Tutorial letter 202/1/2018

Quantitative Modelling 1

DSC1520

Semester 1

Department of Decision Sciences

Solutions to Assignment 2

Bar code

Define tomorrow.

UNISA university of south africa

Dear Student

This tutorial letter contains the solutions to the second compulsory assignment. Please contact me if you have any questions or need any help with the next assignment.

Kind regards

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Question 1

1.
$$\begin{aligned}
& \frac{(2a^2b^3)^2 \times (ab^4)^3}{(2a^3b^2)^4} \\
= & \frac{(4a^4b^6) \times (a^3b^{12})}{16a^{12}b^8} \\
= & \frac{a^{4+3-12}b^{6+12-8}}{4} = \frac{a^{-5}b^{10}}{4} = \frac{b^{10}}{4a^5}.
\end{aligned}$$

2.
$$\begin{aligned}
\sqrt{\tfrac{4x}{y^{-4}}} &= \left(\sqrt{4x} \times \sqrt{y^4}\right) \\
&= (4x)^{\frac{1}{2}} \times y^4)^{\frac{1}{2}} \\
&= (4^{\frac{1}{2}}x^{\frac{1}{2}} \times y^2) \\
&= 2y^2\sqrt{x}
\end{aligned}$$

Question 2

1.
$$-x^2 + 8x - 16.$$

For roots we make use of the quadratic formula

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
x &= \frac{-8 \pm \sqrt{(8)^2 - 4(-1)(-16)}}{2(-1)} \\
&= \frac{-8 \pm \sqrt{0}}{-2} \\
&= 4.
\end{aligned}$$

2.
$$\begin{aligned}
y &= -x^2 + 8x - 16 \Rightarrow a = -1, b = 8, c = -16 \\
x &= \frac{-b}{2a} = \frac{-(8)}{2(-1)} = \frac{-8}{-2} = 4 \text{ units}
\end{aligned}$$

Question 3

1. Revenue or Income is defined as price times quantity or $R = p \times q$ or $p \times x$. Now given is quantity as x and price is given as $p(x) = 10 - \dfrac{x}{1000}$. Thus

$$\begin{aligned}
\text{Revenue} &= p \times x \\
&= (10 - \tfrac{x}{1\,000}) \times x \\
&= 10x - \tfrac{x^2}{1\,000} \text{ or } 10x - 0{,}001x^2.
\end{aligned}$$

2. (a)

$$\begin{aligned}
\text{Profit} &= \text{revenue} - \text{cost}\\
\text{Price} &= -\frac{-b}{2a}\\
&= \frac{-10}{2(-\frac{1}{1000})}\\
&= 5\ 000 \text{ or Price} = (10 - \frac{x}{1\ 000}) = \text{R}5
\end{aligned}$$

(b) Maximum revenue $= (10 - \frac{5\ 000}{1\ 000})(5\ 000) = 25\ 000.$

(c) The number of radios to produce to realize maximum profit

$$\begin{aligned}
\text{profit} &= \text{revenue-cost}\\
&= 10x - \tfrac{x^2}{1\ 000} - 2x - 5\ 000\\
&= -\tfrac{x^2}{1\ 000} + 8x - 5\ 000.
\end{aligned}$$

$$\begin{aligned}
\text{number of radios} &= -\tfrac{-b}{2a}\\
&= \tfrac{-8}{2(-\frac{1}{1000})}\\
&= 4\ 000.
\end{aligned}$$

(d) Maximum profit $= -(\frac{4\ 000^2}{1\ 000}) + 8(4\ 000) - 5\ 000 \ = 11\ 000$

(e) The price that the company should charge for each radio to realize maximum profit

$$\text{At maximum profit, } p = 10 - \frac{4\ 000}{1\ 000} = 6.$$

Question 4

1. Given the function $f(x) = -2x^2 + 10x - 8$. To graph the function we need to determine the vertex, roots and y-intercept of the function.

   The graph has a maximum point since $a < 0$.

   The y-intercept is -8.

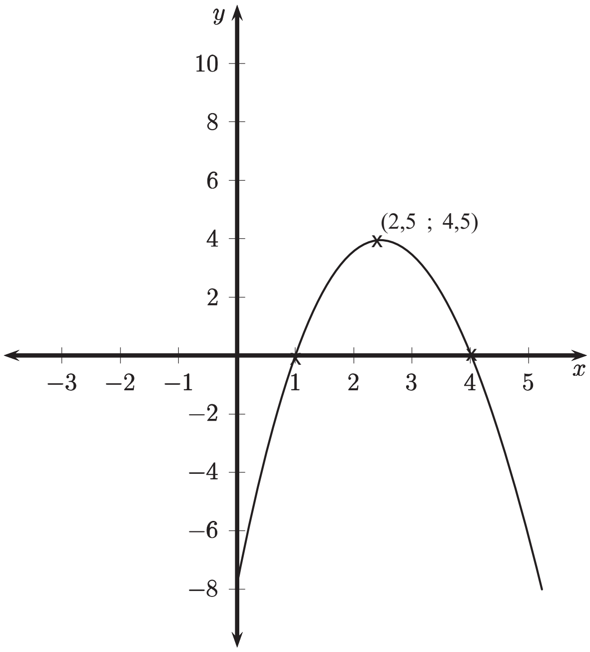
   Vertex is the point:

$$\begin{aligned}
x &= \frac{-b}{2a} = \frac{-10}{2(-2)} = 2{,}5\\
y &= -2(2{,}5)^2 + 10(2{,}5) - 8 = 4{,}5.
\end{aligned}$$

   The roots are:

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-(10) \pm \sqrt{(10)^2 - 4(-2)(-8)}}{2(-2)}\\
&= \frac{-10 \pm 6}{-4}\\
&= 1 \text{ or } 4
\end{aligned}$$

2. Thus the graph of the function $f(x) = -2x^2 + 10x - 8$ is:



Question 5

1. Initial means $t = 0$

$$P = \frac{6\ 000}{1 + 29e^{-0,4\times 0}} = \frac{6\ 000}{30} = 200.$$

2. If $P = 4\ 000$ then

$$\begin{aligned} 4\ 000 &= \frac{6\ 000}{1 + 29e^{-0,4t}} \\ 1 + 29e^{-0,4t} &= \frac{3}{2} \\ e^{-0,4t} &= \frac{1}{58} \end{aligned}$$

$$\ln(e^{-0,4t}) = \ln\left(\frac{1}{58}\right)$$

$$-0,4\,t\ln e = \ln(\tfrac{1}{58})$$

$$\begin{aligned} t &= \frac{\ln(\frac{1}{58})}{-0,4} \\ &= 10,15110753 \\ &= 10,2 \text{ years, rounded to one decimal place.} \end{aligned}$$

Question 6

1.

$$\begin{aligned}
\log(Q) - \log\left(\tfrac{Q}{Q+1}\right) &= \log\left(\frac{Q}{\frac{Q}{Q+1}}\right) = 0{,}8 \\
\log(Q+1) &= 0{,}8 \\
Q+1 &= 10^{0{,}8} \\
Q &= 10^{0{,}8} - 1 \\
Q &= 5{,}309573445 \\
&= 5{,}31, \text{rounded to 2 decimal places.}
\end{aligned}$$

2.

$$\begin{aligned}
\left(\frac{4L^2}{L^{-2}}\right)^2 &= (4L^2 \times L^2)^2 \\
&= (4L^4)^2 = 16L^8.
\end{aligned}$$

Question 7

1.

$$\begin{aligned}
3\ln(2x^2) - 5\ln x &= 7 \\
\ln \tfrac{8x^6}{x^5} &= 7 \\
8x &= e^7 \\
x &= \frac{1096}{8} = 137.
\end{aligned}$$

2.

$$\begin{aligned}
\frac{\log_3 12{,}34}{\ln 12{,}34} &= \frac{\ln 12{,}34}{\ln 3} \times \frac{1}{\ln 12{,}34} \\
&= 0{,}9102.
\end{aligned}$$

Question 8

The cost function is a linear function. We need two points to draw the line of the cost function. Choose the two points where the lines cut through the x-axis ($x-$axis intercept, thus $y = 0$) and y-axis ($y-$axis intercept, $x = 0$). Calculate $(0; y)$ and $(x; 0)$ and draw a line through the two points. Therefore:

If $P = 0$ then

$$\begin{aligned}
C\left(P\right) &= 432\,000 - 1\,800\left(0\right) \\
&= 432\,000
\end{aligned}$$

the point $(0\ ;\ 432\,000)$,

and if $C(P) = 0$ then

$$\begin{aligned} 0 &= 432\,000 - 1\,800\,(P) \\ 1\,800P &= 432\,000 \\ P &= \frac{43200}{1800} = 240 \end{aligned}$$

the point $(240\ ;\ 0)$.

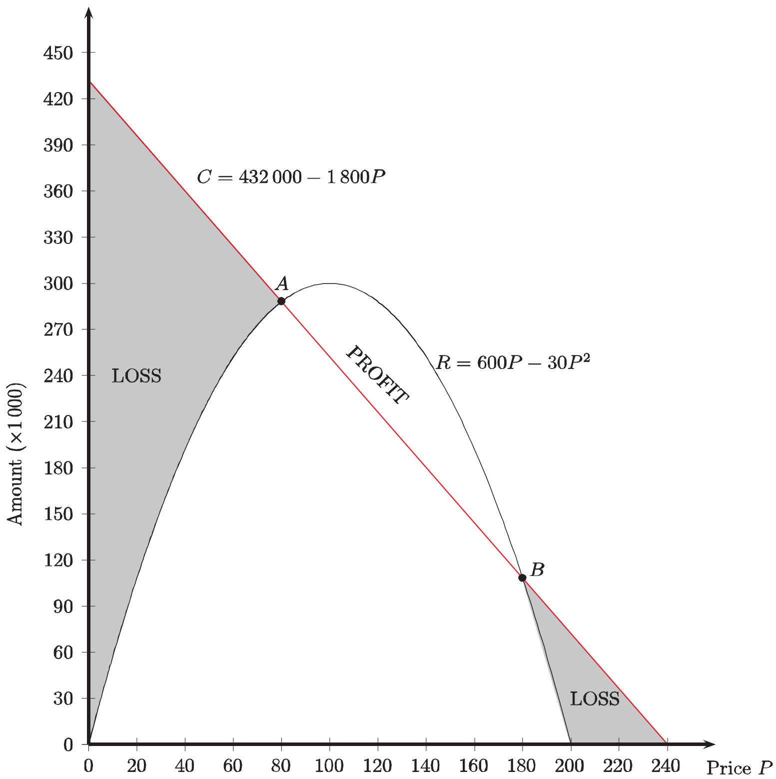
The revenue function is a quadratic function. We need to determine the vertex, roots and y-intercept of the function to draw it. Now given the function $R(P) = 6\,000P - 30P^2$ with the coefficients equal to $a = -30; b = 6\,000$; and $c = 0$.

$$\begin{aligned} x &= \frac{-b}{2a} = \frac{-(6\,000)}{2(-30)} = 100 \\ y &= 6\,000(100) - (30)(100)^2 = 300\,000 \end{aligned}$$

The roots are the value of the quadratic formula with $a = -30; b = 6\,000$; and $c = 0$. Therefore

$$\begin{aligned} x &= \frac{-6\,000 \pm \sqrt{6\,000^2 - 4(-30)}}{2(-30)} \\ &= \frac{-6\,000 \pm 6\,000}{-60} \\ &= 0 \text{ or } 200. \end{aligned}$$

The graphs of the two functions are:

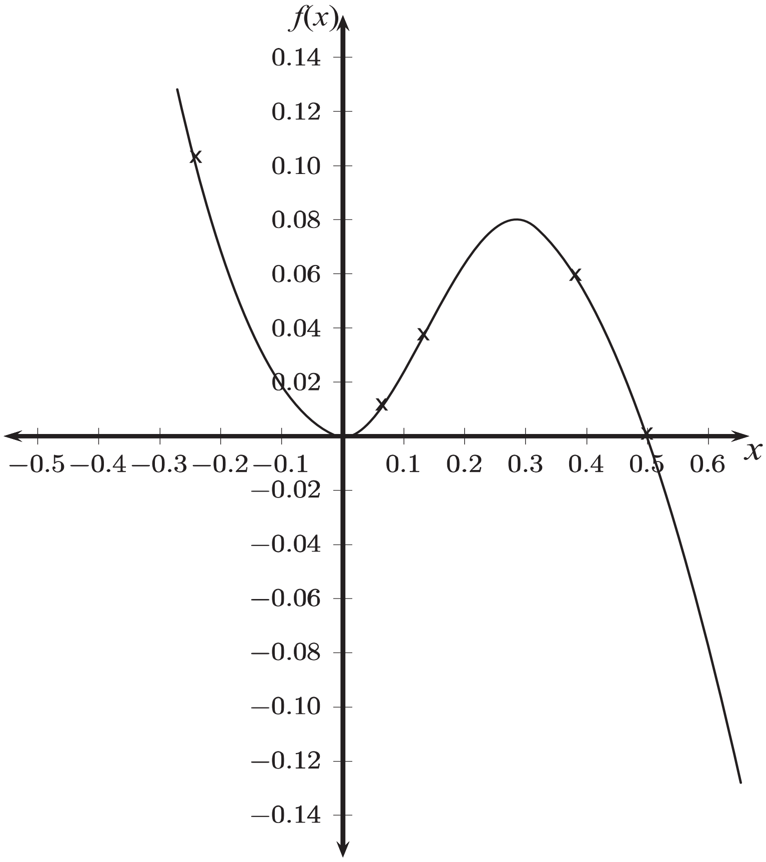


The break-even points are where the cost function is equal to the revenue function. These are the points A and B. Profit is revenue minus cost therefore the profit area is where the revenue is bigger than the cost (revenue function lies above cost function) and the loss is where the cost is bigger than the revenue (cost function lies above revenue function).

Question 9

1.

$$
\begin{aligned}
Q &= -0{,}4;\ f(x) = -4 \times (-0{,}4)^3 + 2 \times (-0{,}4)^2 = 0{,}574 \\
Q &= -0{,}2; f(x) = -4 \times (-0{,}2)^3 + 2 \times (-0{,}2)^2 = 0{,}112 \\
Q &= 0; f(x) = -4 \times (0)^3 + 2 \times (0)^2 = 0 \\
Q &= 0{,}2; f(x) = -4 \times (0{,}2)^3 + 2 \times (0{,}2)^2 = 0{,}048 \\
Q &= 0{,}4; f(x) = -4 \times (0{,}4)^3 + 2 \times (0{,}4)^2 = 0{,}064 \\
Q &= 0{,}6; f(x) = -4 \times (0{,}6)^3 + 2 \times (0{,}6)^2 = -0{,}144
\end{aligned}
$$



2. Roots: $Q = 0$ and $Q = 0{,}5$

   Turning points:

$$
\begin{aligned}
&\text{Maximum } f(x): \ Q \approx 0{,}3 \text{ and } f(x) \approx 0{,}07 \\
&\text{Minimum } f(x): \ Q = 0 \text{ and } f(x) = 0
\end{aligned}
$$

Question 10

Equilibrium when demand and supply are equal, that is $P_d = P_s$ or

$$\begin{aligned}
Q^2 + 2Q + 5 &= 29 - 3Q \\
Q^2 + 5Q - 24 &= 0 \\
(Q + 8)(Q - 3) &= 0 \\
Q &= -8 \text{ or } Q = 3 \\
Q_e &= 3.
\end{aligned}$$

Substitute into P_s or P_d to find

$$P_e = 29 - 3 \times 3 = 20. \quad P = \; 20.$$