PLEASE NOTE: It should be noted that the methods are not unique for some questions. The answering steps are necessary.

QUESTION 1 The closed loop transfer function of a system is

Gc(s)= $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, Find the maximum overshoot σ_p =9.6%, damping ration ξ , and ω_n when the peak time t_p =0.2 second.

[15]

ANSWER:

$\because \sigma_p = e^{\frac{\varepsilon\pi}{\sqrt{1-\varepsilon^2}}} \times$100%=9.6%

$\therefore$ ξ =0.6

$\because t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\therefore \omega_n = \frac{\pi}{t_p\sqrt{1-\xi^2}} = \frac{3.14}{0.2\sqrt{1-0.6^2}} =$ 19.6rad/s

QUESTION 2 FIND TRANSFER FUNCTION (2) [20]

2.1The differential equation of a system is $\ddot{y}$ +2 $\dot{y}$ +3y= $\dot{u}$ +u, find the transfer function $\frac{Y(s)}{U(s)}$. Assume all initial states are zero. (5)

ANSWER:

Laplace transform:

Because the initial states are zero, we have

$L\{\ddot{y}\} = s^2Y(s)$

$L\{\dot{y}\} = sY(s)$

$L\{y\} = Y(s)$

$L\{\dot{u}\} = sU(s)$

$L\{u\} = U(s)$

$(s^2 + 2s + 3)Y(s) = (s+1)U(s)$

$\frac{Y(s)}{U(s)} = \frac{s+1}{(s^2+2s+3)}$

2.2 A system is given in Figure 1. Sketch the signal flow chart and find the transfer function $\frac{C(s)}{R(s)}$. **(15)**

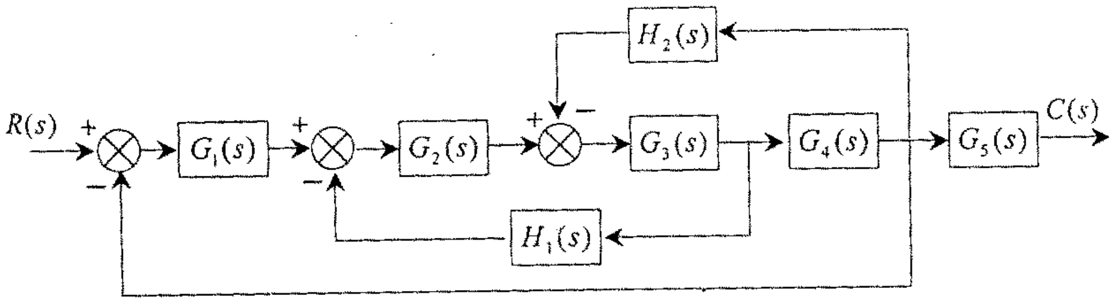
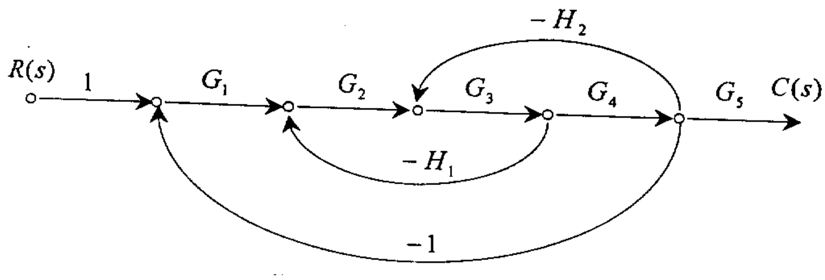


Figure 1.

ANSWER:



$P_1 = G_1G_2G_3G_4G_5$,
$L_1 = -G_1G_2G_3G_4$,
$L_2 = -G_2G_3H_1$,
$L_3 = -G_3G_4H_2$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5}{1 + G_1G_2G_3G_4 + G_2G_3H_1 + G_3G_4H_2}$$

QUESTION 3 The open loop transfer function of a unit negative feedback system is $G(s) = \dfrac{10}{s(s+1)}$.

Find

(a)	ω_n	(2)
(b)	ξ	(2)
(c)	$\delta\%$ (Maximum overshoot)	(2)
(d)	t_p	(2)
(e)	t_s	(2)

[10]

ANSWER:

$$\because \xi = \frac{1}{2\sqrt{10}} = 0.158$$

$$\omega_n = \sqrt{10} = 3.16$$

$$\therefore \sigma\% = e^{-\pi\xi / \sqrt{1-\xi^2}} * 100\% = 60.5\%$$

$$t_s = \frac{3}{\xi\omega_n} = 6$$

$$t_p = \frac{\pi}{\omega_d} = \sqrt{1-\xi^2}\,\omega_n = 1.94 \text{ s}$$

QUESTION 4 The open loop transfer function of a unit negative feedback system is $G(s) = \dfrac{k}{(s+2)(s+4)(s^2+6s+25)}$. Find K for system oscillation using Routh stability criterion.

[20]

ANSWER:
The characteristic equation of the system is

$$(s+2)(s+4)(s^2+6s+25) + k = 0$$

i.e.

$$s^4 + 12s^3 + 69s^2 + 198s + 200 + k = 0$$

$s^4:\ 1 \quad 69 \quad 200+k$

$s^3:\ 12 \quad 198$

$s^2:\ 52.5 \quad 200+k$

$s^1:\ \dfrac{52.5*198-12*(200+k)}{52.5}$

$s^0:\ 200+k$

The system oscillates means

$$\frac{52.5*198-12*(200+k)}{52.5}=0$$

Or

$200+k$

i.e.

$52.5*198-12*(200+k)=0$

$k=$666.25

Or

$k=$-200

QUESTION 5: The open loop transfer function of a system is $G(s)=\frac{K}{s(s+2)(s+4)}$.

Sketch the root locus and find the range of K for stability. **[15]**

ANSWER:

(1) The root loci have 3 branches
Origin from s1=0,s2=-2,s3=-4
Terminate at infinity.

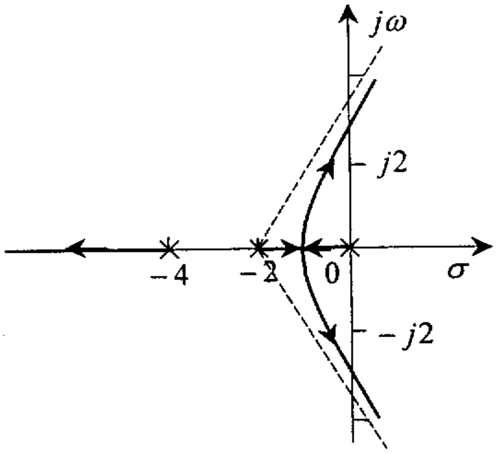
(2) Root locus on the real axis
[0, -2] and [-4, -∞]

(3) Asymptotes：
$\pm 60^\circ$,180°
$\sigma_a=\frac{-2-4}{3}$ =-2

(4) Points of departure
Since $\frac{dK}{ds}$ =0, we have s_1=-0.85,s_2=-3.15. Because it must lie on [0, -2], so s_1=-0.85 is the departure point.

(5) The point cross the imagery axis.
ω_1=0, K=0;
$\omega_{2,3}=\pm 2\sqrt{2}$, K=48

The range of K for stability: 0<K<48.



QUESTION 6: The Bode diagram of a minimum phase system is given in Figure 3.

6.1Find the open loop transfer function of the system; (8)

6.2Find the static step error constant $k_{p,}$, the static velocity error constant k_v, and the static acceleration error constant k_a. (12)

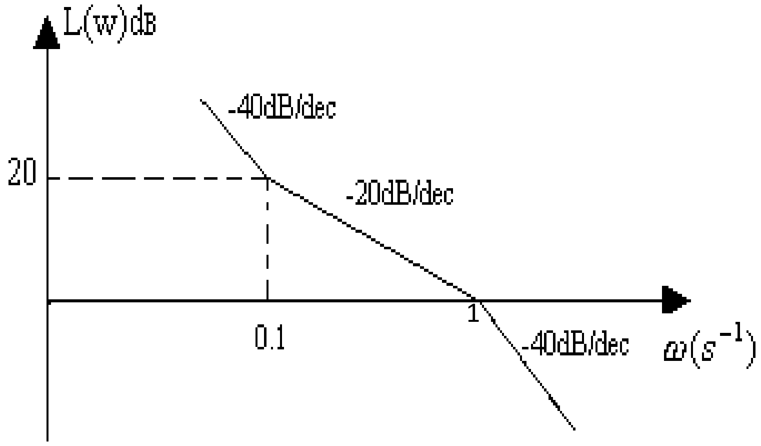


Figure 3.

[20]

ANSWER:

(1)

II-type sytem;

Open loop zero: s=-0.1;

Open loop pole: s=-1.

Must have the form of $G(s)H(s) = \dfrac{K(10s+1)}{s^2(s+1)}$

$$20\log\left|G(j\omega)H(j\omega)\right|_{\omega=0.1} = 20$$

$$20\log(\left|\frac{K(j\omega 10+1)}{-\omega^2(j\omega+1)}\right|_{\omega=0.1}) = 20$$

$$\left|\frac{K(j\omega 10+1)}{-\omega^2(j\omega+1)}\right|_{\omega=0.1} \approx \left|\frac{K}{-\omega^2}\right|_{\omega=0.1} = \frac{K}{0.01} = 10$$

$K = 0.1$

So $G(s)H(s) = \dfrac{0.1(10s+1)}{s^2(s+1)}$

(2) $k_p = \infty, k_v = 100, k_a = 0.1$

[TOTAL: 100]

Annexure A

Laplace transform table:

f(t)	F(s)
$\delta(t)$	1
1(t)	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$t^n (n=1,2,3\cdots)$	$\frac{n!}{s^{n+1}}$
$t^n e^{-at} (n=1,2,3\cdots)$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(b-a)}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}(at-1+e^{=at})$	$\frac{1}{s^2(s+a)}$
$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}t)$	$\frac{{\omega_n}^2}{s^2+2\xi\omega_n s+{\omega_n}^2}$