Tutorial letter 201/1/2018

Quantitative Modelling 1
DSC1520

Semesters 1

Department of Decision Sciences

Solutions to Assignment 1
Dear Student

This tutorial letter contains the solutions to the first assignment. Please contact me if you have any questions or need any help with the next assignment.

Kind regards

Dr Mabe-Madisa
E-mail: mabemgv@unisa.ac.za
Question 1

To determine the slope of the given line

\[ 6 + 3x - 2y = 0 \]

we need to write the given equation in the standard format of a line, namely

\[ y = mx + c. \]

We therefore need to manipulate the equation so that \( y \) is the subject to find the slope \( m \).

Rewriting the equation with \( y \) at the left gives

\[ 2y = 6 + 3x \]

or

\[ y = 3 + \frac{3}{2}x. \]

The slope of the line

\[ 0 = 6 + 3x - 2y \]

is equal to

\[ \frac{3}{2}. \]

[Option 2]

Question 2

\[ P = 100 - 5Q, \]

so if

\[ Q = 8, P = 60. \]

\[ \varepsilon_d = -\frac{1}{b} \times \frac{P}{Q} = -\frac{1}{5} \times \frac{60}{8} = -\frac{60}{40} = -1.5 < -1. \]

[Option 1]

Question 3

Given the demand function

\[ P = 60 - 0.2Q \]

where \( P \) and \( Q \) are the price and quantity respectively, we have to calculate the arc price elasticity of demand when the price decreases from R50 to R40.

The arc price elasticity of a demand function \( P = a - bQ \) between two prices \( P_1 \) and \( P_2 \) is arc elasticity of demand \( = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2} \) with \( b \) the slope of the demand function and \( P_1, P_2 \) and \( Q_1, Q_2 \) the price and quantity demanded. We can now determine \( Q_1 \) and \( Q_2 \) by substituting \( P_1 = 50 \) and \( P_2 = 40 \) into the equation. Therefore if \( P_1 = 50 \) then \( Q_1 = 300 - 5 \times 50 = 50 \) and if \( P_2 = 40 \) then \( Q_2 = 300 - 5 \times 40 = 100 \).
Elasticity of demand \[= \frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2} \]
\[= \frac{1}{0,2} \times \frac{50 + 40}{50 + 100} \]
\[= \frac{1}{0,2} \times \frac{90}{150} = -\frac{90}{30} = -3. \]

**Question 4**

Equilibrium is the price and quantity where the demand and supply functions are equal. Thus \( P_d = P_s \) or \( Q_d = Q_s \). If the demand: \( Q = 50 - 0,1P \) and supply: \( Q = -10 + 0,1P \) then

\[
50 - 0,1P = -10 + 0,1P \\
-0,2P = -60 \\
P = \frac{-60}{-0,2} = 300.
\]

To calculate the quantity at equilibrium, we substitute the value of \( P \) into the demand or supply function and calculate \( Q \). Say we use the demand function, then

\[
Q = 50 - 0,1(300) \\
= 50 - 30 \\
= 20.
\]

The equilibrium price is 300 and the quantity is 20.

**Question 5**

To solve the inequality

\[
-3(x + 1) + 6(x + \frac{1}{3}) \leq 4\left(x - \frac{1}{2}\right) \\
-3x - 3 + 6x + 2 \leq 4x - 2 \\
-3x + 6x - 4x \leq 3 - 2 - 2 \\
-x \leq -1 \\
x \geq 1.
\]

**Question 6**

To draw the line

\[2P = 20 - Q\]

or

\[P = 10 - 0,5Q,\]
we need two points on the line. Therefore select any two values for \( P \) or \( Q \) and find the coordinates.

To simplify the calculations, we choose \( Q = 0 \) to find

\[
P = 10 - 0.5(0) = 10,
\]
giving the coordinate \((0; 10)\).

We also choose \( P = 0 \) to find

\[
0 = 10 - 0.5Q
\]
resulting in

\[
Q = 20.
\]
The second coordinate is \((20; 0)\).  

[Option 2]  

**Question 7**  

One point is \((40; 80)\). Demand decreases by 3 if price increases by R5, therefore slope is \(\frac{3}{5}\).

\[
Q - 80 = -\frac{3}{5}(P - 40)
\]
\[
Q = -0.6P + 24 + 80
\]
\[
= -0.6P + 104.
\]

[Option 1]  

**Question 8**  

\(A = (0; 952), B = (0; 400), C = (119; 0)\)  

[Option 4]  

**Question 9**  

Equilibrium; \(D = (46; 584)\)  

[Option 2]  

**Question 10**  

We need to solve the following system of equations:

\[
x + y + z = 8 \quad (1)
\]
\[
x - 3y = 0 \quad (2)
\]
\[
5y - z = 10 \quad (3)
\]

Make \( x \) the subject of equation (2) and \( z \) the subject of equation (3):

\[
x = 3y \quad (4)
\]
\[
z = -10 + 5y \quad (5)
\]
Substitute equation (4) and equation (5) into equation (1):

\[
x + y + z = 8 \\
(3y) + y + (-10 + 5y) = 8 \\
y = \frac{18}{9} = 2.
\]

Substitute \( y = 2 \) into equation (4) and equation (5):

\[
x = 3y = 3 \times 2 = 6
\]

and

\[
z = -10 + 5y = -10 + 5(2) = -10 + 10 = 0
\]

Therefore

\[
x = 6; \ y = 2
\]

and

\[
z = 0.
\]

The sum is 8.

[Option 4]

**Question 11**

\[
136 - 4Q = 14 + 5Q \\
9Q = 122 \\
Q_c = 13,56
\]

\[
P_c = 136 - 4Q \\
= 136 - 4(13,56) \\
= 81,76.
\]

Producer surplus:

\[
P_s = \frac{1}{2}(13,56)(81,78 - 14) \\
= 459,41.
\]

[Option 1]

**Question 12**

The system of equations is

\[
10X + 20Y + 30Z \geq 1,800 \\
30X + 20Y + 40Z \leq 2,800 \\
20X + 40Y + 25Z \geq 2,200.
\]

[Option 3]
Question 13

[Option 2]

Question 14

\[
\begin{align*}
2x + 6y & \geq 30 \quad (1) \\
4x + 2y & \geq 20 \quad (2) \\
y & \geq 2 \quad (3)
\end{align*}
\]

The corner points of the feasible region are the points A, B, C.

Point A:
The point where the line (2) cuts the \(y\)-axis.
Point A is the point \((0; 10)\).

Point B:
The point where lines (1) and (2) intersect, therefore
\[
4x + 2y = 20 \quad \text{and} \quad 2x + 6y = 30
\]
\[
\begin{align*}
-2x + \frac{2}{6}x & = 5 - 10 \\
-\frac{10}{6}x & = -5 \\
x & = 3.
\end{align*}
\]

Substituting the value of \(x = 3\) into equation (2) gives
\[
\begin{align*}
y & = 10 - 2x \\
& = 10 - 2(3) \\
& = 4.
\end{align*}
\]

Point B is the point \((3; 4)\).

Point C is the point where lines (3) and (1) intersect.
\[
y = 2 \quad \text{and} \quad 2x + 6y = 30
\]
Substituting the value of \( y = 2 \) into equation (1) gives

\[
2x + 6(2) = 30
\]

\[
x = \frac{18}{2}
\]

\[
x = 9.
\]

Point C is the point (2; 9).

<table>
<thead>
<tr>
<th>Corner points of feasible region</th>
<th>Value of ( Z = 18x + 12y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: ( x = 0; \ y = 10 )</td>
<td>( Z = 1,8(0) + 1,2(10) = 12,0 )</td>
</tr>
<tr>
<td>B: ( x = 3; \ y = 4 )</td>
<td>( Z = 1,8(3) + 1,2(4) = 10,2 ) ← Minimum</td>
</tr>
<tr>
<td>C: ( x = 9; \ y = 2 )</td>
<td>( Z = 1,8(9) + 1,2(2) = 18,6 )</td>
</tr>
</tbody>
</table>

Minimum of \( Z \) is at point B where \( x = 3, y = 4 \) with \( Z = 10,2 \).

[Option 2]

**Question 15**

We need to find the consumer surplus for the demand function

\[
P = 90 - 5Q
\]

when the market price \( P = 20 \).

\( CS \) = Amount willing to pay – Amount actually paid.

This is determined by calculating the area of the triangle \( P_0E_0a \) in the following graph which is equal to

\[
CS = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times Q_0(a - P_0)
\]

with

\>
\( P_0 \) the market price,

\>
\( Q_0 \) the number of units demanded at price \( P_0 \), and

\>
\( a \) the \( y \)-intercept of the demand function \( P = a - bQ \).

In general we can summarise the steps of determining the consumer surplus as follows:

1. Calculate \( Q_0 \) if \( P_0 \) is given.

2. Draw a rough graph of the demand function.

3. Read the value of \( a \) from the demand function – that is the \( y \)-intercept of the demand function.

4. Calculate the area of \( CS = \frac{1}{2} \times Q_0(a - P_0) \).
First we need to determine $Q$ from the demand function $P = 90 - 5Q$ if $P = 20$. That is

$$20 = 90 - 5Q,$$

giving

$$Q = 14.$$

Draw the demand function by using the point $(14; 20)$ found before and $(0; a) = (0; 90)$.

The value of $a$ is found by substituting $Q = 0$ into the demand function, that is

$$P = 90 - b0 = 90.$$

The consumer surplus is the area of the shaded triangle in the sketch, that is

$$\text{CS} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 14 \times (90 - 20)$$

$$= 490.$$

The consumer surplus is equal to 490 if the price $P = 20$. 

[Option 2]