Tutorial letter 202/1/2017

Quantitative Modelling 1
DSC1520

Semester 1

Department of Decision Sciences

Solutions to Assignment 2
Dear Student

Here are the solutions to the second compulsory assignment. Please contact me if you have any questions or need any help with the third assignment. My contact details are as follows:

Office: Room 4-37, Hazelwood Campus, Unisa
Tel: +27 12 433 4602
E-mail: mabemgv@unisa.ac.za

Kind regards

Dr Victoria Mabe-Madisa
Question 1

1. Non-linear

2. Cubic function / quadratic

Question 2

\[ \frac{50}{x+2} = 2x + 10 \]
\[ (2x + 10)(x + 2) = 50 \]
\[ 2x^2 + 14x - 30 = 0 \]
\[ x^2 + 7x - 15 = 0. \]

Using quadratic formula: 
\[ x = \frac{-7 \pm \sqrt{7^2 - 4(-15)}}{2} = \frac{-7 \pm 10.44}{2} = 1.72 \text{ or } -8.72. \]

Question 3

(a) \[ \frac{x^2 + 7x + 12}{x + 3} = \frac{(x + 3)(x + 4)}{x + 3} = x + 4; \]
(b) \[ \sqrt{\frac{x^3 y^4}{x^2}} = x^2 y^2. \]
(c) \[ \frac{(3^{-1}a^4 b^{-3} c^{-2})}{(6a^2 b^3 c^{-2})^2} = \frac{9b^8 c^4}{36a^4} = \frac{b^8 c^4}{4a^4}. \]

Question 4

(a) \[ \ln (x - 1) + \ln 3 = \ln (3(x - 1)) = \ln (3x - 3) = \ln x. \] Taking \( e^y \) on both sides, gives \( 3x - 3 = x \) or \( x = \frac{3}{2}. \)
(b) \[
3 \ln(5x) - 2 \ln x = 7 \\
\ln(5x)^3 - \ln x^2 = 7 \\
\ln \left( \frac{125x^3}{x^2} \right) = 7 \\
\frac{125x^3}{x^2} = e^7 \\
125x = 1096 \\
x = 8.77.
\]

**Question 5**

(a) Turning point at \( x = -\frac{b}{2a} = -\frac{6}{2 \times 2} = -\frac{6}{4} = -1.5 \). The function value at this point is \( f(-1.5) = 2(-1.5)^2 + 6(-1.5) - 20 = -24.5 \). The coordinates are \((-1.5; -24.5)\).

(b) The roots of \( f \) are the \( x \) values for which \( f(x) = 0 \), that is \( 2x^2 + 6x - 20 = (2x - 4)(x + 5) = 0 \). Solving this gives the roots of \( f \) as \( x = 2 \) or \( x = -5 \).

(c)

**Question 6**

(a) The roots are at \( x = -2 \) and \( x = 1 \), and the turning points are \((1; 0)\) and \((-1; 4)\).

(b) The roots are at \( x = -2 \) and \( x = 1 \), and the turning points are \((1; 0)\) and \((-1; 4)\).

**Question 7**

(a) From the given demand function, \( P = 200 - \frac{Q}{30} \). From this, \( TR = PQ = (200 - \frac{Q}{30})Q = 200Q - \frac{Q^2}{30} \).

(b) Profit \( \pi = TR - TC = 200Q - \frac{Q^2}{30} - (5000 + 20Q) = -\frac{Q^2}{30} + 180Q - 5000 \).
Question 8

(a) \( TR = 100Q - 20Q^2, TC = 10Q + 40 \) and \( \pi = -20Q^2 + 90Q - 40 \).

(b) Break-even where \( TR = TC \) or \( \pi = 0 \), that is when \(-20Q^2 + 90Q - 40 = 0 \). This gives \( Q = 4 \) and \( Q = 0,5 \) units. When 500 and 4000 units are produced, the company breaks even.

(c) Maximum profit at the turning point of the profit function, that is at \( Q = \frac{-b}{2a} = \frac{-90}{2(-20)} = 2.25 \). They should produce 2250 units for max profit. The optimum price is \( P(2,25) = 100 - 20(2,25) = 55 \), or R55.

(d) The required graph:

![Graph](image)

Question 9

(a) (i) \[
F(t) = 100 - 95e^{-0.15t}
\]
\[
F(0) = 100 - 95e^{-0.15(0)} = 5\%.
\]

(ii) \[
F(20) = 100 - 95e^{-0.15(20)}
\]
\[
= 100 - 95e^{-3}
\]
\[
= 95.25\%.
\]

(iii) Work out with any number of weeks of your choice greater than 20.

(b) \[
\frac{50 - 100}{-9.5} = e^{-0.15t}
\]
\[
\ln\left(\frac{10}{19}\right) = -0.15t \ln e
\]
\[
t = \frac{\ln\left(\frac{10}{19}\right)}{-0.15} = \frac{-0.6419}{-0.15} = 4.28 \text{ weeks}.
\]

Question 10

(a) Equilibrium when demand and supply are equal, that is \( P_d = P_s \) or
\[
Q^2 + 2Q + 7 = 25 - Q
\]
\[
Q^2 + 3Q - 18 = 0
\]
\[
(Q + 6)(Q - 3) = 0
\]
This gives $Q = -6$ or $Q = 3$. From the demand function we find $P = 25 - Q$. At equilibrium $P = 22$. They should therefore produce 3 units.

(b) The required graph: