error for the approximation in (6.1).

(6.3) Use the spline constructed in (6.1) to approximate \( \cos 0.34 \).

(6.4) Use the spline constructed in (6.1) to approximate

\[ \int_{0.30}^{0.35} \sin x \, dx \]

Question 7: 18 Marks

Consider the following set of data points in the table below:

<table>
<thead>
<tr>
<th>i</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>105</td>
<td>140</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>140</td>
</tr>
<tr>
<td>7</td>
<td>190</td>
<td>120</td>
</tr>
<tr>
<td>8</td>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>130</td>
<td>80</td>
</tr>
</tbody>
</table>

(7.1) Using guide-points of your choice from the data set, construct the connected Bezier curve from the set of points. (Hint: divide the set of points into three parts).

(7.2) Draw the connected Bezier polynomial

(7.3) Why is the graph smoothly connected at points 3 and 6?
Question 8: 12 Marks

Consider the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.050446</td>
</tr>
<tr>
<td>0.3</td>
<td>0.098426</td>
</tr>
<tr>
<td>0.6</td>
<td>0.33277</td>
</tr>
<tr>
<td>0.9</td>
<td>0.72660</td>
</tr>
<tr>
<td>1.1</td>
<td>1.0972</td>
</tr>
<tr>
<td>1.3</td>
<td>1.5697</td>
</tr>
<tr>
<td>1.4</td>
<td>1.8487</td>
</tr>
<tr>
<td>1.6</td>
<td>2.5015</td>
</tr>
</tbody>
</table>

Find the least squares polynomials of degree one, two and three for the data in the above table. Compute the total error in each case. Draw a graph of the data and the polynomials.
Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.