DSC1520

DEPARTMENT OF DECISION SCIENCES
QUANTITATIVE MODELLING I

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS MF IMMELMAN
SECOND DR G DAVIE

Programmable pocket calculator is permissible

This examination paper remains the property of the University of South Africa
and may not be removed from the examination venue.

This examination question paper consists of 18 pages, including 4 pages for rough work. The paper
comprises 25 questions that count a total of 100 marks.

THE USE OF A PROGRAMMABLE CALCULATOR IS PERMITTED.

The paper is divided into two parts Section A and Section B

SECTION A

Answer ALL the questions in this section on the mark-reading sheet supplied. Follow the instructions
for completing the mark-reading sheet carefully.
Also pay attention to the following information. Suppose you are asked the question

\[ 3 + 2 \times -1 + 4 \div 2 = \]

[1] 7
[2] 1
[3] 3
[4] 4
[5] None of the above

The correct answer is 3 and therefore option [3] should be marked. Only one option (indicated as [1],
[2], [3], [4], [5]) per question is correct. If you mark more than one option, you will receive no marks
for that question. For a correct answer you will receive 3 MARKS. Marks WILL NOT be deducted
for incorrect answers.

Section A consists of 20 questions and counts 60 marks. Hand in the completed mark-reading sheet
with your answers for Section B. DO NOT STAPLE IT!

[TURN OVER]
SECTION B

This section must be completed in the space provided below each question. Section B consists of five questions and counts 40 marks.

Remember to include your MARK-READING SHEET IN YOUR ANSWER BOOK.

SECTION A

[60]

Question 1

Simplify

\[ 1 + \frac{36}{45} \times \frac{5}{12} - \frac{2}{3} \]

[1] 2
[2] \(\frac{11}{9}\)
[3] \(\frac{2}{3}\)
[4] 1\(\frac{1}{2}\)
[5] None of the above

Question 2

The 2011 price of a mountain bike is 35% lower than in 2010. If the price is R3 315 in 2011, what was the price in 2010?

[1] R2 154,75
[2] R5 100,00
[4] R4 714,3
[5] None of the above

Question 3

Simplify the following expression

\[ \sqrt[4]{\frac{4x^2}{y^{-4}}} \]

[1] 16\(x^4y^8\)
[2] \(\frac{2x^2}{y^{-4}}\)
[3] The expression cannot be simplified
[4] 2\(xy^2\)
[5] None of the above

[TURN OVER]
Question 4
A company’s profit function can be represented by the function

\[ P = -0.2x^2 + 20x + 20 \]

where \( x \) is a decision variable that can assume any value. What is the value of the company’s maximum profit?

[1] 50
[2] 31
[3] 220
[4] 520
[5] None of the above

Question 5
\( \log_{20} \left( \frac{410}{1234} \right) \) approximated to four decimal places equals

[1] 0.0423
[2] 0.0049
[3] –1.1019
[4] –0.3678
[5] None of the above

Question 6
Suppose the quantity demanded for a certain soft drink is given by \( Q = 100 - 2P \), where \( P \) is the price per can and \( Q \) is the number of cans demanded. Determine the value of \( P \) for which the marginal revenue is equal to zero.

[1] 2
[2] 50
[3] 25
[4] 0
[5] None of the above
Question 7
The roots of the function \[ y = x^2 - 4x - 12 \]
are

[1] \( x = 6, x = -2 \)
[2] \( x = -4, x = 2 \)
[3] \( x = 2, x = -16 \)
[4] \( x = 12, x = 1 \)
[5] none of the above

Question 8
Assume that \( t \) is a value so that the lines \( 2y - 4x + 8 = 1 \) and \( y = tx - 4 \) are parallel. What is the value of \( t^2 \)?

[1] 2
[3] -4
[4] 8
[5] None of the above

Question 9
Evaluate the following definite integral
\[
\int_1^3 (2x + 3x^2) \, dx
\]

[1] 34
[2] 12
[3] 188
[4] 9
[5] None of the above

Question 10
Differentiate the following function \( f(x) = x^3 (50 + x) \)

[1] \( f'(x) = 3x^2 (50 + x) \)
[2] \( f'(x) = 150x^2 + 4x^3 \)
[3] \( f'(x) = 50x^2 \)
[4] \( f'(x) = 1 + 150x^3 \)
[5] None of the above
Question 11

Determine the following indefinite integral

\[ \int x (x^2 + 2) \, dx \]

[1] \( \frac{x^4}{4} + x^2 + c \)

[2] \( \frac{x^4}{4} + 2x + c \)

[3] \( 3x^2 + 2 + c \)

[4] \( \frac{x^2}{2} + c \)

[5] None of the above

Question 12

If the demand function is \( P = 90 - 0.05Q \), where \( P \) is the price and \( Q \) is the quantity, determine the expression for the price elasticity of demand in terms of \( P \) only

[1] \( \frac{P - 1800}{P} \)

[2] \( \frac{P - 1800}{P} \)

[3] \( \frac{P}{P - 90} \)

[4] \( \frac{1800}{P - 90} \)

[5] None of the above

Question 13

Solve the following system of linear equations

\[
\begin{align*}
S_1 - 3S_2 &= 0 \\
5S_2 - S_3 &= 10 \\
S_1 + S_2 + S_3 &= 8
\end{align*}
\]

[1] \( S_1 = 6, S_2 = 2, S_3 = 0 \)

[2] \( S_1 = -2, S_2 = 1, S_3 = 1 \)

[3] \( S_1 = 5, S_2 = 1, S_3 = 2 \)

[4] Impossible to calculate

[5] None of the above
Question 14
The demand function of a certain product is \( P = 120 - 3Q \). Determine the consumer surplus of the product if the market price is equal to \( P = 90 \).

[1] 150
[2] 90
[3] 10
[4] 450
[5] None of the above

Question 15
The number of guest houses in Limpopo has been declining over the years and can be approximated by the equation

\[ y = -3,14x + 502, \]

where \( x \) is the number of years since 2000 and \( y \) is the number of guest houses. Approximately how many guest houses will there be in Limpopo in the year 2015 if \( x = 10 \) corresponds to the year 2010?

[1] 470
[2] 455
[3] 5825
[4] 502
[5] None of the above

Question 16
The \( y \)-intercept of the line

\[ 3y - 10x + 5 = 5y \]

equals

[1] \( \frac{1}{2} \)
[2] \( 2\frac{1}{2} \)
[3] 5
[4] -10
[5] None of the above

Question 17
Find the values of \( x \) for which the function \( f(x) = x^4 + 3x^2 \) has a minimum or maximum value.

[1] \( x = 0, x = -2 \)
[2] \( x = 0, x = 2 \)
[3] \( x = 0, x = -6 \)
[4] \( x = -3, x = -6 \)
[5] None of the above
Question 18

In the graph below the following set of inequalities

\[
2x + 6y \geq 30 \\
4x + 2y \geq 20 \\
y \geq 2 \\
x, y \geq 0
\]

was drawn and the feasible region of the set of inequalities shaded in grey

Determine the minimum value of the cost function

\[
Z = 18x + 12y
\]

subject to the set of inequalities above. The minimum cost is equal to

[1] 120
[2] 102
[3] 186
[4] 96
[5] none of the above

[TURN OVER]
Question 19
A trader sells $x$ number of watches at a flea market per day. He has a fixed cost of R120 per day. If he buys a watch wholesale at R30 and sells it for R60, how many watches must he sell per day in order to break even?

[1] 2
[2] 18
[3] 4
[4] 10
[5] None of the above

Question 20
The fish population in a certain lake at time $t$ months is given by the function

$$p(t) = 3000e^{0.1t}$$

After approximately how many months will the fish population reach 10,000?

[1] 12 months
[2] 8 months
[3] 89 months
[4] 33 months
[5] None of the above

Please turn over for Section B
SECTION B

Remember to include your MARK-READING SHEET in your answer paper.

Please fill in your answer to Section A in the spaces provided below before you move on to Section B in case your mark-reading sheet gets misplaced.

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Option

This section must be completed in the space provided below each question.

Question 21

Susan takes Maxivite and Health-high vitamin tablets every day to increase her energy levels. One Maxivite tablet contains 40 mg of iron and 10 mg of vitamin B-1 while, one Health-high tablet provides 10 mg of iron and 15 mg of vitamin B-1. How many tablets of each vitamin should she take if her doctor has recommended that she takes at most 2400 mg of iron and at least 2100 mg of vitamin B-1 per day to increase her energy levels? Write down the system of linear inequalities that describes Susan’s appropriate constraints. Please define your decision variables clearly.
Question 22

Consider the market for a certain homemade jam product defined by the following function

Demand \[ P = 50 - 3Q \]
Supply \[ P = 14 + 1.5Q \]

where \( P \) and \( Q \) are the price and quantity respectively of the product

(a) Calculate the equilibrium price and quantity of the jam
(b) What is the difference between the quantity supplied and that demanded if the price is equal to R40? (5)
Question 23

Draw the lines representing the following constraints

\[ x + y \leq 16 \]
\[ x + 3y \leq 36 \]
\[ x \leq 10 \]
\[ x, y \geq 0 \]

on the grid below and indicate the region where all the constraints are satisfied (feasible region)
Question 24

It is known that the number of soccer tickets demanded at a rural soccer stadium is 1500 if the price per ticket is R40 and the demand is 1000 when the price per ticket is R60. Determine the linear demand function, if it is assumed that a linear relationship exists between the number of tickets sold and the price.
Question 25

The Lunchbox Company sells lunchboxes at a price \( p = 10 - 0.001x \) rand each where \( x \) is the number of lunchboxes produced per week. The cost to produce \( x \) plastic lunchboxes per week is given by \( C = 5000 + 2x \) rand.

(a) Write down the mathematical equations of the revenue and profit functions of the company. (4)

(b) Determine the profit if the company sold 2000 lunchboxes in one week. (2)

(c) How many lunchboxes have been sold in a week if the company made a loss of R5000? (4)

TOTAL [100]
ROUGH WORK
ROUGH WORK
ROUGH WORK
ROUGH WORK
MARK READING SHEET INSTRUCTIONS

Your mark reading sheet is marked by computer and should therefore be filled in thoroughly and correctly.

USE ONLY AN HB PENCIL TO COMPLETE YOUR MARK READING SHEET

PLEASE DO NOT FOLD OR DAMAGE YOUR MARK READING SHEET

Consult the illustration of a mark reading sheet on the reverse of this page and follow the instructions step by step when working on your sheet.

Instruction numbers 1 to 10 refer to spaces on your mark reading sheet which you should fill in as follows:

1. Write your paper code in these eight squares, for instance

   | P | S | Y | 1 | 0 | 0 | - | X |

2. The paper number pertains only to first-level courses consisting of two papers

   WRITE 0 1 for the first paper and 0 2 for the second. If only one paper, then leave blank.

3. Fill in your initials and surname

4. Fill in the date of the examination

5. Fill in the name of the examination centre

6. WRITE the digits of your student number HORIZONTALLY (from left to right). Begin by filling in the first digit of your student number in the first square on the left, then fill in the other digits, each one in a separate square.

7. In each vertical column mark the digit that corresponds to the digit in your student number as follows [-]

8. WRITE your unique paper number HORIZONTALLY.

   NB Your unique paper number appears at the top of your examination paper and consists only of digits (e.g. 403326)

9. In each vertical column mark the digit that corresponds to the digit number in your unique paper number as follows [-]

10. Question numbers 1 to 140 indicate corresponding question numbers in your examination paper. The five spaces with digits 1 to 5 next to each question number indicate an alternative answer to each question. The spaces of which the number correspond to the answer you have chosen for each question and should be marked as follows [-]

   For official use by the invigilator. Do not fill in any information here.