DSC1520 (478679) October/November 2016

QUANTITATIVE MODELLING I

EXAMINERS
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SECOND MS CH VAN DER MERWE

Programmable pocket calculator is permissible

Closed book examination.

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This paper consists of 21 pages, including paper for rough work, plus instructions for completing a mark-reading sheet. The paper comprises 30 questions that count a total of 100 marks.

Please complete the attendance register on the back page, tear it off and hand it to the invigilator.

Answer ALL questions on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet. Also pay attention to the following:

- Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. Do not mark more than one option per question on the mark-reading sheet.
- Marks will not be deducted for incorrect answers.

You are strongly advised to write your name on the mark-reading sheet. Then, if you have entered your student number incorrectly, we will still be able to link you to the mark-reading sheet.
Question 1
A line cuts the x-axis at 4 and goes through the point (2, 4). The equation of the line is

[1] \( y = 4 \)
[2] \( y = 4 - 2x \)
[3] \( y = -2x + 8 \)
[4] \( y = \frac{1}{2}x + 3 \)
[5] none of the above

Question 2
Simplify the following expression \( \frac{(2a^2b^3)^2 \times (ab^4)^3}{(2a^3b^2)^4} \).

[1] \( \frac{a^{10}}{b^5} \)
[2] \( \frac{b^{10}}{4a^5} \)
[3] \( \frac{a^{10}}{4b^5} \)
[4] \( \frac{b^{10}}{a^5} \)
[5] None of the above

**Questions 3 and 4 are based on the following information**
The demand and total cost functions for a commodity are

\[ Q = 6000 - 30P \quad \text{and} \quad TC = 5000 + 2Q, \]

where \( P \) and \( Q \) are the price and quantity respectively.

Question 3
The total revenue function (\( TR \)) in terms of \( P \) and the company's quantity when the commodity sells for R202 per unit are

[1] \( TR = 6000 - 30P, \; Q = 24 \)
[2] \( TR = P(6000 - 30P), \; Q = 24 \)
[3] \( TR = 6000P - 30P^2, \; Q = 25 \)
[4] \( TR = 6000 - 30P, \; Q = 25 \)
[5] none of the above

Question 4
What is the company's profit function?

[1] Profit = \(-30P^2 + 5940P - 17000\)
[2] Profit = \(-30P^2 + 5940P + 7000\)
[3] Profit = \(-30P^2 + 6060P - 17000\)
[4] Profit = \(-30P^2 + 6060P - 7000\)
[5] None of the above
Question 5
Solve the equation \( 2\ln x - \ln(x + 2) = 0 \)

[1] \( x = 1.99 \)
[2] \( x = 0; \ x = 1 \)
[3] \( x = 1, \ x = 2 \)
[4] \( x = -1, \ x = 2 \)
[5] None of the above

Questions 6 and 7 are based on the following information
Consider the market defined by the following demand and supply functions
\( P_d = 50 - 3Q \) and \( P_s = 14 + 1.5Q, \)
where \( P \) and \( Q \) are the price and quantity respectively

Question 6
Calculate the equilibrium price and quantity
[1] \( P = 26, \ Q = 30 \)
[2] \( P = 30, \ Q = 8 \)
[3] \( P = 8, \ Q = 26 \)
[4] \( P = 26, \ Q = 8 \)
[5] None of the above

Question 7
What is the difference between the quantity supplied and the quantity demanded, if the price is R20?  
[1] \( Q = 2 \)
[2] \( Q = 4 \)
[3] \( Q = 6 \)
[4] \( Q = 10 \)
[5] None of the above

Question 8
If the demand function of a commodity is
\( P = 215 - 5Q, \)
where \( P \) and \( Q \) are the price and quantity respectively, determine the point price elasticity of demand if \( P = 15 \) Is demand elastic or inelastic at this price?
[1] \( \frac{4}{15}, \) elastic
[2] \( -\frac{9}{15}, \) elastic
[3] \( \frac{3}{40}, \) inelastic
[4] \( -\frac{3}{40}, \) inelastic
[5] None of the above
Question 9

Which set of graphs represents the following

A $y - x^2 - 2 = 0$, B $y - x^2 = 0$, C $y - x^2 + 3 = 0$; D $y + x^2 = 0$?

[5] None of the above
Question 10

Choose the correct graphical representation of the following set of inequalities

\[ x + y \leq 13 \quad (1) \]
\[ 2x - y \leq 8 \quad (2) \]
\[ -2x + 3y \leq 12 \quad (3) \]
\[ x, y \geq 0 \quad (4) \]

[5] None of the above
Question 11
The x-intercepts of the curve $2x^3 = -3x^2 + 9x$ are

[1] $x = 0$, $x = \frac{3}{2}$ and $x = -3$
[2] $x = 0$, $x = \frac{3}{2}$ and $x = 3$
[3] $x = 0$, $x = -\frac{3}{2}$ and $x = -3$.
[4] $x = 0$, $x = -\frac{3}{2}$ and $x = 3$
[5] None of the above

Question 12
The coordinates of the turning point of the graph of 

$$y = 4x^2 - x - 3$$

are

[1] $(0,125, -3,0625)$
[2] $(-0,125, -3,0625)$.
[3] $(-3,0625, 0,125)$
[4] $(3,0625, -0,125)$
[5] None of the above

Question 13
A company supplying fitted kitchen units has the following pricing strategy

$$P = 5504 - 0.8Q \quad \text{and} \quad TC = 608580 + 120Q,$$

where $P$ is the price per unit and $Q$ the number of units supplied

Calculate the price and quantity for which profit is a maximum

[1] $P = 1510,40, Q = 4992$
[2] $P = 3365,00, Q = 4992$
[3] $P = 2812,00, Q = 3365$
[4] $P = 1456,80; Q = 5059$
[5] None of the above.

Question 14
The demand function of a product is $P = 58 - 0.4Q$, where $P$ and $Q$ represent the price and quantity respectively. Determine the consumer surplus of the product if the market price $P = 10$

[1] 1296
[2] 1566
[3] 2880
[4] 3480
[5] None of the above
ROUGH WORK
Question 15
Solve the following system of linear equations

\[ \begin{align*}
-p + q + 2r &= 8 \\
p + q + 2r &= 0 \\
2q - 2r &= 2
\end{align*} \]

The sum of the values of \( p, q \) and \( r \) of the solution is

[1] -7
[3] 1
[4] 7
[5] none of the above

Question 16
Find the values of \( x \) for which the function

\[ f(x) = x^3 + 3x^2, \]

has turning points. For each point, state whether it is a minimum or maximum

[1] \( x = -3 \) (maximum), \( x = -6 \) (maximum)
[2] \( x = 0 \) (minimum), \( x = 2 \) (maximum)
[3] \( x = 0 \) (maximum), \( x = -6 \) (minimum)
[4] \( x = 0 \) (minimum); \( x = -2 \) (maximum)
[5] None of the above

Questions 17 and 18 are based on the following information

The daily rate of sales of a product (in units per day) is approximated by the exponential function

\[ S(t) = 1800 + 1500e^{-0.3t+1.5}, \]

with \( t \) the number of days it has been on the market

Question 17
After 20 days the rate of sales of the product is approximately

[1] 37
[3] 1783
[4] 1817
[5] none of the above
Question 18
After how many days, rounded to a whole number, will the rate of sale be 2000 units per day?
[1] 4
[2] 5
[3] 11
[4] 12
[5] None of the above

Question 19
You are baking for a street bazaar and are given 18 kg flour, 36 eggs and 10 kg sugar. You are planning to bake two types of cakes. Each unit of Cake 1 uses 1.8 kg flour, 3 eggs and 0.4 kg sugar. Each unit of Cake 2 uses 0.75 kg flour, 2 eggs and 0.6 kg sugar. If \( x \) is the number of units of Cake 1 to be baked and \( y \) the number of units of Cake 2, choose the system of linear inequalities that describes your appropriate constraints.

[1] \( 1.8x + 0.75y \leq 18, 3x + 2y \leq 36, 0.4x + 0.6y \leq 10, x, y \geq 0 \)
[2] \( 1.8x + 0.75y \geq 18, 2x + 3y \geq 36, 0.4x + 0.6y \leq 10, x, y \geq 0 \)
[3] \( 0.75x + 1.8y \leq 18, 3x + 2y \geq 36, 0.4x + 0.6y \geq 10, x, y \geq 0 \)
[4] \( 1.8x + 0.75y \leq 18; 3x + 2y \leq 36, 0.6x + 0.4y \leq 10, x, y \geq 0 \)
[5] None of the above

Question 20
Total revenue is given by

\[ TR = 2x^5 - \frac{1}{2}x^2 + 10x + 15, \]

where \( x \) is the number of units sold. What is the marginal revenue when five units are sold?
[1] 55
[2] 630
[3] 6255
[4] 6270
[5] None of the above

Question 21
Lighting Warehouse produces \( q \) solar lamps at a fixed cost of R10 000 per week. Each lamp costs R150 to produce and is sold for R350. The total revenue, total cost and profit functions are

[1] \( TR = 350q, TC = 150q + 10 000, \text{ Profit } = 200q - 10 000 \)
[2] \( TR = 150q + 10 000, TC = 200q - 10 000; \text{ Profit } = 350q \)
[3] \( TR = 350q, TC = 200q - 10 000, \text{ Profit } = 150q + 10 000 \)
[4] \( TR = 200q - 10 000, TC = 350q, \text{ Profit } = 150q + 10 000 \)
[5] None of the above
ROUGH WORK
Question 22
The supply function of a certain product is \( P = 80 + 5Q \), where \( P \) is the price and \( Q \) is the number of units produced. Find the producer surplus if the market price \( P = 200 \)

[1] 960
[2] 1 440
[3] 2 400
[4] 3 360
[5] None of the above

Question 23
Find the derivative of the function 

\[ P(x) = x^5 e^{3x} + \frac{x + 1}{x} \]

[1] \( 3x^5 e^{3x} + 5x^4 e^{3x} - \frac{1}{x^2} \)
[2] \( 3x^5 e^{3x} + 5x^4 e^{3x} - \frac{1}{x^2} \)
[3] \( x^5 e^{3x} + 5x^4 e^{3x} - \frac{1}{x^2} \)
[4] \( x^5 e^{3x} + x^4 e^{3x} + \frac{1}{x^2} \)
[5] None of the above

Question 24
Find the derivative

\[ \frac{d}{dQ} (20 + 20 \ln Q)^2 \]

[1] \( 2 (20 + 20 \ln Q) \)
[2] \( \frac{800}{Q} (1 + \ln Q) \)
[3] \( 40 \left( 1 + \frac{1}{Q} \right) \)
[4] \( \frac{20}{Q} (1 + \ln Q) \)
[5] None of the above

Question 25
Integrate the following

\[ \int \sqrt{9x - 5} \, dx \]

[1] \( \frac{1}{6} \sqrt{(9x - 5)^3} + c \)
[2] \( \frac{1}{6} \sqrt{(9x - 5)^3} + c \)
[3] \( \frac{2}{27} \sqrt{(9x - 5)^3} + c \)
[4] \( \frac{2}{27} \sqrt{(9x - 5)^3} + c \)
[5] None of the above
Question 26

Calculate the consumer surplus for the demand function

\[ P = \frac{25}{Q+2}, \]

when the market price \( P = 5 \)

[1] 2,3  
[2] 7,9  
[3] 15,0  
[4] 25,2  
[5] None of the above

Question 27

The marginal cost function for a good is given by

\[ MC = -Q^2 + 80Q. \]

Find the total cost function if fixed costs are 500

[1] \(-\frac{Q^3}{3} + 40Q^2 + 500\)  
[2] \(\frac{Q^3}{3} + 40Q^2 + C\)  
[3] \(-\frac{Q^3}{3} + 80Q^2 + C\)  
[4] \(Q^3 + 80Q^2 + 500\)  
[5] None of the above

Question 28

Determine the following definite integral.

\[ \int_{-1}^{2} (-2x + 3) \, dx \]

[1] -5  
[2] -3  
[3] 3  
[4] 6  
[5] None of the above

Question 29

Find the equilibrium price \( (P) \) and quantity \( (Q) \) for the demand and supply functions

\[ P_d = \frac{21}{Q+2} \] and \[ P_s = 5 + 2Q. \]

[1] \( Q = 7, \ P = 1 \)  
[2] \( Q = 2, \ P = 9 \)  
[3] \( Q = 9, \ P = 2 \)  
[4] \( Q = 1, \ P = 7 \)  
[5] None of the above
Question 30

In the graph below, the following set of inequalities is represented and the feasible area is shaded in grey

\[
\begin{align*}
  & x + y \leq 10 \\
  & 5x + 4y \geq 40 \\
  & x \leq 4 \\
  & x; y \geq 0
\end{align*}
\]

If the profit function is

\[ P = 18x + 12y, \]

determine the maximum profit subject to the set of inequalities above

\[ \begin{align*}
[1] & \quad 120 \\
[2] & \quad 132 \\
[3] & \quad 144 \\
[4] & \quad 154 \\
[5] & \quad \text{None of the above}
\end{align*} \]

TOTAL: [100]

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ROUGH WORK
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**EXAMINATION MARK READING SHEET**

**PART 1 (GENERAL/ALGEMEEN) DEEL 1**

1. **STUDY UNIT NG.** P|T|X
2. **PAPER NUMBER.** VRAESTELNUMMER
3. **INITIALS AND SURNAME.** VOORLETTERS EN VAN
4. **DATE OF EXAMINATION.** DATUM VAN EKSMEN
5. **EXAMINATION CENTRE (E.G. PRETORIA).** EKSMENSENTRUM (V. PRETORIA)

**STUDENT NUMBER.** STUDENTNUMMER
**UNIQUE PAPER NO.** UNIEKE VRAESTEL NR

**IMPORTANT**
1. USE ONLY AN HB PENCIL TO COMPLETE THIS SHEET
2. MARK LIKE THIS .++
3. CHECK THAT INITIALS AND SURNAME HAS BEEN FILLED IN CORRECTLY
4. ENTER YOUR STUDENT NUMBER FROM LEFT TO RIGHT
5. CHECK THAT YOUR STUDENT NUMBER HAS BEEN FILLED IN CORRECTLY
6. CHECK THAT THE UNIEQUE NUMBER HAS BEEN FILLED IN CORRECTLY
7. CHECK THAT ONLY ONE ANSWER PER QUESTION HAS BEEN MARKED
8. DO NOT FOLD

**BELANGRIJK**
1. GEBrUIK SLEGS 'n HB-potlood om hierdie blad te voltooi!
2. MERK AS VOLG ++
3. KONTROLEER DAT U VOORLETTERS EN VAN REG INGEVUL IS
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**PART 2 (ANSWERS/ANTWOORDE) DEEL 2**

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**Specimen only**