Tutorial letter 201/0/2017

Control Systems III (Theory)

CSY3601

Year Module

Department of Electrical and Mining Engineering

IMPORTANT INFORMATION:

The examination will be a partial open book examination paper.

ONLY THE PRESCRIBED BOOK IS ALLOWED IN THE EXAMINATION VENUE

BAR CODE

Learn without limits.

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CSY3601 2017： Assignment memo

It should be noted that the method to solve the questions is not unique.

ASSIGNMENT 1

Question 1, Answer: 4).

Question 2, Answer: 2)

Question 3, Answer: 2)

Question 4, Answer: 4)

Question 5, Answer: 1)

Question 6, Answer: 2)

Question 7, Answer: 1)

Question 8, Answer: 3)

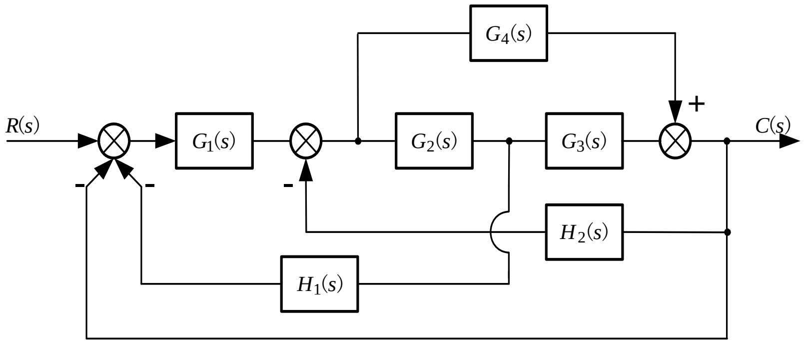
Question 9, Answer: 3)

Question 10, Answer: 3)

ASSIGNMENT 2

Question 1

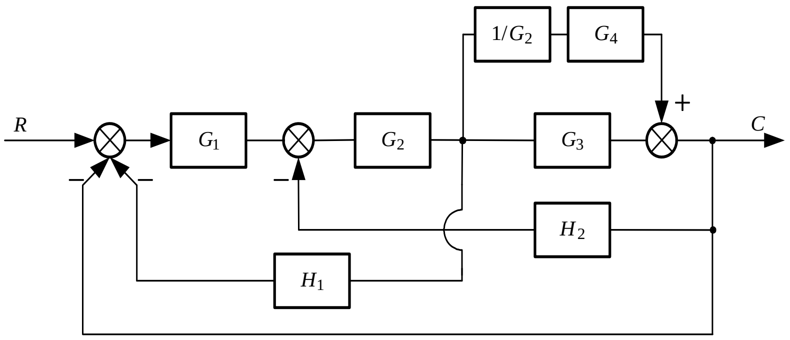
Use block-diagram-reduction techniques to find the transfer function $T(s)=\dfrac{C(s)}{R(s)}$ of the system represented by the following block diagram.



[20]

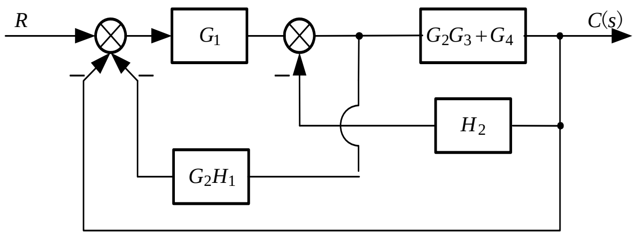
Answer:

Step 1 Move G2 to the left side of the pickoff point



✓✓✓✓

Step 2 Move G2 block to the right side of pickoff point



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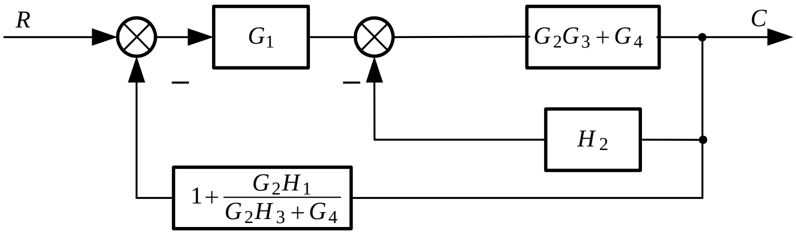
Step 3

Move G2G3 +G4 to the left side of the pickoff point and combine the two feedback paths



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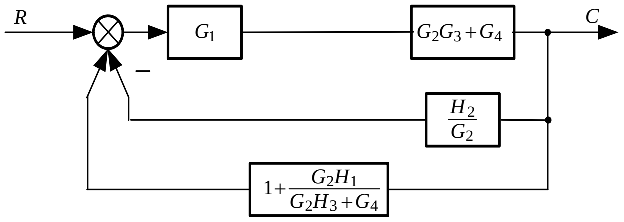
Step 4 Move block G1 to the right side of the summing junction



✓✓✓✓

Step 5 The final result $\frac{C(s)}{R(s)} = \frac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + G_1(G_2H_1 + G_2G_3 + G_4)}$

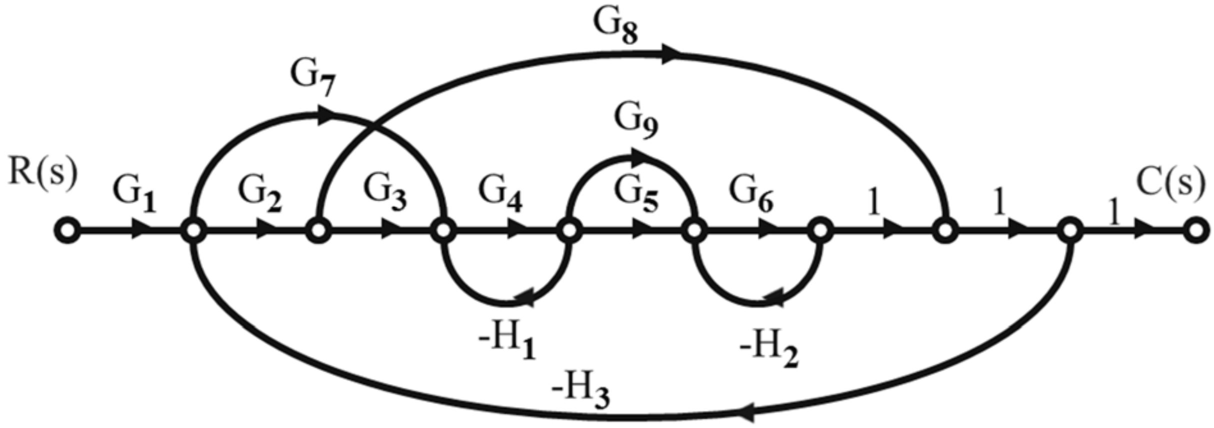
✓✓✓✓

Question 2

Use Mason's gain formula to calculate the close-loop transfer function of the following figure.

Hint: use the website materials to find how you can use Mason's gain formula.

[30]



Solution:

Forward paths:

$T_1=G_1 G_2 G_3 G_4 G_5 G_6$ ✓✓

$T_2=G_1 G_2 G_8$ ✓✓

$T_3=G_1 G_7 G_4 G_5 G_6$ ✓✓

$T_4=G_1 G_2 G_3 G_4 G_9 G_6$ ✓✓

$T_5=G_1 G_7 G_4 G_9 G_6$ ✓✓

Loops:

$L_1=- G_4 H_1$
$L_1=- G_6 H_2$
$L_3=- G_2 G_3 G_4 G_5 G_6 H_3$
$L_4=- G_2 G_3 G_4 G_9 G_6 H_3$
$L_5=- G_7 G_4 G_5 G_6 H_3$
$L_6=- G_7 G_4 G_9 G_6 H_3$
$L_7=- G_2 G_8 H_3$

✓✓✓✓✓✓✓

There are three non-touching loops taken two at a time:

$L_1 L_2= G_4 G_6 H_1 H_2$

$L_1 L_7= G_2 G_4 G_8 H_1 H_3$

$L_2 L_7= G_2 G_6 G_8 H_2 H_3$

✓✓✓

There is one non-touching loops taken three at a time:

$L_1 L_2 L_7= - G_2 G_4 G_6 G_8 H_1 H_2 H_3$

✓

$\Delta=1-(L_1+ L_2 + L_3+ L_4 + L_5+ L_6 + L_7)+ L_1 L_2+ L_1 L_7+ L_2 L_7- L_1 L_2 L_7$

$=1+ G_4 H_1 + G_6 H_2 + G_2 G_3 G_4 G_5 G_6 H_3 + G_2 G_3 G_4 G_9 G_6 H_3$

$+ G_7 G_4 G_5 G_6 H_3 + G_7 G_4 G_9 G_6 H_3 +G_2 G_8 H_3 + G_4 G_6 H_1 H_2$

$+ G_2 G_4 G_8 H_1 H_3 + G_2 G_6 G_8 H_2 H_3 + G_2 G_4 G_6 G_8 H_1 H_2 H_3$

✓✓

Moreover,

$\Delta_1=1$,

Δ_2=1-（ L_1+L_2）+L_1L_2 =1+ $G_4 H_1$ + $G_6 H_2$ + $G_4 G_6 H_1 H_2$

Δ_3=1

Δ_4=1

Δ_5=1

✓✓✓✓✓

$$T = \frac{1}{\Delta}\sum_{k=1}^{5} T_k \Delta_k$$

✓✓

Question 3

A system is described by the following differential equation:

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$$

Find the expression for the transfer function of the system $Y(s)/X(s)$.

[10]

Answer:

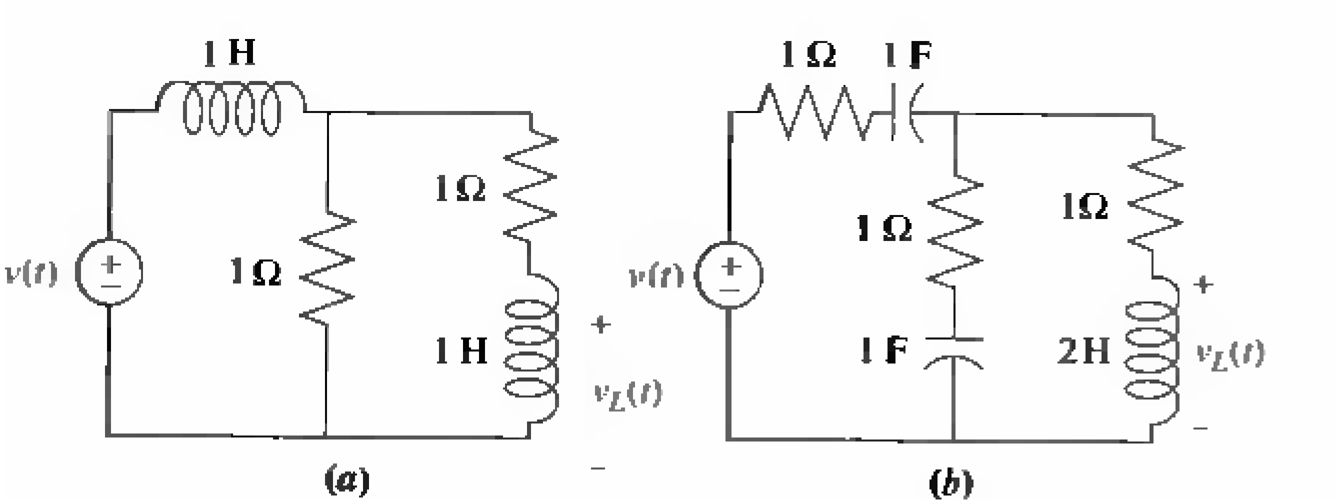
The Laplace transform of the differential equation, assuming zero initial conditions, is,

$(s^3+3s^2+5s+1)Y(s) = (s^3+4s^2+6s+8)X(s)$. ✓✓✓✓✓

Solving for the transfer function, $\frac{Y(s)}{X(s)} = \frac{s^3+4s^2+6s+8}{s^3+3s^2+5s+1}$. ✓✓✓✓✓

Question 4

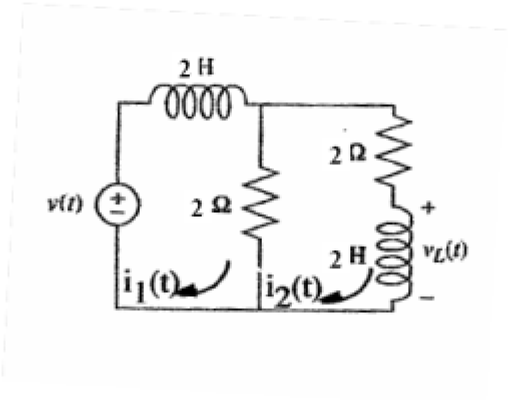
Find the transfer functions, $V_L(s)/V(s)$,for the following circuits.



[20]

Answer:

a.



Writing mesh equations

$$(2s+2)I_1(s) - 2\,I_2(s) = V_i(s)$$

$$-2I_1(s) + (2s+4)I_2(s) = 0$$ ✓✓✓✓✓

But from the second equation, $I_1(s) = (s+2)I_2(s)$. Substituting this in the first equation yields,

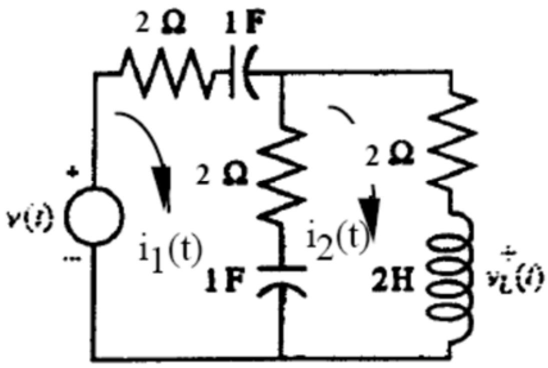
$$(2s+2)(s+2)I_2(s) - 2\,I_2(s) = V_i(s)$$

or

$$I_2(s)/V_i(s) = 1/(2s^2 + 6s + 2)$$

But, $V_L(s) = 2sI_2(s)$. Therefore, $V_L(s)/V_i(s) = s/(s^2 + 3s + 1)$. ✓✓✓✓✓

b.



$$(4+\frac{2}{s})I_1(s)-(2+\frac{1}{s})I_2(s)=V(s)$$

$$-(2+\frac{1}{s})I_1(s)+(4+\frac{1}{s}+2s)=0$$

✓✓✓✓✓

Solving for $I_2(s)$:

$$I_2(s)=\frac{\begin{vmatrix}\frac{4s+2}{s} & V(s)\\ \frac{-(2s+1)}{s} & 0\end{vmatrix}}{\begin{vmatrix}\frac{4s+2}{s} & \frac{-(2s+1)}{s}\\ \frac{-(2s+1)}{s} & \frac{(2s^2+4s+1)}{s}\end{vmatrix}}=\frac{sV(s)}{4s^2+6s+1}$$

Therefore, $\frac{V_L(s)}{V(s)}=\frac{2sI_2(s)}{V(s)}=\frac{2s^2}{4s^2+6s+1}$

✓✓✓✓✓

Question 5

Given the following differential equation, $\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=2u(t)$

Calculate $y(t)$ if all initial conditions are zero using the Laplace transform method.

[20]

Total: 100

Answer:

The Laplace transform is $s^2Y(s)+4sY(s)+3Y(s)=\frac{2}{s}$,✓✓✓

Solving for the response, $Y(s)$, yields $Y(s)=\frac{2}{s(s^2+4s+3)}=\frac{2}{s(s+3)(s+1)}$,✓✓✓

$$Y(s)=\frac{2}{s(s+3)(s+1)}=\frac{K_1}{s}+\frac{K_2}{s+3}+\frac{K_3}{s+1}$$, ✓✓✓✓✓

$$K_1=\left.\frac{2}{(s+3)(s+1)}\right|_{s\to 0}=\frac{2}{3}.\ K_2=\left.\frac{2}{s(s+1)}\right|_{s\to -3}=\frac{1}{3}.\ K_3=\left.\frac{2}{s(s+3)}\right|_{s\to -1}=-1$$ ✓✓✓

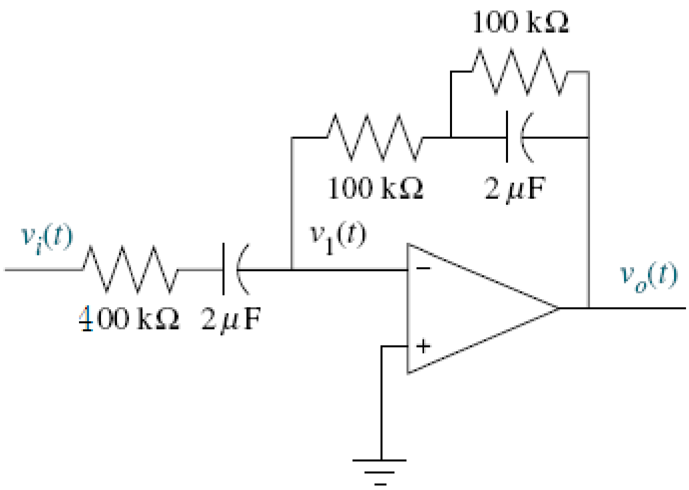
$$Y(s)=\frac{2/3}{s}+\frac{1/3}{s+3}+\frac{-1}{s+1}$$ ✓✓✓

$$y(t)=2/3+1/3\,e^{-3t}-e^{-t}$$ ✓✓✓

ASSIGNMENT 3

Question 1

Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for the operational amplifier circuits as shown in the following figure.



[12]

Answer:

$Z_1(s) = 10^5(\frac{5}{s} + 4)$ ✓✓✓✓

$Z_2(s) = 10^5(1 + \frac{5}{s+5}) = 10^5(\frac{s+10}{s+5})$,✓✓✓✓

Therefore, $-\frac{Z_2(s)}{Z_1(s)} = -\frac{s(s+10)}{(s+5)(4s+5)}$ ✓✓✓✓

Question 2

Assume that the motor, whose transfer functions is shown in the following figures, is used as the forward path of a closed-loop, unity feedback system.

a. Calculate the percent overshoot and settling time that could be expected.

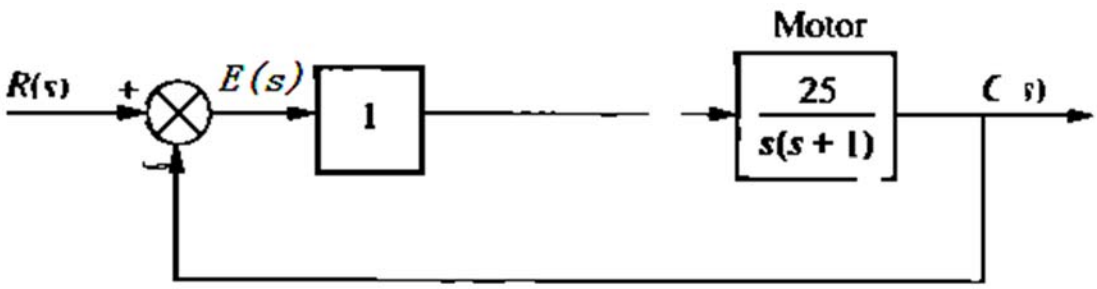
(15)

b. Find the values of K_1 and K_2 to yield a 16% overshoot and a settling time of 0.2 second.

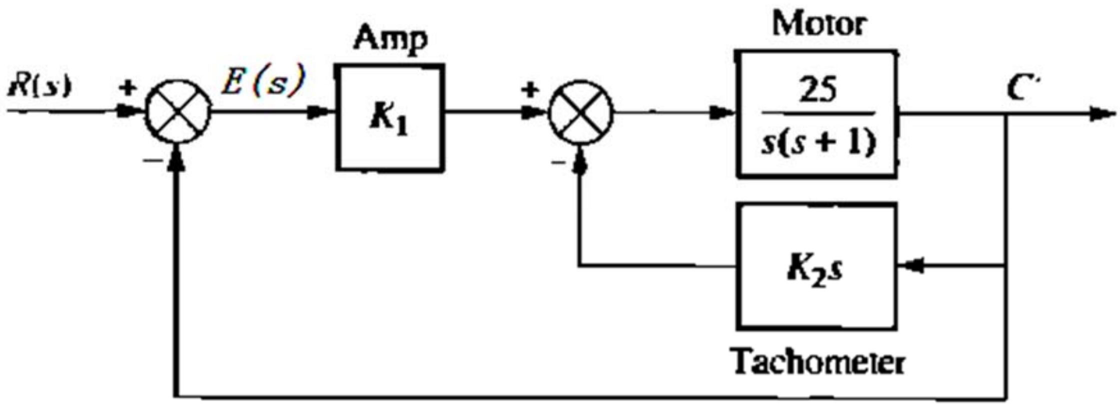
(17)

[32]

Here 2% criterion is used.



(a)



(b)

Answer:

a. $T(s) = \frac{25}{s^2+s+25}$; from which, $2\zeta\omega_n = 1$ and $\omega_n = 5$. Hence, $\zeta = 0.1$. Therefore, ✓✓✓✓✓✓✓✓

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 72.92\%$; $T_s = \frac{4}{\zeta\omega_n} = 8$. ✓✓✓✓✓✓✓

b. $T(s) = \frac{25K_1}{s^2+(1+25K_2)s+25K_1}$; from which, $2\zeta\omega_n = 1+25K_2$ and $\omega_n = 5\sqrt{K_1}$. Hence, ✓✓✓✓✓✓

$\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.504$. Also, $T_s = \frac{4}{\zeta\omega_n} = 0.2$, Thus, $\zeta\omega_n = 20$; from which $K_2 = \frac{39}{25}$ and

✓✓✓✓✓✓✓✓✓✓✓

$\omega_n = 39.68$. Hence, $K_1 = 62.98$.

Question 3

For the unity negative feedback system with open loop transfer function

$$G(s) = \frac{2}{2s^4 + s^3 + s^2 + 2s}$$

Determine the closed-loop system is stable or not based on Routh's stability criterion.

[16]

Answer:

The closed-loop transfer function is

$$T(s) = \frac{2}{2s^4 + s^3 + s^2 + 2s + 2}$$

✓✓✓✓

The routh array is

$$\begin{array}{llll} s^4 & 2 & 1 & 2 \\ s^3 & 1 & 2 & \\ s^2 & -3 & 2 & \\ s^1 & 8/3 & & \\ s^0 & 2 & & \end{array}$$

s^4	2	1	1
s^3	5	2	0
s^2	1	5	
s^1	-23	0	
s^0	5		

✓✓✓✓✓✓✓✓✓✓

Hence this closed-loop system is unstable. ✓✓

Question 4

For the unity negative feedback system with open loop transfer function

$$G(s) = K\frac{(s+2)}{s(s-1)(s+3)}$$

Find the range of K for closed-loop stability based on Routh's stability criterion.

[17]

Answer:

The characteristic equation is:

$$1+K\frac{(s+2)}{s(s-1)(s+3)}=0 \text{ or}$$

$$s(s-1)(s+3)+K(s+2)=0 \text{ or}$$

$$s^3+2s^2+(K-3)s+2K=0$$

✓✓✓✓✓

The Routh array is:

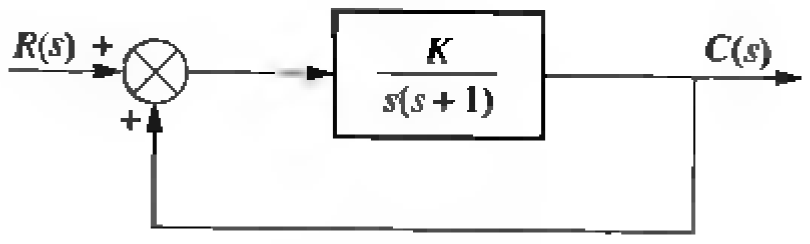
s^3	1	$K-3$
s^2	2	$2K$
s	-3	
1	$2K$	

The first column will always have a sign change regardless of the value of K. There is no value of K that will stabilize this system.

✓✓✓✓✓✓✓✓✓✓✓✓

Question 5

Sketch the root locus for the **positive-feedback** control system shown in the following figure.

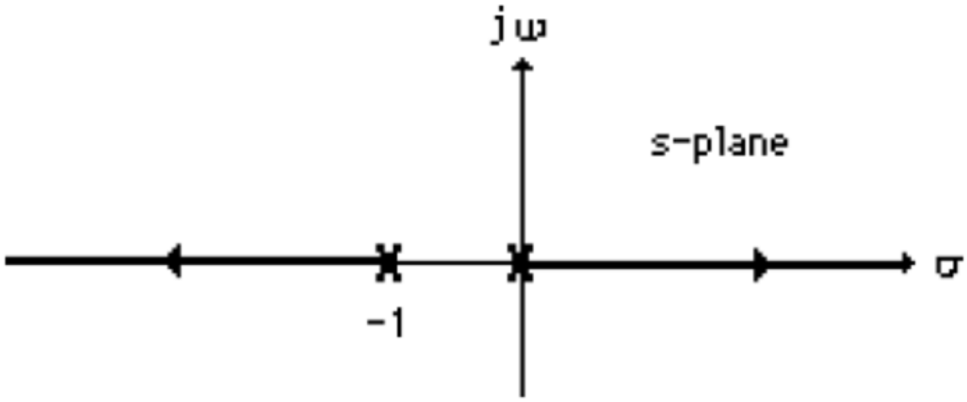


[8]

Answer:

Poles: s1=0, s2=-1

The root loci exit on the real axis is (-inf, -1) and [0, +inf)

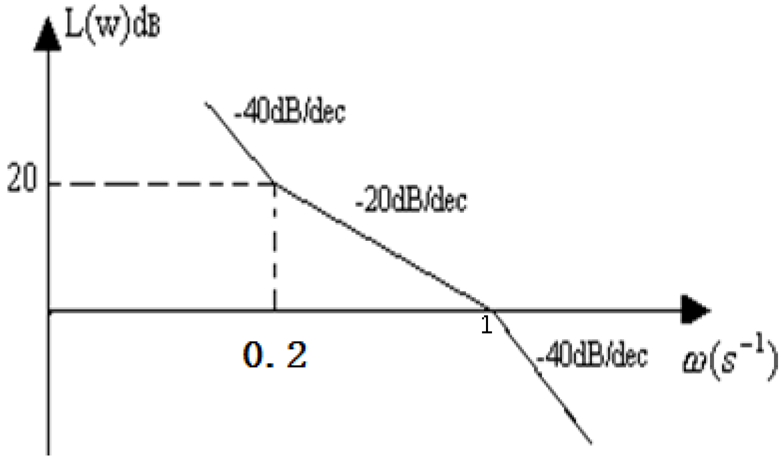


✓✓✓✓✓✓✓✓

Question 6

The Bode diagram of a minimum phase system is given in the following figure.

Find the open loop transfer function of the system;



[15]
Total:100

Answer:

II-type sytem;

Open loop zero: s=-0.2; ✓✓✓

Open loop pole: s=-1. ✓✓✓

Must have the form of $G(s)H(s) = \dfrac{K(5s+1)}{s^2(s+1)}$ ✓✓✓

$$20\log\left|G(j\omega)H(j\omega)\right|_{\omega=0.2} = 20$$

$$20\log\left(\left|\frac{K(j\omega 5+1)}{-\omega^2(j\omega+1)}\right|_{\omega=0.2}\right) = 20$$

$$\left|\frac{K(j\omega 5+1)}{-\omega^2(j\omega+1)}\right|_{\omega=0.2} = 10$$

$K = 0.29$ ✓✓✓✓

So $G(s)H(s) = \dfrac{0.29(5s+1)}{s^2(s+1)}$ ✓✓