Tutorial letter 201/2/2012

QUANTITATIVE MODELLING

DSC1520

Semester 2

Department Decision Sciences

IMPORTANT INFORMATION:
This tutorial letter contains Solutions to Assignment 01.
Dear Student

I hope at this stage you have worked through chapter 1 to 2 of the textbook and completed your first assignment. As the assignments contain questions from old examination papers you are already in a way preparing for the examination. Practice makes perfect! Try and do as many examples as possible. The more examples you do, the better you will be able to recognise a problem and know how to solve it.

Remember help is just a phone call or e-mail away. Please contact me if you need any help with the second assignment. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)

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ASSIGNMENT 1: SOLUTIONS

Question 1

\[ \frac{2}{3} \div \frac{5}{6} + \frac{5}{4} \div -1 \times 6 \]

\[ \frac{2}{3} \div \frac{5}{6} + \frac{5}{4} \div -1 \times 1 \]

change the mixed fraction to an improper fraction

\[ \frac{2 \times 6}{3} + \frac{5 \times 4}{1} - \frac{4 \times 6}{3} \]

\[ \frac{a \times c}{b} \div \frac{d}{c} = \frac{a \times d}{bc} \]

\[ \frac{12}{15} + \frac{20}{5} - \frac{24}{3} \]

multiply fractions: \( \frac{a \times d}{b} \div \frac{c}{bc} \)

\[ \frac{12 + 60 - 120}{15} \]

common denominator

\[ -48 \]

simplify by dividing nominator and denominator by 3

\[ -16 \]

change the improper fraction to mixed fraction

\[ -3 \frac{1}{5} \]

[Option 4]

Question 2

We have to determine which mark out of 20 is equal to 75%. Let the mark be \( x \). Therefore 75% is equal to

\[ \frac{x}{20} = \frac{75}{100} \]

\[ x = \frac{75}{100} \times 20 \]

\[ x = 15 \]

You scored 15 out of 20.

[Option 3]

Question 3

The \( x \)-intercept is where the line cuts the \( x \)-axis. This means where \( y = 0 \). Thus, the coordinate of the \( x \)-intercept of the line is \((20 ; 0)\). The \( y \)-intercept is where the line cuts the \( y \)-axis. This means where \( x = 0 \). Thus, the coordinate of the \( y \)-intercept of the line is \((0 ; 40)\).

We have to find the equation of the line passing through the points \((20 ; 0)\) and \((0 ; 40)\). The equation of a linear line is: \( y = mx + c \)

Let \((x_1 ; y_1) = (20 ; 0)\) and \((x_2 ; y_2) = (0 ; 40)\)
The slope $m$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40 - 0}{0 - 20} = \frac{40}{-20} = -2$$

Therefore $y = -2x + c$.

Substitute any one of the points in the equation of the line to determine $c$. Let’s choose the point $(0 ; 40)$. Then

$$y = -2x + c$$
$$40 = -2 \times 0 + c$$
$$40 = c$$

The equation of the line is $y = -2x + 40$.

**Question 4**

The cost $y$ of manufacturing $x$ bicycles is given as:

$$y = 240x + 720$$

Now we have to solve the linear equation when the cost or $y = R30 000$. Thus

$$30 000 = 240x + 720$$
$$30 000 - 720 = 240x$$
$$29 280 = 240x$$
$$\frac{29 280}{240} = x$$
$$122 = x$$

If the cost was R30 000, 122 bicycles have been manufactured.

**Question 5**

The cost function is defined as the sum of the variable cost and the fixed cost of the operation.

It is given that a company’s fixed cost is R 560 000 and the variable cost is R9 000 per unit. The total cost is thus: $Cost = \text{Variable cost} + \text{Fixed cost}$ or $Cost = 9 000x + 560 000$.

We have to determine the total cost of producing 140 units, which means we have to solve the cost function when $x$ is 140.
\[ Cost = 9\,000x + 560\,000 \]
\[ Cost = 9\,000(140) + 560\,000 \]
\[ Cost = 1260\,000 + 560\,000 \]
\[ Cost = 1820\,000 \]

The total cost to produce 140 units is R1 820 000.

[Option 2]

**Question 6**

We need to determine the price of the jacket in 2010.

Let the price of the jacket be \( x \).

The cost of a jacket is R800 in 2000. The price in 2004 is 21% higher than in 2000, and the price in 2010 is 25% higher than in 2004.

Therefore, in 2004 the price the price of the jacket was:

\[
x = \text{original price in 2000} + \text{increase of 21%} \\
x = 800 + \left( \frac{21}{100} \times 800 \right) \\
= 800 + 168 \\
= R968.
\]

In 2010 the price of the jacket was:

\[
x = \text{original price in 2004} + \text{increase of 25%} \\
x = 968 + \left( \frac{25}{100} \times 968 \right) \\
= 968 + 242 \\
= 1210
\]

The price of the jacket in 2010 was R1 210.

[Option 1]

**Question 7**

The demand function is given as \( P = 250 - 5Q \).

Now the price elasticity of demand is \( \varepsilon_d = \frac{1}{b} \times \frac{P}{Q} \) with \( a \) and \( b \) the values of the demand function \( P = a - bQ \).
To determine the price elasticity of demand we thus need to determine the values of \( b, Q \) and \( P \). It is given that \( P = 250 - 5Q \) and we have been asked to calculate it in terms of \( P \); thus \( P = P \). Comparing \( P = 250 - 5Q \) with \( P = a - bQ \), we can say that \( a = 250 \) and \( b = 5 \). At this stage \( a, b \) and \( P \) are known and \( Q \) is unknown.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore, if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving for \( Q \).

To determine the value of \( Q \), we need to change the equation of the demand function \( P = 250 - 5Q \) so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \).

Now

\[
\begin{align*}
P &= 250 - 5Q \\
\frac{P - 250}{-5} &= Q \\
Q &= \frac{P - 250}{-5}.
\end{align*}
\]

As we have determined the values of \( b, P \) and \( Q \) we can now substitute them into the formula for elasticity of demand:

\[
\varepsilon_d = -\frac{1}{5} \times \frac{P}{\frac{P - 250}{-5}} = -\frac{1}{5} \times \frac{P}{\frac{P - 250}{1}} = \frac{P}{P - 250}
\]

Or alternatively,

you can use the given formula of price elasticity of demand in terms of \( P \) of a demand function in the form \( P = a - bQ \), given in the textbook on page 78; equation 2.14 (2nd ed) and on page 89; equation 2.14 (3rd ed).

\[
\varepsilon_d = \frac{P}{P - a}
\]

Now \( a = 250 \) (intercept on the \( y \)-axis of the demand function)

\[
\varepsilon_d = \frac{P}{P - 250}.
\]

[Option 3]

Question 8

In general the slope of a line in standard format \( y = mx + c \) has the value \( m \).
To determine the slope of the given line $2x = 3y - 5$, we first need to change the given function to the general format of a line, namely $y = mx + c$.

We need to change the equation so that $y$ is the subject of the equation. This means we write it on its own on one side of the equation. We start with $2x = 3y - 5$ as given.

Move 5 to the left-hand side by adding 5 on both sides of the equation, then divide by 3 to get the coefficient of $y$ to be 1:

$$2x + 5 = 3y - 5 + 5$$
$$2x + 5 = 3y$$
$$\frac{2x}{3} + \frac{5}{3} = \frac{3y}{3}$$
$$\frac{2x}{3} + \frac{5}{3} = y$$

The slope of the line $2x = 3y - 5$ is thus the value of $m$ in the rewritten equation in the form $y = mx + c$ of the given line. Comparing the rewritten formula $\frac{2x}{3} + \frac{5}{3} = y$ of the line with the standard format of a line $y = mx + c$, we conclude that the slope of the line $2x = 3y - 5$ is equal to $\frac{2}{3}$.

[Option 4]

**Question 9**

Profit is equal to revenue (sales) minus total cost. The cost to produce $x$ number of sport hats is given as: $c = 200 + 25x$, and the profit as R3 000. Thus

Profit = Sales – cost or $3 000 = 45x - (200 + 25x)$

Now we have to solve $x$:

$$3 000 = 45x - (200 + 25x)$$
$$3 000 = 45x - 200 - 25x$$
$$3 000 = 20x - 200$$
$$3 000 + 200 = 20x$$
$$3 200 = 20x$$
$$\frac{3 200}{20} = x$$
$$160 = x$$

160 hats were sold to make a profit of R3 000.

[Option 2]
Question 10

The demand function is given as $P = 70 - 0.5Q$. Now the price elasticity of demand is $\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q}$ with $a$ and $b$ the values of the demand function $P = a - bQ$.

To determine the price elasticity of demand, we thus need to determine the values of $b$, $Q$ and $P$. It is given that $P = 70 - 0.5Q$ and the question is asked in terms of $P$. Thus $P = P$. Comparing $P = 70 - 0.5Q$ with $P = a - bQ$, we can say that $a = 70$ and $b = 0.5$. At this stage $a$, $b$ and $P$ are known, and $Q$ is unknown.

The demand function denotes the relationship between the price $P$ and the demand $Q$. Therefore, if $P$ is given, we can derive $Q$ by substituting $P$ into the demand function and solve $Q$.

To determine the value of $Q$ we need to change the equation of the demand function $P = 70 - 0.5Q$ so that $Q$ is the subject of the equation. That means we write $Q$ in terms of $P$. Now

$$P = 70 - 0.5Q$$

$$P - 70 = -0.5Q$$

$$\frac{P - 70}{-0.5} = Q$$

$$Q = \frac{P - 70}{-0.5}.$$  

As we have determined the values of $b$, $P$ and $Q$ we can now substitute them in the formula for elasticity of demand and solve when $P = 20$:

$$\varepsilon_d = -\frac{1}{0.5} \times \frac{P}{P - 70}$$

$$= -\frac{1}{0.5} \times \frac{P}{P - 70} \times \frac{-0.5}{1}$$

$$= \frac{P}{P - 70}$$

$$= \frac{20}{70 - 20}$$

$$= \frac{20}{50} = -0.4$$

The point price elasticity of demand is 0.4 or 0.40 as the zero to the right of the comma is insignificant.

[Option 4]