DSC1520 (488537) October/November 2017

QUANTITATIVE MODELLING I

Duration 2 Hours 100 Marks

EXAMINERS
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Programmable pocket calculator is permissible.

Closed book examination.

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This paper consists of 21 pages, including paper for rough work, plus instructions for completing a mark-reading sheet. The paper comprises 30 questions that count a total of 100 marks.

Please complete the attendance register on the back page, tear it off and hand it to the invigilator.

Answer ALL questions on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet. Also pay attention to the following:

- Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. Do not mark more than one option per question on the mark-reading sheet.
- Marks will not be deducted for incorrect answers.

You are strongly advised to write your name on the mark-reading sheet. Then, if you have entered your student number incorrectly, we will still be able to link you to the mark-reading sheet.
Question 1
Suppose the cost of manufacturing 10 units of a product is R40 and the cost of manufacturing 20 units is R70. If the cost $C$ is linearly related to output $Q$ (units produced), the cost of producing 35 items is

[1] R71.67  
[2] R113.33  
[3] R115.00  
[4] R15.00  
[5] none of the above

Questions 2, 3 and 4 are based on the following information

The demand and cost functions of a commodity are given by the equations

\[
\text{demand function: } Q = 6000 - 30P \\
\text{cost function: } C(Q) = 5000 + 2Q,
\]

where $P$ and $Q$ are the price and quantity respectively.

Question 2
The total revenue function ($TR$) in terms of $P$ is

[1] $TR = 6000 - 30P$  
[3] $TR = 6000P - 30P^2$  
[4] $TR = 6000 - 30P$  
[5] none of the above

Question 3
What is the company’s profit function?

[1] profit $= -30P^2 + 5940P - 17000$  
[2] profit $= -30P^2 + 5940P + 7000$  
[3] profit $= -30P^2 + 6060P - 17000$  
[4] profit $= -30P^2 + 6050P - 7000$  
[5] None of the above
Question 4
At what price is revenue at a maximum?

[1] 100
[2] 200
[3] 5940
[4] 5970
[5] None of the above

Question 5
A company manufactures bicycle pumps. Each pump sells for R50, and the variable cost of producing each unit is 9% of the selling price. The monthly fixed cost incurred by the company is R50 000. What is the break-even point (rounded off to the nearest integer) for the company?

[1] 917 units
[2] 1 099 units
[3] 1 220 units
[4] 10 000 units
[5] None of the above

Question 6
Employees at a certain company prefer buying hamburgers at their cafeteria. There is usually a demand for 100 hamburgers when the price is R25. It is known that for each R1 increase in price, demand decreases by one hamburger. Determine the demand function with quantity demanded (Q) as a function of price (P).

[1] \( Q = 75 - P \)
[2] \( Q = 125 - P \)
[3] \( Q = -25 - P \)
[4] \( Q = 100(P - 25) \)
[5] None of the above

Question 7
The coordinates of the point of intersection of the lines

\[ 7x + 3y = 45 \]
\[ 10x + 5y = 70 \]

are approximately equal to

[1] \((-1, -7.5)\)
[2] \((8, 3)\)
[3] \((2.88, -5.21)\)
[4] \((3, 8)\)
[5] none of the above
ROUGH WORK
Question 8
Solve the following system of linear equations
\[\begin{align*}
x + y - z &= 3 \quad (1) \\
2x + y - z &= 4 \quad (2) \\
2x + 2y + z &= 12 \quad (3)
\end{align*}\]
The sum of the values of \(x, y\) and \(z\) of the solution is
[1] \(-3\)
[2] \(-1\)
[3] \(7\)
[4] \(19\)
[5] None of the above

Question 9
Simplify \(\frac{(2a^2b^3)^2 \times (ab^4)^3}{(2a^3b^2)^4}\)
[1] \(\frac{b^{10}}{2a^2}\)
[2] \(\frac{b^{10}}{4a^2}\)
[3] \(\frac{b^{20}}{4}\)
[4] \(\frac{b^{10}}{a^2}\)
[5] None of the above

Question 10
Use rules of logarithms to solve the equation
\[3 \ln(2r^2) - 5 \ln r = 7\]
[1] \(0,007\)
[2] \(56\)
[3] \(1,37\)
[4] \(182,67\)
[5] None of the above

Question 11
Evaluate \(\frac{\log_3 12.34}{\ln 12.34}\)
[1] \(3\)
[2] \(-0,9102\)
[3] \(0,9102\)
[4] \(\frac{1}{3}\)
[5] None of the above
ROUGH WORK
Question 12
The demand and supply functions of a product are given by
\[ P_d = Q^2 + 2Q + 5 \text{ and } P_s = 29 - 3Q, \]
where \( P \) and \( Q \) are the price and quantity respectively. Determine the equilibrium price and quantity of the product.

[1] \( P = 20, Q = 3 \)
[2] \( P = 3, Q = 20 \)
[3] \( P = 5, Q = 8 \)
[4] \( P = 8, Q = 5 \)
[5] None of the above

Question 13
The demand function of a product is
\[ P = 58 - 0.4Q \]
where \( P \) and \( Q \) are the price and quantity, respectively. Calculate the consumer surplus for the demand function when the market price is \( P = 10 \).

[1] 1 296
[2] 1 566
[3] 2 880
[4] 3 480
[5] None of the above

Question 14
Calculate the consumer surplus for the demand function
\[ P = \frac{25}{Q + 2} \]
when the market price is \( P = 5 \).

[1] 0.3
[2] 1.5
[3] 7.9
[4] 11.5
[5] None of the above
ROUGH WORK
Question 15
The supply function of a certain product is

\[ P = 50 + 2Q \]

where \( P \) is the price and \( Q \) is the number of units produced. Find the producer surplus at market equilibrium if the market price is \( P = 90 \)

[1] 290
[2] 400
[3] 500
[4] 625
[5] None of the above

Questions 16 and 17 are based on the following information The sales of a product (in units per day) is approximated by the function

\[ S(t) = 1800 + 1500e^{-0.5t} \]

with \( t \) the number of days it has been on the market

Question 16
After how many days, rounded to a whole number will the sales be 2 010 units per day?

[1] 4
[2] 7
[3] 11
[4] 12
[5] None of the above

Question 17
The rate of change in sales 10 days after the product has been on the market, is

[1] 0.0498
[2] -3.32
[4] 74.68
[5] None of the above

Question 18
If the demand function is

\[ P = 50 - 2.5Q \]

where \( P \) and \( Q \) are the price and quantity, respectively, determine the point price elasticity of demand if \( P = 10 \). Is demand elastic or inelastic at this price?

[1] -2.50, demand is elastic
[2] -0.25, demand is inelastic
[3] 0.25, demand is inelastic
[4] 2.50, demand is elastic
[5] -0.25 demand is elastic
ROUGH WORK
Question 19

The roots of the function \( 5x^3 - 6x + 1 = 0 \) are as follows:

1. \( x = 1, x = 0.2 \)
2. \( x = -0.2, x = -1 \)
3. \( x = -0.148, x = 1.348 \)
4. \( x = -1348, \text{ and } x = 0.148 \)
5. None of the above

Question 20

Approximately how many units must be manufactured to maximise a profit defined by the function \( -x^2 + 8x - 16 \)?

1. 1 unit
2. 2 units
3. 3 units
4. 4 units
5. None of the above

Question 21

Find the values of \( x \) for which the function \( y = -x^3 + 9x^2 - 24x + 26 \) has a maximum or a minimum value.

1. \( x = -4, x = 2 \)
2. \( x = 4, x = 2 \)
3. \( x = 4, x = -2 \)
4. \( x = -4, x = 2 \)
5. None of the above

Question 22

Differentiate \( y = 5x^{-1} + 3x^{\frac{1}{3}} + 300 \)

1. \( -\frac{5}{x^2} + \frac{1}{2} \sqrt[3]{x^3} \)
2. \( -5 + \frac{1}{2} \sqrt[3]{x^3} \)
3. \( -\frac{5}{x^2} + \frac{1}{2} \sqrt[3]{x} \)
4. \( -\frac{5}{x^2} + \frac{1}{2} \sqrt[3]{x^3} \)
5. None of the above
Question 23
Find the derivative
\[
\frac{d}{dQ} (20 + 20 \ln Q)^2
\]
\[\begin{array}{ll}
[1] & 40 + 40 \ln Q \\
[2] & \frac{800}{Q} (1 + \ln Q) \\
[3] & \frac{40}{Q} (1 + \ln Q) \\
[4] & \frac{10}{Q} (1 + 2\ln Q) \\
[5] & \text{None of the above}
\end{array}\]

Question 24
Suppose the total cost (in rand) of manufacturing radios is given by
\[TC = 2Q^4 - Q^2 + 8Q + 150,\]
where \(Q\) is the number of radios manufactured. What is the marginal cost if 10 radios are manufactured?
\[\begin{array}{ll}
[1] & \text{R}80 \\
[2] & \text{R}660 \\
[3] & \text{R}700 \\
[4] & \text{R}2850 \\
[5] & \text{None of the above}
\end{array}\]

Question 25
Integrate the function
\[f(x) = \frac{2}{(2x + 1)^3}\]
\[\begin{array}{ll}
[1] & -\frac{1}{(2x+1)^2} + c \\
[2] & -\frac{2}{(2x+1)^3} + c \\
[3] & -\frac{1}{3(2x+1)^3} + c \\
[4] & -\frac{2}{3(2x+1)^3} + c \\
[5] & \text{None of the above}
\end{array}\]
ROUGH WORK
Question 26
Evaluate the following definite integral
\[ \int_{x=1}^{x=5} (2x + x^3) \, dx . \]

[1] 69.25
[2] 132
[3] 180
[4] 180.50
[5] None of the above

Question 27
The marginal labour cost function is given by the equation
\[ MLC = 3 + 4L \]
Calculate the cost of employing the first seven labourers

[1] 31
[2] 114
[3] 114
[4] 119
[5] None of the above

Question 28
The Bargain Furniture Store pays R300 for a chair (C) and R240 for a small side table (T). A recent shipment of 22 units costs the store at least R6,180. If this information is written as a system of equations, it will be represented by

[1] \[ C + T \leq 22, \quad 300C + 240T \leq 6,180 \quad C, T \geq 0 \]
[2] \[ C + T = 22, \quad 300C + 240T \leq 6,180 \quad C, T \geq 0 \]
[3] \[ C + T = 22, \quad 300C + 240T \geq 6,180, \quad C, T \geq 0 \]
[4] \[ C + T \geq 22, \quad 300C + 240T \leq 6,180, \quad C, T \geq 0 \]
[5] None of the above
ROUGH WORK
Question 29

In the graph below the set of inequalities

\begin{align*}
2x + y - 5 &\leq 0 \quad (1) \\
x - 2 &\leq 0 \quad (2) \\
y - 4 &\leq 0 \quad (3) \\
x, y &\geq 0
\end{align*}

were drawn and the feasible region of the set of inequalities is shaded in grey. Determine the maximum value of the function

\[ C = 20x + 30y, \]

subject to the set of inequalities above.

[1] 70
[2] 120
[3] 130
[4] 150
[5] None of the above.
ROUGH WORK
Consider the following system of inequalities:

\[
\begin{align*}
    x + 8y &\leq 400 \quad (1) \\
    x + 2y &\geq 200 \quad (2) \\
    x &\geq 0, \quad y &\geq 0
\end{align*}
\]

The feasible area of this system is given by

[1] - The shaded region on graph 1.

[5] None of the above

TOTAL. [100]
ROUGH WORK
### Part 1 (General/Algemeen), Deel 1

1. **Student Number:** 1234567890
2. **Surname:** Smith
3. **Initials:** J. M. K.
4. **Date of Examination:** 12/03/2023
5. **Examination Centre:** University of South Africa

### Instructions

1. **Use only an HB pencil to complete this sheet.**
2. **Mark like this:**
3. **Check that your initials and surname have been filled in correctly.**
4. **Enter your student number from left to right.**
5. **Check that your student number has been filled in correctly.**
6. **Check that the unique number has been filled in correctly.**
7. **Check that only one answer per question has been marked.**
8. **Do not fold.**

### Part 2 (Answers/Antwoorde), Deel 2

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**Score:** 1200

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**For use by examination invigilator**

**Vir gebruik deur eksamenopsiener**

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**Specimen only**

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