DSC1520 (491597) May/June 2016

QUANTITATIVE MODELLING I

Duration 2 Hours 100 Marks

EXAMINERS:
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Programmable pocket calculator is permissible

Closed book examination

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This paper consists of 21 pages, including paper for rough work, plus instructions for completing a mark-reading sheet. The paper comprises 30 questions that count a total of 100 marks.

Please complete the attendance register on the back page, tear it off and hand it to the invigilator.

Answer ALL questions on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet. Also pay attention to the following:

- Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. Do not mark more than one option per question on the mark-reading sheet.
- Marks will not be deducted for incorrect answers.

You are strongly advised to write your name on the mark-reading sheet. Then, if you have entered your student number incorrectly, we will still be able to link you to the mark-reading sheet.
Question 1
A line cuts the x-axis at 3 and goes through the point (1, 3). The equation of the line is

1. \( y = 3 \)
2. \( y = 3 - \frac{3}{2}x \)
3. \( y = -\frac{3}{2}x + \frac{9}{2} \)
4. \( y = \frac{9}{2}x + \frac{9}{2} \)
5. None of the above

Question 2
Evaluate the following expression

\[
\frac{a^6 \sqrt{a^b}}{2a^{0.3}}
\]

1. \( a^{24.7} \)
2. \( 2a^{5.2} \)
3. \( \frac{a^{7.2}}{2} \)
4. \( 2a^{25} \)
5. None of the above

Question 3
A firm has a total cost function

\[
TC = \sqrt{4 + 6Q},
\]

with \( Q \) the number of units produced. If the firm sells each unit for R2, determine the firm’s break-even point.

1. \( Q = 0.5 \)
2. \( Q = 1.5 \)
3. \( Q = 2.0 \)
4. \( Q = 3.0 \)
5. None of the above

Question 4
Evaluate the following expression

\[
\log_2 32x - \log_2 8x
\]

1. \(-4\)
2. \(-2\)
3. \(2\)
4. \(4\)
5. None of the above
Question 5
Consider a market defined by the following demand and supply functions

\[ P_d = 50 - 0.6Q \quad \text{and} \quad P_s = 20 + 0.4Q, \]

where \( P \) and \( Q \) are the price and quantity respectively. Calculate the equilibrium price and quantity.

[1] \( P = 32, Q = 30 \)
[2] \( P = 30, Q = 32 \)
[3] \( P = 68, Q = 30 \)
[4] \( P = 8, Q = 70 \)
[5] None of the above

Question 6
If the demand function of a commodity is

\[ P = 70 - 0.5Q, \]

where \( P \) and \( Q \) are price and quantity respectively, determine the arc price elasticity of demand when the price decreases from \( P \) to \( P - 5 \). Indicate whether demand is elastic or not.

[1] \( \varepsilon_d = -0.4 \), elastic
[2] \( \varepsilon_d = -1.5 \), inelastic
[3] \( \varepsilon_d = \), inelastic
[4] \( \varepsilon_d = 1.5 \); elastic
[5] None of the above

Question 7
Price elasticity of demand measures

[1] the changes in a good's own price
[2] the sensitivity of quantity demanded to change in price
[3] percentage change in quantity demanded
[4] the relationship between price and quantity demanded
[5] none of the above
ROUGH WORK
Question 8

Choose the correct graphical representation of the following set of inequalities

\[ y \geq 5 - 2.5x \quad (1) \]
\[ y \leq 3 - x \quad (2) \]
\[ x \geq 0 \quad (3) \]
\[ y \geq 0 \quad (4) \]

[5] None of the above
ROUGH WORK
Question 9

Which one of the following graphs represents the function

\[ f(x) = -2x^2 + 10x - 8 \]

[1] [2] [3] [4]

[5] None of the above
ROUGH WORK
Question 10
Find the equilibrium price \((P)\) and quantity \((Q)\) for the demand and supply functions

\[ P_d = \frac{50}{Q + 2} \quad \text{and} \quad P_s = 10 + 2Q \]

[1] \(Q = 2, P = 14\)
[2] \(Q = 14, P = 2\)
[3] \(Q = 21, P = 52\)
[4] \(Q = 52, P = 21\)
[5] None of the above

Question 11
Determine the roots of the function

\[ f(x) = x^3 - 3x^2 - 4x \]

rounded to integers

[1] \(x = 0, x = 4, x = -1\)
[2] \(x = 4, x = -1\)
[3] \(x = 0, x = 2, x = -2\)
[4] \(x = 0, x = -4, x = 1\)
[5] None of the above

Question 12
The \( y \)-intercept and the coordinates of the turning point (rounded to two decimal places), of the graph of

\[-4y + 1 = -2x^2 + x\]

are

[1] \(-0.25 \text{ and } (0.25, 0.22) \text{ respectively}\)
[2] \(-0.25 \text{ and } (0.22, 0.25) \text{ respectively}\)
[3] \(0.25 \text{ and } (0.25, 0.22) \text{ respectively}\)
[4] \(0.25 \text{ and } (0.22, 0.25) \text{ respectively}\)
[5] none of the above

Question 13
If the demand function of a commodity is

\[ Q = 40 - 0.2P \]

fixed cost is R\(1\,000\) and variable cost is R\(1\,500\) per unit produced, then the profit function in terms of \(P\) is

[1] \(\text{Profit} = -0.2P^2 - 43P + 1600\)
[2] \(\text{Profit} = -0.2P^2 + 43P - 1600\)
[3] \(\text{Profit} = -0.2P^2 + 1000P - 1600\)
[4] \(\text{Profit} = -0.2P^2 + 37P - 1600\)
[5] none of the above
Question 14

The demand function of a certain product is

\[ P = 120 - 4Q \]

Determine the consumer surplus of the product if the market price is \( P = 80 \)

[1] 90
[2] 150
[3] 200
[4] 450
[5] None of the above

Question 15

Solve the following system of linear equations

\[ \begin{align*}
    x + 2y - z &= 5 \\
    2x - y + z &= 2 \\
    y + z &= 2 \\
\end{align*} \]

The sum of the values of \( x, y \) and \( z \) of the solution is

[1] 0
[2] 0.25
[3] 3.25
[4] 3.75
[5] none of the above

Question 16

Find the values of \( x \) for which the function

\[ f(x) = x^3 - 12x - 6 \]

has a minimum or maximum value

[1] \( x = -3.46, x = 3.46 \)
[2] \( x = 0, x = 6 \)
[3] \( x = -3, x = 3 \)
[4] \( x = -2, x = 2 \)
[5] None of the above
ROUGH WORK
Questions 17 and 18 are based on the following information

The number of people who contracted a contagious disease $t$ days after an epidemic started is approximated by the exponential equation

$$Q(t) = \frac{5000}{2 + 1249e^{-0.3t}}$$

**Question 17**

Approximately how many people had contracted the disease after 15 days?

1. 83
2. 461
3. 564
4. 565
5. None of the above

**Question 18**

After how many days, rounded to an integer, would 1000 people have contracted the disease?

1. 17
2. 18
3. 19
4. 20
5. None of the above

**Question 19**

A furniture manufacturing company manufactures dining room tables and chairs. A table requires 540 minutes for assembly and 180 minutes for finishing. A chair requires 150 minutes for assembly and 60 minutes for finishing. The number of hours available per week for assembly is at least 150 and for finishing it is at most 100. If $x$ is the number of tables and $y$ the number of chairs produced per week, choose the system of inequalities that best describes the situation

1. $9x + 2.5y \leq 150$, $3x + y \geq 100$, $x, y \geq 0$
2. $9x + 2.5y \geq 150$, $3x + y \leq 100$, $x, y \geq 0$
3. $2.5x + 9y \geq 150$, $3x + y \leq 100$, $x, y \geq 0$
4. $540x + 150y \geq 150$, $180x + 60y \leq 100$, $x, y \leq 0$
5. None of the above

**Question 20**

If the roots of a quadratic function are 3 and -2, the coordinates of the turning point are

1. (0.5, -6.25)
2. (-0.5, -6.25)
3. (0.5, 5.25)
4. (-0.5, -5.25)
5. none of the above
ROUGH WORK
Question 21
The demand function of a firm is

\[ Q = 90 - 1.5P, \]

where \( P \) and \( Q \) represent the price and quantity respectively. At what price is revenue a maximum?

[1] 15
[2] 30
[3] 90
[4] 270
[5] None of the above

Question 22
The supply function of a certain product is

\[ P = 50 + 2Q, \]

where \( P \) is the price and \( Q \) is the number of units produced. Find the producer surplus at market equilibrium if the market price is \( P = 90 \)

[1] 260
[2] 290
[3] 400
[4] 600
[5] None of the above

Question 23
Find the derivative of the function

\[ P(Q) = 15 - Qe^Q + \frac{Q^2}{2} \]

[1] \(-e^Q(Q + 1) + Q\)
[2] \(-e^Q(Q - 1) + Q\)
[3] \(e^Q(Q - 1) + Q\)
[4] \(e^Q(1 - Q) + 2Q\)
[5] None of the above

Question 24
Find the derivative of the function

\[ f(x) = 5x^{-1} + 3\sqrt{x^3} \]

[1] \(-\frac{5}{x^2} + \frac{15}{2}\sqrt{x^3}\)
[2] \(-5 + 15\sqrt{x}\)
[3] \(-\frac{5}{x^2} + \frac{15}{2}\sqrt{x}\)
[4] \(-\frac{5}{x^2} + \frac{5}{2}\sqrt{x^3}\)
[5] None of the above
ROUGH WORK
Question 25
Suppose the total cost (in rand) of manufacturing radios is given by

\[ TC = 2Q^4 - Q^2 + 80Q + 150, \]

where \( Q \) is the number of radios manufactured. What is the marginal cost if 10 radios are manufactured?

[1] R80
[2] R660
[3] R700
[4] R2850
[5] None of the above

Question 26
Find the derivative

\[ \frac{d}{dx} \ln 3x^5 \]

[1] \( \ln 3x^5 \)
[2] \( 15x^4 \)
[3] \( 5x^{-1} \)
[4] \( \frac{1}{3x^5} \)
[5] None of the above

Question 27
Evaluate the following definite integral rounded to one decimal

\[ \int_0^1 3x\sqrt{1-2x^2} \, dx \]

[1] 0.5
[2] 1.5
[3] 2.1
[4] 2.6
[5] None of the above

Question 28
Calculate the consumer surplus for the demand function

\[ P = \frac{40}{Q + 3}, \]

when the market price is \( P = 10 \)

[1] 0.3
[2] 1.5
[3] 11.5
[5] None of the above
ROUGH WORK
Question 29
The marginal labour cost function is given by the equation

\[ MLC = 3 + 4L \]

Calculate the cost of employing the first seven labourers

[1] -119
[2] -114
[3] 114
[4] 119
[5] None of the above

Question 30
In the graph below the following set of inequalities is represented, with the feasible area shaded in grey

\[ 10x_1 + 15x_2 \leq 300 \quad (1) \]
\[ x_1 \leq 6 \quad (2) \]
\[ x_2 \geq 12 \quad (3) \]
\[ x_1, x_2 \geq 0 \quad (4) \]

Determine the maximum value of the objective function

\[ P = 6x_1 + 20x_2, \]

subject to the set of inequalities above

[1] 276
[2] 456
[3] 400
[4] 476
[5] None of the above

TOTAL: [100]
ROUGH WORK