Tutorial letter 201/1/2017

Quantitative Modelling 1
DSC1520

Semesters 1

Department of Decision Sciences

Solutions to Assignment 1
Dear Student

This tutorial letter contains the solutions to the assignment. Please contact me if you have any questions or need any help with the next assignment.

Kind regards

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**Question 1**

The general equation of a line is $y = mx + c$ where $m$ is the slope and $c$ a constant. Two points on the line are given as $(x_1; y_1) = (4; 0)$ and $(x_2; y_2) = (2; 4)$.

The slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{2 - 4} = \frac{4}{-2} = -2$.

Using the $(4; 0)$, we find $0 = -2(4) + c$ which gives $c = 8$. Therefore the line is $y = -2x + 8$. [Option 3]

**Question 2**

At equilibrium, $P_d = P_s$

$50 - 3Q = 14 + 1.5Q$

$-3Q - 1.5Q = 14 - 50$

$4.5Q = 36$

$Q = 8.$

Substituting this into $P_d$ gives $P = 50 - 3(8) = 26$ [Option 4]

**Question 3**

From $P = 215 - 5Q$ we find $Q = \frac{215 - P}{5} = 43 - 0.2P$. At $P = 15$, $Q = 43 - 0.2(15) = 40$ giving $P_0 = 15$ and $Q_0 = 40$.

From the general demand function $P = a - bQ$ we find that $a = 215$ and $b = 5$. Therefore,

$\varepsilon_d = -\frac{1}{b} \frac{P_0}{Q_0} = -\frac{1}{5} \times \frac{15}{40} = -\frac{3}{40}$ or $\varepsilon_d = \frac{P}{P - a} = \frac{15}{15 - 215} = \frac{15}{-200} = -\frac{3}{40}$

[Option 4]

**Question 4**

Since $|\varepsilon_d| = \left| -\frac{3}{40} \right| = \frac{3}{40} < 1$, demand is inelastic at $P = 15$, meaning that the percentage change in demand is less than the percentage change in price. [Option 3]

**Question 5**

Subtract 2x equation (2) from equation (1) and solve for $y$.

$4x + 3y = 11$ (1)

$-2(2x + y) = 5$ (2)

$y = 1$
Substitute value of \( y \) into any one of equations and solve \( x \). Substitute the value of \( y = 1 \) into say equation (2):

\[
\begin{align*}
2x + y &= 5 \\
2x + 1 &= 5 \\
2x &= 4 \\
x &= 2.
\end{align*}
\]

[Option 2]

**Question 6**

Equilibrium is the price and quantity where the demand and supply functions are equal. Thus \( P_d = P_s \) or \( Q_d = Q_s \). If the demand: \( P_d = 255 - 4Q \) and supply: \( P_s = 25 + 7.5Q \) then

\[
\begin{align*}
255 - 4Q &= 25 + 7.5Q \\
(-7.5 - 4)Q &= -255 + 25 \\
-11.5Q &= -230 \\
Q &= 20.
\end{align*}
\]

To calculate the price at equilibrium, we substitute the value of \( Q \) into the demand or supply function and calculate \( P \). Say we use the demand function, then

\[
\begin{align*}
P_d &= 255 - 4(20) \\
&= 255 - 80 \\
&= 175.
\end{align*}
\]

We need to find the consumer surplus for the demand function

\[
P_d = 255 - 4Q
\]

when the market price \( P = 175 \).

From the textbook page 128, we know that the consumer surplus is calculated as

\[
CS = \text{Amount willing to pay} - \text{Amount actually paid}.
\]

This is determined by calculating the area of the triangle

\[
P_0E_0a
\]

in the following graph which is given by

\[
CS = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times Q_0(a - P_0),
\]

with

\[
\begin{align*}
\& P_0 \text{ the market price,} \\
\& Q_0 \text{ the number of units demanded at price } P_0 \text{ and} \\
\& a \text{ the } y \text{-intercept of the demand function } P = a - bQ.
\end{align*}
\]
In general, we can summarise the steps of determining the consumer surplus as follows:

1. Calculate $Q_0$ if $P_0$ is given (or vice versa).

2. Draw a rough graph of the demand function, with $a$ the $y$-intercept and going through $(Q_0; P_0)$.

3. Calculate the area of $CS = \frac{1}{2} \times Q_0(a - P_0)$.

Draw the demand function by using the point $(20; 175)$ that we found before and $(0; a) = (0; 255)$.

The consumer surplus is the area of the shaded triangle in the sketch, that is

\[
\text{Consumer surplus} = \left[ \frac{1}{2} \times 20 \times (255 - 175) \right] = \left[ \frac{1}{2} \times 20 \times 80 \right] = 800.
\]

The price is equal to 175 and the consumer surplus is equal to 800.

[Option 4]
Question 7

Tax imposed, supplier’s price decreases. Supply function becomes $P - 11.5 = 25 + 7.5Q$ or $P = 36.5 + 7.5Q$. $P_d = 255 - 4Q$ and supply: $P_s = 36.5 + 7.5Q$ then

\[
255 - 4Q = 36.5 + 7.5Q \\
(-7.5 - 4)Q = -255 + 36.5 \\
-11.5Q = -218.5 \\
Q = 19.
\]

And

\[
P_d = 255 - 4(19) \\
= 255 - 76 \\
= 179.
\]

Equilibrium is at $P = 179$ and $Q = 19$. [Option 3]

Question 8

$TR = 350q$, $TC = 150q + 10000$ and $\pi = TR - TC = 350q - (150q + 10000) = 200q - 10000$. [Option 1]

Question 9

The correct graph is the third one (see chapter 9). [Option 3]

Question 10

From the given selling prices we can find total revenue; The welding time for both types of gate cannot be more than the available hours and the same for finishing time. The number of gates produced cannot be negative.

Maximise $TR = 7200x + 665y$

subject to

\[
4.5x + 2y \leq 900 \text{ (Welding time)} \\
x + 2y \leq 400 \text{ (Finishing time)} \\
x, y \geq 0 \text{ (Non-negativity)}
\]

[Option 2]

Question 11

When $P = 200$, then $Q = \frac{200 - 80}{5} = 24$. The producer surplus is the area of the triangle above $P = 200$ and the supply function. That is $PS = 0.5 \times 24 \times (200 - 80) = 1440$. [Option 2]
Question 12

\[ TR = 4.75Q, \; VC = (2.5 + 1.0)Q = 3.5Q \; \text{and} \; FC = 1000. \]; Break even when \( TR = TC \), that is when \( 4.75Q = 3.5Q + 1000 \). This gives \( Q = 800 \). [Option 3]

Question 13

Demand function has negative slope and supply function positive slope. [Option 2]

Question 14

We are given one point on the demand line: \((45; 80)\). Since demand decreases by 2 for each R1 increase in price, the slope of the line is \( m = -2 \). Now, using the point \((45; 80)\), we find \( Q - 80 = -2(P - 45) \) and the equation is \( Q = -2P + 170 \). [Option 1]

Question 15

Was not marked, because it was incorrectly formulated.