DSC1520 (475501) May/June 2018

Quantitative Modelling I

Duration 2 Hours 100 Marks

EXAMINERS
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Programmable pocket calculator is permissible

Closed book examination.

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This paper consists of 23 pages, including paper for rough work, plus instructions for completing a mark-reading sheet. The paper comprises 30 questions that count a total of 100 marks.

Please complete the attendance register on the back page, tear it off and hand it to the invigilator.

Answer ALL the questions on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet. Also pay attention to the following:

- Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. Do not mark more than one option per question on the mark-reading sheet.
- Marks will not be deducted for incorrect answers.

You are strongly advised to write your name on the mark-reading sheet. Then, if you have entered your student number incorrectly, we will still be able to link you to the mark-reading sheet.
Question 1

If the demand function for a commodity is $Q = 40 - 0.2P$, fixed costs are R1000 and variable costs are R15 per unit produced, then the profit function in terms of $P$ is

1. $\pi = 0.2P^2 + 43P - 1600$
2. $\pi = -0.2P^2 + 43P - 1600$
3. $\pi = -0.2P^2 + 1000P - 1600$
4. $\pi = -0.2P^2 + 37P - 1600$
5. None of the above

Question 2

If fixed costs are R4, variable costs per unit are R1 and the demand function is

$$P = 10 - 2Q,$$

where $P$ is price per item and $Q$ is the number of units produced (in thousands), for what values of $Q$ does the firm break even?

1. 0.5 or 4
2. 500
3. 500 or 4000
4. 4000
5. None of the above

Question 3

The firm's total cost function is given as $TC = 4 + 8Q - Q^2$ and the total revenue as $50Q - 4Q^2$ where $Q$ is the number of units produced and sold (in thousands). How many units should be produced to maximise profit?

1. 7
2. 252
3. 7000
4. 115904
5. None of the above
Question 4
The demand function of a firm is

\[ Q = 90 - 1.5P, \]

where \( P \) and \( Q \) represent the price and quantity respectively. At what price is revenue at a maximum?

1. 15
2. 30
3. 90
4. 270
5. None of the above

Questions 5 and 6 are based on the following information:
Consider the market defined by the following demand and supply functions

\[ P_d = 50 - 0.6Q \quad \text{and} \quad P_s = 20 + 0.4Q, \]

where \( P \) and \( Q \) are the price and quantity respectively. The following graph represents these functions.

![Graph showing demand and supply curves]

Question 5
The coordinates of the intercept are

1. (32, 30)
2. (30, 32)
3. (68, 30)
4. (8, 70)
5. None of the above

Question 6
At the intercept,

1. 30 or 32 units are produced to break even
2. equilibrium is reached when 30 units are produced at R32 per unit
3. consumer surplus is 68 and producer surplus is 30
4. point elasticity of demand is 8 and arc elasticity of demand is 70
5. None of the above
ROUGH WORK
Question 7
Find the equilibrium price \((P)\) and quantity \((Q)\) for the demand and supply functions,

\[
P_d = -Q^2 - 10Q + 150
\]

and

\[
P_s = Q^2 + 14Q + 22
\]

[1] \(Q = 4, P = 94\)
[2] \(Q = 94, P = 4\)
[3] \(Q = 16, P = 320\)
[4] \(Q = 320, P = 16\)
[5] None of the above

Question 8
If the demand function is \(P = 80 - 0.5Q\) where \(P\) and \(Q\) are the price and quantity respectively, determine the expression for price elasticity of demand in terms of \(P\) only

[1] \(\frac{P - 80}{P}\)
[2] \(\frac{P}{P - 50}\)
[3] \(\frac{P}{P - 1600}\)
[4] \(\frac{1000}{P^2 - 50}\)
[5] None of the above

Question 9
If the demand function is

\[
P = -Q^2 - 4Q + 96,
\]

where \(P\) and \(Q\) are the price and quantity respectively, determine the point price elasticity of demand if \(P = 51\). Is demand elastic or inelastic at this price?

[1] \(\varepsilon_d = -0.075\), inelastic
[2] \(\varepsilon_d = -0.53\), elastic
[3] \(\varepsilon_d = 0.73\), inelastic
[4] \(\varepsilon_d = 1.3\), elastic
[5] None of the above

Question 10
Price elasticity of demand measures

[1] the changes in a good's own price
[2] the sensitivity of quantity demanded to change in price
[3] percentage change in quantity demanded
[4] the relationship between price and quantity demanded
[5] none of the above
ROUGH WORK
Question 11
Simplify the expression

$$\frac{e^{x}(1 + e^{1-x})}{e}$$

[1] $e^{x-1} + 1$
[2] $e^{x+1} + 1$
[3] $\frac{e^x}{e} - 1$
[4] $\frac{e + x^2 - e^x}{e}$
[5] None of the above

Question 12
Use the rules of logarithms to solve the equation

$$\log (x - 3) + \log (x + 2) - \log (x - 5) = 0$$

[1] $x = -0.41, x = -2.41$
[2] $x = 2.41, x = -0.41$
[3] $x = 6$
[4] $x = -4$
[5] None of the above

Question 13
An economy is forecast to grow continuously so that the gross national product (GNP) measured in billions of rands, after $t$ years is given by

$$GNP = 80e^{0.02t}$$

After how many years (rounded to two decimal places) is GNP forecast to be R88 billion?

[1] 0.11 years
[2] 0.99 years
[3] 1.11 years
[4] 4.77 years
[5] None of the above
Questions 14 and 15 are based on the following information

Consider the market defined by the following demand and supply functions

\[ P_d = 50 - 3Q \]

and

\[ P_s = 14 + 1.5Q, \]

where \( P \) and \( Q \) are the price and quantity respectively.

**Question 14**
The producer surplus of the product is

- [1] 8
- [2] 48
- [3] 96
- [4] 192
- [5] none of the above

**Question 15**
The consumer surplus of the product is

- [1] 8
- [2] 48
- [3] 96
- [4] 192
- [5] none of the above

**Question 16**
Demand for a certain product is given by the function

\[ P = \frac{40}{Q + 3} \]

Determine the consumer surplus if only one product is demanded

- [1] 0,3
- [2] 1,5
- [3] 11,5
- [4] 21,5
- [5] None of the above
ROUGH WORK
Question 17
Solve the following system of linear equations

\[ \begin{align*}
4x - y + 3z &= 8 \\
-2x + 5y + z &= 4 \\
3x + 2y + 4z &= 9
\end{align*} \]

The sum of the values of \( x, y \) and \( z \) of the solution is

[1] 0
[2] 0.25
[3] 3
[4] 3.75
[5] none of the above

Question 18
You are baking for a street bazaar and are given 18 kg of flour and 10 kg of sugar. You want the cakes to be high protein and therefore want to use at least 36 eggs. You are planning to bake two types of cakes. The chocolate cake uses 1.8 kg of flour, three eggs and 0.4 kg of sugar for each unit of cake. The cheese cake uses 0.75 kg flour, two eggs and 0.6 kg sugar for each unit of cake. If \( x \) is the number of units of chocolate cake and \( y \) the number of units of cheese cake, choose the system of linear inequalities that describes the appropriate constraints

[1] \( 1.8x + 0.75y \leq 18, 3x + 2y \geq 36, 0.4x + 0.6y \leq 10, x, y \geq 0 \)
[2] \( 1.8x + 0.75y \leq 18, 2x + 3y \geq 36, 0.4x + 0.6y \leq 10, x, y \geq 0 \)
[3] \( 0.75x + 1.8y \leq 18, 3x + 2y \leq 36, 0.4x + 0.6y \geq 10, x, y \geq 0 \)
[4] \( 1.8x + 0.75y \leq 18, 3x + 2y \leq 36, 0.6x + 0.4y \geq 10, x, y \geq 0 \)
[5] None of the above
Question 19

\[ 6x + 2y \leq 840 \]
\[ 2x + y \geq 300 \]
\[ x + y \leq 250 \]
\[ x, y \geq 0 \]

The feasible region represented by the above constraints is

[1] 
[2] 
[3] 
[4] 

[5] none of the above
Question 20
Determine the maximum value of $P = 120x + 95y$, subject to the feasible region below

\[ \begin{array}{c}
1 & 20,100 \\
2 & 24,525 \\
3 & 28,800 \\
4 & 42,750 \\
5 & None of the above
\end{array} \]

Question 21
Consider the following graph. The equation and the turning point of the above graph are

\[ \begin{array}{c}
1 & y = -2x - x^2 - 3 and (-5, 1) \\
2 & y = -2x - x^2 - 3 and (1, -4) \\
3 & y = 2x + x^2 - 3 and (1, -5) \\
4 & y = -2x + x^2 - 3 and (1, -5) \\
5 & None of the above
\end{array} \]
Question 22
Differentiate the function

\[ P(x) = x^5 e^{3x} \]

[1] \( 5x^4 e^{3x} \)
[2] \( e^{3x}(3x^5 - 5x^4) \)
[3] \( 6e^{3x} + 8x^4 x^5 \)
[4] \( e^{3x}(3x^5 + 5x^4) \)
[5] None of the above

Question 23
Find the derivative of the function

\[ P(x) = 15 - x\sqrt{3x + 1} + \frac{x}{1 + x} \]

[1] \( -\frac{9x+1}{\sqrt{3x+1}} + \frac{1}{(1+x)^2} \)
[2] \( -\frac{3x}{\sqrt{3x+1}} + \sqrt{3x + 1} + \frac{1}{1+x} \)
[3] \( \frac{3x}{\sqrt{3x+1}} + \frac{1}{(1+x)^2} \)
[4] \( \frac{3x+1}{\sqrt{3x+1}} + \frac{1-2x}{(1+x)^2} \)
[5] None of the above

Question 24
Find the derivative

\[ \frac{d}{dx} \ln 3x^6 \]

[1] \( \ln 3x^5 \)
[2] \( 15x^4 \)
[3] \( 5x^{-1} \)
[4] \( \frac{1}{3x^3} \)
[5] None of the above

Question 25
If the production function is given by

\[ Q = 300\sqrt{L} - 4L, \]

where \( Q \) denotes output and \( L \) denotes the size of the workforce, calculate the value of marginal product of labour if \( L = 9 \)

[1] 11
[2] 16
[3] 46
[4] 146
[5] None of the above
ROUGH WORK
Question 26
Evaluate the definite integral
\[ \int_{0}^{1} 18e^{2x+1} \, dx \] (round to an integer)

[1] 311  
[2] 322  
[3] 932  
[4] 965  
[5] None of the above

Question 27
Evaluate the following definite integral to one decimal place
\[ \int_{0}^{1} 3x \sqrt{1-2x^2} \, dx \]

[1] 0.5  
[2] 1.5  
[3] 2.1  
[4] 2.6  
[5] None of the above

Question 28
What is the marginal cost when \( Q = 8 \), if the total cost function is given by
\[ TC = \frac{1}{3} Q^3 - 8Q^2 + 210Q + 800 \]

[1] 146  
[2] 946  
[3] 1616  
[4] 2138.67  
[5] None of the above

Question 29
If marginal labour costs are given by the equation
\[ MLC = 20 + 8L, \]
calculate the cost of employing the first two labourers

[1] 16  
[2] 36  
[3] 56  
[4] 72  
[5] None of the above
ROUGH WORK
Question 30

Find the values of $x$ for which the function

$$f(x) = x^3 - 12x + 6,$$

has a minimum or maximum value

[1] $x = -3.46, x = 3.46$
[2] $x = 0, x = 6$
[3] $x = -3, x = 3$
[4] $x = -2, x = 2$
[5] None of the above

TOTAL: [100]
ROUGH WORK
For use by examination invigilator

VR GEBRUIK DEUR EKSENAI\nENOPSIENER

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**PART 8 ANSWERS/ANTWOORDDEEL**

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