DSC1520 (494099) May/June 2012

DEPARTMENT OF DECISION SCIENCES
QUANTITATIVE MODELLING I

Duration 2 Hours 100 Marks

EXAMINERS
FIRST MRS MF IMMELMAN
SECOND DR G DAVIE

Programmable pocket calculator is permissible.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This examination question paper consists of 17 pages, including 4 pages for rough work as well as instructions for completion of a mark reading sheet. The paper comprises 25 questions that count a total of 100 marks

Please complete the attendance register on the back page, tear off and hand to the invigilator.

THE USE OF A PROGRAMMABLE CALCULATOR IS PERMITTED.
The paper is divided into two parts Section A and Section B

SECTION A [60]
Answer ALL the questions in this section on the mark-reading sheet supplied. Follow the instructions for completing the mark-reading sheet carefully.
Also pay attention to the following information. Suppose you are asked the question

\[ 3 + 2 \times -1 + 4 - 2 = \]

[1] 7
[2] 1
[3] 3
[4] 4
[5] None of the above

(TURN OVER)
The correct answer is 3 and therefore option [3] should be marked. Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. If you mark more than one option, you will receive no marks for that question. For a correct answer, you will receive 3 MARKS. Marks WILL NOT be deducted for incorrect answers.

Section A consists of 20 questions and counts 60 marks. Hand in the completed mark-reading sheet with your answers for Section B. DO NOT STAPLE IT!

SECTION B

This section must be completed in the space provided below each question. Section B consists of five questions and counts 40 marks.

Remember to include your MARK-READING SHEET IN YOUR ANSWER BOOK.

SECTION A

Question 1

A bicycle cost R1 200 in 2005. The price of the bicycle increased by 10% in 2007 and then again by 25% in 2010 due to an increase in the price of aluminium. What is the current price of the bicycle?

[1] R1 200
[2] R1 230
[3] R1 650
[4] R1 980
[5] None of the above

Question 2

Solve for \(x\)

\[
\frac{x - 1}{x^2 + 5x + 11} = 0
\]

[1] \(x = 0\)
[2] \(x = 1\)
[3] \(x = 2\)
[4] Impossible to solve
[5] None of the above

Question 3

A line cuts the \(x\)-axis at 4 and cuts through the point (2, 4). The equation of the line is

[1] \(y = 4\)
[2] \(y = 4 - 2x\)
[3] \(y = -2x + 8\)
[4] \(y = \frac{1}{2}x + 3\)
[5] None of the above

[TURN OVER]
Question 4
If the variable cost of a product increases by R4 for each unit produced and the fixed cost is R64, determine what the total cost of producing 200 units will be.

[1] R736
[2] R864
[3] R12 796
[4] R13 600
[5] None of the above

Question 5
If the demand function of a product is equal to \( P = 40 - Q \), with \( P \) and \( Q \) being the price and quantity respectively, determine the price elasticity of demand if \( P = 20 \).

[1] -0.025
[2] -0.500
[3] -1.000
[4] -2.000
[5] None of the above

Question 6
Simplify
\[
\frac{3 \frac{1}{2}}{2} \times \frac{3}{4} - 2(\frac{1}{4} + \frac{3}{8})
\]

[1] \( \frac{105}{128} \)
[2] \( 1 \frac{2}{7} \)
[3] \( 2 \frac{1}{10} \)
[4] 3
[5] None of the above
Question 7
Solve the following system of simultaneously linear equations

\[ x + y - z = 3 \]
\[ 2x + y - z = 4 \]
\[ 2x + 2y + z = 12 \]

The sum of the values of \(x, y\) and \(z\) of the solution is

[1] 3
[2] 7
[3] 19
[4] impossible to calculate
[5] none of the above

Question 8
Suppose the total cost (in rand) of manufacturing grass brooms is

\[ TC = 10t^2 - 4t + 14, \]

where \(t\) is the number of grass brooms manufactured. What is the marginal cost if 100 brooms are manufactured?

[1] 5
[2] 996
[3] 1 996
[5] None of the above

Question 9
Evaluate the following definite integral

\[ \int_{x=1}^{x=10} \left( \frac{1}{2}x + 1 \right) dx. \]

[1] 4,50
[3] 33,75
[4] 58,50
[5] None of the above
Question 10
Simplify the following expression

\[ \frac{a^5 \sqrt{a^5}}{2a^{8,3}} \]

[1] \[ a^{24,7} \]
[2] \[ 2a^{5,2} \]
[3] \[ a^{7,2} \]
[4] \[ \frac{a^7}{2} \]
[5] None of the above

Question 11
\[ \log_5 \left( \frac{1200}{\sqrt{25}} \right) \] approximated to five decimal places equals

[1] 0.61584
[2] 0.88106
[3] 2.38021
[4] 3.40531
[5] None of the above

Question 12
The demand function of a product is \( P = 58 - 0.4Q \), where \( P \) and \( Q \) represent the price and quantity respectively. Calculate the consumer surplus for the demand function if the market price is \( P = 10 \)

[1] 1296
[2] 1566
[3] 2280
[4] 3480
[5] None of the above

Question 13
Determine the following indefinite integral

\[ \int (x^2 + 2x^3) \, dx \]

[1] \[ 2x + 6x^2 + c \]
[2] \[ \frac{x^3}{3} + \frac{x^4}{2} + c \]
[3] \[ x^3 + 2x^4 + c \]
[4] \[ \frac{2x^3}{3} + \frac{6x^4}{4} + c \]
[5] None of the above
Question 14
Bacteria in a specific biology experiment grow according to the following formula

\[ B(t) = \frac{8500}{1 + 15e^{-0.4t}} \]

where \( t \) is time in hours and \( B \) is the number of bacteria. How many bacteria are present after four hours?

[1] 206.21
[3] 8 503.03
[4] 8 540.22
[5] None of the above

Question 15
In the graph below, the set of inequalities

\[
\begin{align*}
10P + 15D &\leq 300 \\
P &< 6 \\
D &\geq 12
\end{align*}
\]

were drawn and the feasible region of the set of inequalities is shaded in grey. Determine the maximum value of the function \( F = 6P + 20D \) subject to the set of inequalities above.

[1] 276
[2] 356
[3] 400
[4] 476
[5] None of the above
Question 16
Differentiate the following function $f(x) = x(x^2 + 3x)$

1. $3x^2 + 6x$
2. $x^3 + 3x^2$
3. $3x^2 + 3$
4. $\frac{x^4}{4} + x^3$
5. None of the above

Question 17
The roots of the function $y = x^2 + x - 6$ are

1. $x = 2$ and $x = -3$
2. $x = 3$ and $x = 2,5$
3. $x = -0,5$ and $x = -6,25$
4. $x = -2$ and $x = 3$
5. None of the above

Question 18
The $y$-intercept of the function $-4y = -2x^2 + x - 8$ is equal to

1. $-8$
2. $-4$
3. $-2$
4. $2$
5. None of the above

Question 19
Approximately how many units must be manufactured to maximise a profit defined by the function $y = -2x^2 + 10x - 8^9$

1. 1 unit
2. 3 units
3. 4 units
4. 5 units
5. None of the above
Question 20
Find the values of $x$ for which the function $y = x^3 - 12x + 6$ has a minimum or maximum value

[1] $x = -3.46$ and $x = +3.46$
[2] $x = 0$ and $x = 6$
[3] $x = -2$ and $x = 2$
[4] No maximum or minimum values exist
[5] None of the above

Please turn over for Section B.
SECTION B

Remember to include your MARK-READING SHEET in your answer paper.

*Please fill in your answer to Section A in the spaces provided below before you move on to Section B in case your mark-reading sheet gets misplaced*

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|----------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| Option   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |

This section must be completed in the space provided below each question.

**Question 21**

A furniture manufacturer makes two types of chairs: diningroom chairs and lounge chairs. A diningroom chair takes two hours of sanding and four hours of staining. A lounge chair takes two hours of sanding and three hours of staining. The profit margin on a diningroom chair is R42, while the profit on a lounge chair is R46. If $x$ is the number of the diningroom chairs and $y$ the number of the lounge chairs manufactured, determine the system of inequalities that best describes this situation if only 78 hours are available for sanding and 96 hours are available for staining the chairs. **Do not solve the system of inequalities**
Question 22

The rate of sales of a new product is given by

\[ S(t) = 1800 + 1500e^{-0.3t+0.5} \]

where \( t \) is the number of months the product has been on the market. If the product is taken off the market when the rate of sales drops below 2000, approximately how long will the product be on the market?
Question 23

Draw the following set of inequalities on the grid below and determine the feasible region where all the inequalities are satisfied simultaneously:

\[
\begin{align*}
25x + 40y &\geq 2000 \\
10x + 4y &\geq 400 \\
y &\geq 40 \\
x, y &\geq 0
\end{align*}
\]
Question 24

Suppose the demand function of a product is \( P = 50 - 0.6Q \) and the supply function is \( P = 20 + 0.4Q \), with \( P \) and \( Q \) being the price and quantity, respectively.

(a) Determine the price and quantity for which the market of the product is in equilibrium.

(b) Determine the expression for the price elasticity of demand in terms of \( P \) only.
Question 25

A company has fixed costs of R450, variable costs of R5 per unit and a selling price of R10 per unit.

(a) Determine the expression for profit in terms of the number of units $Q$ sold.

(b) What is the value of $Q$ that will achieve a profit target of R225?

(c) How many units $Q$ should be sold in order for the firm to break even?

TOTAL [100]
ROUGH WORK
ROUGH WORK
ROUGH WORK
ROUGH WORK
### PART 1 (GENERAL/ALGEMEEN) DEEL 1

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For use by examination invigilator
Vir gebruik deur eksamenopsiener

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### IMPORTANT

1. USE ONLY AN HB PENCIL TO COMPLETE THIS SHEET
2. MARK THE BOXES CORRECTLY
3. CHECK THAT YOUR INITIALS AND SURNAME HAS BEEN FILLED IN CORRECTLY
4. ENTER YOUR STUDENT NUMBER FROM LEFT TO RIGHT
5. CHECK THAT YOUR STUDENT NUMBER HAS BEEN FILLED IN CORRECTLY
6. CHECK THAT THE UNIQUE NUMBER HAS BEEN FILLED IN CORRECTLY
7. CHECK THAT ONLY ONE ANSWER PER QUESTION HAS BEEN MARKED
8. DO NOT FOLD

### BELANGRIK

1. GEBRUIK SLEGS N HB POTlood OM HIERdie BLAD TE VOLLOOD!
2. MERK AS VOLG...
3. KONTROLEER DAT U VOORLEKTERS EN VAN REG INGEVUL IS
4. VUL U STUDENTENOMMER VAN LINKS NA REGS IN
5. KONTROLEER DAT U DIE KORRekte STUDENTENOMMER VERSTREEK HET
6. KONTROLEER DAT DIE UNIEKE NOOMER REG INGEVUL IS
7. MAAK SIEKER DAT HET EEN ALTERNATIEF PER VRAAG GEMERK IS
8. MOENIE VOU NIE

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### PART 2 (ANSWERS/ANTWOORDE) DEEL 2

| 1 | c₁ + c₂ + c₃ + c₄ | 26 | c₁ + c₂ + c₃ + c₄ |
| 2 | c₁ + c₂ + c₃ + c₄ | 27 | c₁ + c₂ + c₃ + c₄ |
| 3 | c₁ + c₂ + c₃ + c₄ | 28 | c₁ + c₂ + c₃ + c₄ |
| 4 | c₁ + c₂ + c₃ + c₄ | 29 | c₁ + c₂ + c₃ + c₄ |
| 5 | c₁ + c₂ + c₃ + c₄ | 30 | c₁ + c₂ + c₃ + c₄ |
| 6 | c₁ + c₂ + c₃ + c₄ | 31 | c₁ + c₂ + c₃ + c₄ |
| 7 | c₁ + c₂ + c₃ + c₄ | 32 | c₁ + c₂ + c₃ + c₄ |
| 8 | c₁ + c₂ + c₃ + c₄ | 33 | c₁ + c₂ + c₃ + c₄ |
| 9 | c₁ + c₂ + c₃ + c₄ | 34 | c₁ + c₂ + c₃ + c₄ |
| 10 | c₁ + c₂ + c₃ + c₄ | 35 | c₁ + c₂ + c₃ + c₄ |

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**Specimen only**