Tutorial Letter 201/1/2012

QUANTITATIVE MODELLING

Semester 1

Department of Decision Sciences

This tutorial letter contains the solutions for assignment 01.
Dear student

I hope you are doing well and that you are enjoying this module. At this stage you should have worked through Chapter 1 to 2 of the textbook and completed your first assignment. As the assignments contain questions from old examination papers you are already in a way preparing for the examination. Practice makes perfect! Try and do as many examples as possible. The more examples you work through the more you will be able to recognise a problem and know how to solve it.

Remember help is just a phone call or e-mail away. Please contact me if you need any help with the second assignment. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)

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Lastly, I wish you everything of the best with your preparation for the second assignment.

Ms Victoria Mabe-Madisa
ASSIGNMENT 1: SOLUTIONS

Question 1

\[
\begin{aligned}
\frac{3}{4} \div 2\left(\frac{5}{6} - \frac{1}{2}\right) + \frac{3}{2} \times \frac{5}{2}
&= \frac{3}{4} \div 2\left(\frac{11}{6} - \frac{1}{2}\right) + \frac{3}{2} \times \frac{5}{2} \\
&= \frac{3}{4} \div 2\left(\frac{11-3}{6}\right) + \frac{3}{2} \times \frac{5}{2} \\
&= \frac{3}{4} \div 2\left(\frac{8}{6}\right) + \frac{3}{2} \times \frac{5}{2} \\
&= \frac{3}{4} \div 2\left(\frac{15}{4}\right) \\
&= \frac{3}{4} \times \frac{3}{4} + \frac{15}{4} \\
&= \frac{9 + 120}{32} \\
&= \frac{129}{32} \\
&= 4 \frac{1}{32}
\end{aligned}
\]

[Option 1]

Question 2

\[-3(x + 1) + 6\left(\frac{x}{3} + \frac{1}{3}\right) \leq 4\left(x - \frac{1}{2}\right)\]

multiply the value outside the bracket

with values inside bracket

\[-3x - 3 + 2x + 2 \leq 4x - 4\]

add the like terms

\[-3x + 2x + 2x - 4x \leq -3 + 2 + 2\]

\[-x \leq -1\]

multiplying by – sign changes \(\leq \) to \(\geq\)

\[x \geq +1\]

[Option 3]
**Question 3**

Let the price of the luxury car in 2007 be \( x \).

The price in 2010 is 25% higher than in 2007, and given as R634 000.

Now the price in 2010 = the price in 2007 + 25% of the price in 2007.

Therefore

\[
634000 = x + (25\% \text{ of } x)
\]

\[
634000 = x + \left( \frac{25}{100} \times x \right)
\]

\[
634000 = x + 0.25x
\]

Taking \( x \) out as a common factor in both terms on the right-hand side we get

\[
634000 = x(1 + 0.25)
\]

\[
634000 = x(1.25)
\]

\[
x = \frac{634000}{1.25}
\]

\[
x = R507200
\]

In 2007 the price of the luxury car was R507 200.

[Option 2]

**Question 4**

Let the original price of the T-shirt before the mark-up be \( x \). The current price after the mark-up of 20% is R36.

Now the

\[
\text{current price} = \text{the original price} + \text{20\% mark-up on original price.}
\]

\[
36 = x + (0.2 \times x)
\]

\[
36 = x(1 + 0.2)
\]

\[
36 = 1.2x
\]

\[
\frac{36}{1.2} = x
\]

\[
x = 30
\]

The price of the T-shirt before the mark-up is R30,00

[Option 3]
Question 5

Let \((x_1 ; y_1) = (4 ; 2)\) and \((x_2 ; y_2) = (2 ; 4)\). To determine the equation of a line \(y = mx + c\) we need to determine the slope \(m\) and \(y\)-intercept \(c\) of the line.

Now the slope \(m\) is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{2 - 4} = \frac{-2}{-2} = 1
\]

Therefore \(y = -1x + c\) or \(y = -x + c\).

Next we determine the \(y\)-intercept \(c\) of the line. Now both the given points \((x_1 ; y_1)\) and \((x_2 ; y_2)\) lie on the line. We can thus substitute any one of the points into the equation of the line to determine \(c\). Let’s choose the point \((4 ; 2)\) then

\[
y = -1x + c \\
2 = (-1 \times 4) + c \\
2 = -4 + c \\
2 + 4 = c \\
c = 6
\]

The equation of the line is \(y = -x + 6\).

[Option 3]

Question 6

If two lines are parallel the slopes of the two lines are equal. Meaning that the value of the slope namely \(m\) is the same for both lines.

Now given the equation of the line to be \(y = -3x\). Comparing the given equation with the general format of a straight line namely \(y = mx + c\) we can conclude that slope of the given line is \(m = -3\). Now as stated the new line must, because it is parallel to the line \(y = -3x\), also have a slope of \(m = -3\). Thus the new line has an equation of \(y = -3x + c\).

Next we need to determine the value of \(c\). It was given that the line passes through the point \((1;4)\). That means that the point \((1;4)\) is a realisation of the line meaning that the \(y\)-value is equal to 4 if the \(x\)-value is equal to 1. Therefore

\[
y = -3x + c \\
4 = -3(1) + c \\
4 = -3 + c
\]
Solve $c$ by adding 3 to both sides of the equation:

$$
4 + 3 = -3 + 3 + c \\
7 = c
$$

Thus the equation of the line is $y = -3x + 7$ or $y = 7 - 3x$.

The equation of the line that is parallel to the line $y = -3x$ and that passes through the point (1;4) is equal to $y = 7 - 3x$.

[Option 1]

**Question 7**

Given the line $P = 10 + 0,5Q$, we need two points to draw a line. Select any $P$ or $Q$ value and calculate the value of the point.

Say we choose $Q = 0$, then

$$
P = 10 + 0,5Q \\
P = 10 + 0,5(0) \\
P = 10.
$$

Therefore point 1 = (0; 10).

Choose $P = 0$ then

$$
0 = 10 + 0,5Q \\
-10 = 0,5Q \\
-10 \\
0,5 = Q \\
Q = -20
$$

Therefore point 2 = (–20; 0).

Please note that you can use any $P$ and/or $Q$ value to calculate the two points.

Normally $P = 0$ and $Q = 0$ are used to simplify the calculation.

Next we plot the two calculated points of the line and draw the line. Now as $P$ is the subject of the equation we draw the $P$ value on the y-axis of the graph end $Q$ on the x-axis of the graph:

[Option 3]

**Question 8**

The demand function is $P = 70 - 0,5Q$.

Now the price elasticity of demand is $\varepsilon_d = -\frac{1}{b} \times \frac{P}{Q}$ with $a$ and $b$ the values of the demand function

$$
P = a - bQ.
$$
To determine the price elasticity of demand we thus need to determine the values of \( b \), \( Q \) and \( P \). It is given that \( P = 70 - 0.5Q \) and question asked in terms of \( P \) thus \( P = P \). Comparing \( P = 70 - 0.5Q \) with \( P = a - bQ \), we can say that \( a = 70 \) and \( b = 0.5 \). At this stage \( a \), \( b \) and \( P \) are known and \( Q \) is unknown.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving for \( Q \).

To determine the value of \( Q \) we need to change the equation of the demand function \( P = 70 - 0.5Q \) so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \).

Now

\[
P = 70 - 0.5Q \\
P - 70 = -0.5Q \\
\frac{P - 70}{-0.5} = Q \\
Q = \frac{P - 70}{-0.5}.
\]

As we have determined the values of \( b \), \( P \) and \( Q \) we can now substitute them into the formula for elasticity of demand:

\[
\varepsilon_d = -\frac{1}{0.5} \times \frac{P}{P - 70} \\
= -\frac{1}{0.5} \times \frac{P}{P - 70} \times \frac{-0.5}{1} \\
= \frac{P}{P - 70}.
\]

Or alternatively,

you can use the given formula of price elasticity of demand in terms of \( P \) of a demand function in the form \( P = a - bQ \), given in the textbook on page 78, equation 2.14 (Edition 2) and page 89, equation 2.14 (Edition 3).

\[
\varepsilon_d = \frac{P}{P - a}
\]

Now \( a = 70 \) (intercept on the \( y \)-axis of the demand function)

\[
\varepsilon_d = \frac{P}{P - 70}.
\]

[Option 3]
Question 9

Given is that the cost \( c \) is linear related to the output \( Q \) or \( cost = f(Q) \). Asked is to determine the cost if 35 items were produced or \( cost = f(35) \).

First we thus need to determine the linear cost function \( cost = f(Q) \). We need two points to determine the equation of the cost function \((x_1,y_1)\) and \((x_2,y_2)\) with \( x \) the quantity and \( y \) the cost. It is given that the cost of manufacturing 10 units is R40. Thus \((x_1,y_1) = (10;40)\). Secondly it is given that the cost of manufacturing 20 units is R70. Thus \((x_2,y_2) = (20;70)\).

The cost function \( y = mx + c \) or in terms of cost and quantity \( cost = mQ + c \). We need to determine the slope \( m \) and \( y \)-intercept \( c \) of the line. Now the slope \( m \) is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 40}{20 - 10} = \frac{30}{10} = 3
\]

Therefore \( cost = 3x + c \).

Now both \((x_1 ; y_1)\) and \((x_2 ; y_2)\) lie on the line. We can thus substitute any one of the points into the equation of the line to determine the \( y \)-intercept \( c \). Let’s choose the point \((10;40)\) then

\[
y = 3x + c
40 = 3(10) + c
40 = 30 + c
40 - 30 = c
\]

\[
c = 10
\]

The equation of the cost function is \( cost = 3Q + 10 \).

We need to determine \( cost(35) \):

\[
cost = 3Q + 10
= 3(35) + 10
= 105 + 10
= 115
\]

The cost of manufacturing 35 items is R115,00.

[Option 1]
Question 10

The cost function is defined as the sum of the variable cost and the fixed cost of the swimming club.

It is given that daily fixed cost is R 1 250 and the variable cost R50 per lesson. If the swimming club present \( x \) number of lessons a day then the variable cost is 50 times \( x \) or 50\( x \).

The linear equation for the cost function is thus:

\[
Cost = 50x + 1 250
\]