Tutorial Letter 201/1/2013

QUANTITATIVE MODELLING

Semester 1

Department of Decision Sciences

This tutorial letter contains the solutions for assignment 01.
Dear student

I hope that by this stage you have worked through chapters 1 and 2 of the textbook and have completed your first assignment. As the assignments contain questions from old examination papers you are already, in a way, preparing for the examination. Practice makes perfect! Try to do as many examples as possible, as the more examples you work through the more you will be able to recognise a problem and know how to solve it.

Remember help is just a phone call or e-mail away. You are welcome to contact me if you need any help with the second assignment. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)

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Lastly, I wish you everything of the best with your preparation for the second assignment.

Ms Victoria Mabe-Madisa
Question 1

We have to simplify the fraction

\[
\frac{(x - 4)(x + 3)}{(x - 4)(x + 5)}
= \frac{x + 3}{x + 5}.
\]

The \((x - 4)\) above and below the line cancels out.

[Option 2]

Question 2

\[-6 - 3x \geq 2x\]
\[-3x - 2x \geq 6\]
\[-5x \geq 6\]
\[-x \geq \frac{6}{5}\]
\[x \leq -\frac{6}{5}\]

Multiplying both sides of the inequality by \(-1\). The inequality sign changes.

[Option 2]

Question 3

\[
\frac{2}{3} \div \frac{5}{6} + \frac{5}{4} - \frac{1}{3} \times 6
= \frac{2}{3} \times \frac{6}{5} + \frac{5}{1} \times \frac{4}{5} \times \frac{6}{1}
= \frac{12}{15} + \frac{20}{5} - \frac{24}{3}
= \frac{12}{15} + \frac{60}{15} - \frac{120}{15}
= \frac{-48}{15}
= -3 \frac{3}{15} = -3 \frac{1}{5}
\]

Because \(\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \times \frac{d}{b}\).

Multiply fractions.

Common denominator.

Subtract fractions.

Simplify.

[Option 4]
Question 4

Let the price in 2007 be $x$.

The price in 2009 is 35% lower than in 2007, and is given as R3 315.

Now

$$\text{price in 2009} = \text{price in 2007} - 35\% \text{ of price in 2007}.$$ 

Therefore

$$3315 = x - (35\% \text{ of } x)$$

$$3315 = x - \left( \frac{35}{100} \times x \right)$$

$$3315 = x - 0.35x$$

$$3315 = 1x - 0.35x$$  Take $x$ out as common factor

$$3315 = x(1 - 0.35)$$

$$3315 = 0.65x$$

$$\frac{3315}{0.65} = x$$

$$x = 5100.$$ 

The price of the computer in 2007 was R5 100.

[Option 2]

Question 5

Let the price of the suit be $x$.

The cost of a suit is 800 in 2000. The price in 2001 is 21% higher than in 2000 and, in 2002, it is 25% higher than in 2001.

Therefore, in 2001 the price is

$$x = 800 + \left( \frac{21}{100} \times 800 \right)$$

$$= 800 + 168$$

$$= 968.$$
In 2002 the price is

\[
x = 968 + \left( \frac{25}{100} \times 968 \right)
\]

\[
= 968 + 242
\]

\[
= 1210
\]

Therefore the price in 2002 is R1 210.

[Option 1]

**Question 6**

In general the slope of a line in standard format \(y = mx + c\) has the value \(m\).

To determine the slope of the given line \(2y - 10x + 5 = 0\), we first need to change the given equation to the general format of a line, namely \(y = mx + c\).

We change the equation so that \(y\) is the subject of the equation. This means that we write \(y\) on its own on one side of the equation. We start with \(2y - 10x + 5 = 0\) as given.

Move 5 to the right-hand side by subtracting 5 from both sides of the equation:

\[
2y - 10x + 5 = 0
\]

\[
2y - 10x + 5 - 5 = 0 - 5
\]

\[
2y - 10x = -5
\]

Next we move \(10x\) to the right-hand side by adding \(10x\) to both sides of the equation:

\[
2y - 10x = -5
\]

\[
2y - 10x + 10x = -5 + 10x
\]

\[
2y = -5 + 10x
\]

Lastly to change the equation to the standard format of \(y = mx + c\), we want \(y\) by itself on one side of the equation. Therefore, we divide both sides of the equation by 2:

\[
2y = -5 + 10x
\]

\[
\frac{2y}{2} = \frac{-5 + 10x}{2}
\]

\[
y = \frac{-5}{2} + \frac{10x}{2}
\]

\[
y = \frac{-5}{2} + 5x
\]
The slope of the line \(2y - 10x + 5 = 0\) is thus the value of \(m\) in the rewritten equation in the form \(y = mx + c\) of the given line. Comparing the rewritten formula \(y = \frac{-5}{2} + 5x\) of the line with the standard format of a line \(y = mx + c\), we conclude that the slope of the line \(2y - 10x + 5 = 0\) is equal to 5.

[Option 3]

**Question 7**

The total cost for a wholesaler to purchase \(x\) units is given as \(c(x) = 300 + 0.92x\).

The total revenue from selling \(x\) products is the price \(x\) quantity = 3.10\(x\)

At breakeven point revenue = cost

Therefore \(3.10x = 300 + 0.92x\)

Solving for \(x\) to obtain the number of units: make \(x\) the subject of the equation

\[3.10x = 300 + 0.92x\]
\[3.10x - 0.92x = 300\]
\[2.18x = 300\]
\[x = \frac{300}{2.18} = 137.61 \approx 138\]

138 units should be sold in order to break even.

[Option 4]

**Question 8**

Given the line \(P = 10 + 0.5Q\), we need two points to draw a line. Select any \(P\) or \(Q\) value and calculate the value of the point.

Say we choose \(Q = 0\), then

\[P = 10 + 0.5Q\]
\[= 10 + 0.5(0)\]
\[= 10\]

Therefore point 1 = (0; 10).
Choose \( P = 0 \) then

\[
\begin{align*}
0 &= 10 + 0.5Q \\
-10 &= 0.5Q \\
\frac{-10}{0.5} &= Q \\
Q &= -20
\end{align*}
\]

Therefore point 2 = (–20; 0).

**Please note that you can use any \( P \) and/or \( Q \) value to calculate the two points. Normally \( P = 0 \) and \( Q = 0 \) are used to simplify the calculation.**

Next we plot the two calculated points of the line and draw the line. Now as \( P \) is the subject of the equation we draw the \( P \) value on the y-axis of the graph and \( Q \) on the x-axis of the graph.

**Question 9**

The demand function is \( P = 80 - 2Q \).

Now the price elasticity of demand is

\[
\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q} \quad \text{with } a \text{ and } b \text{ being the values of the demand function } P = a - bQ.
\]

To determine the price elasticity of demand we thus need to determine the values of \( b \), \( Q \) and \( P \). It is given that \( P = 80 - 2Q \) and \( P = P \). By comparing \( P = 80 - 2Q \) with \( P = a - bQ \), we can say that \( a = 80 \) and \( b = 2 \). At this stage \( a \), \( b \) and \( P \) are known and \( Q \) is unknown.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving for \( Q \).

To determine the value of \( Q \) we need to change the equation of the demand function \( P = 80 - 2Q \) so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \). Now

\[
\begin{align*}
P &= 80 - 2Q \\
P - 80 &= -2Q \\
\frac{P - 80}{-2} &= Q \\
Q &= \frac{P - 80}{-2}.
\end{align*}
\]
As we have determined the values of \( b, P \) and \( Q \) we can now substitute them into the formula for elasticity of demand:

\[
\varepsilon_d = -\frac{1}{2} \times \frac{P}{P-80}
\]

\[
= -\frac{1}{2} \times \frac{P}{P-80} \times -\frac{2}{1}
\]

\[
= \frac{P}{P-80}.
\]

Or alternatively,

you can use the given formula of price elasticity of demand in terms of \( P \) of a demand function in the form \( P = a - bQ \), which is given in the textbook on page 78, equation 2.14 (2nd edition) and page 89, equation 2.14 (3rd edition),

\[
\varepsilon_d = \frac{P}{P-a}.
\]

Now \( a = 80 \) (intercept on the \( y \)-axis of the demand function)

\[
\varepsilon_d = \frac{P}{P-80}.
\]

[Option 4]

Question 10

We need to simplify or write \( \sqrt{(x^8)^6} \) in a different way. Now

\[
\sqrt{(x^8)^6}
\]

\[
= \sqrt{x^{8 \times 8}}
\]

because \((a^b)^c = a^{b \times c}\)

\[
= \sqrt{x^{64}}
\]

\[
= x^{64 \times \frac{1}{2}}
\]

because \(\sqrt{a} = a^{\frac{1}{2}}\)

\[
= x^{32}
\]

[Option 3]