Tutorial letter 202/2/2018

Quantitative Modelling 1

DSC1520

Semester 2

Department of Decision Sciences

Solutions to Assignment 2

Bar code

Define tomorrow.

UNISA university of south africa

Dear Student

This tutorial letter contains the solutions to the second compulsory assignment. Please contact me if you have any questions or need any help with the next assignment.

Kind regards

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Question 1

1.

$$\begin{aligned}\frac{x^2+x-12}{x^2-x-20} &= \frac{(x+4)(x-3)}{(x+4)(x-5)}\\ &= \frac{(x-3)}{(x-5)}.\end{aligned}$$

2.

$$\begin{aligned}\frac{\frac{4(0{,}6)\,K^{0{,}4}\,L^{-0{,}4}}{L}}{\frac{4(0{,}4)\,K^{-0{,}6}\,L^{0{,}6}}{K}} &= \frac{4(0{,}6)\,K^{0{,}4}\,L^{-0{,}4}\,K}{4(0{,}4)\,K^{-0{,}6}\,L^{0{,}6}\,L}\\ &= \frac{(0{,}6)\,K^{1{,}4}\,L^{-0{,}4}}{0{,}4\,K^{-0{,}6}\,L^{1{,}6}}\\ &= \frac{0{,}3K^2}{0{,}2L^2}\\ &= \frac{3K^2}{2L^2}.\end{aligned}$$

Question 2

1. The quantity demanded is $Q = 6\ 000 - 30P$, the fixed costs of R72 000 and the variable costs per unit of R60 are given. Now

$$\begin{aligned}\text{Total revenue} &= \text{Price} \times \text{Quantity}\\ TR &= PQ\\ &= P(6\,000 - 30P)\\ &= 6\,000P - 30P^2\end{aligned}$$

$$\begin{aligned}\text{Total cost} &= \text{Fixed cost} + \text{Variable cost}\\ TC &= 72\ 000 + 60Q\\ &= 72\,000 + 60(6\,000 - 30P)\\ &= 432\,000 - 1\,800P.\end{aligned}$$

2. Profit is total revenue minus total cost. Thus

$$\begin{aligned}\text{Profit} &= TR - TC\\ &= 6\,000P - 30P^2 - (432\,000 - 1\,800P)\\ &= -30P^2 + 7\,800P - 432\,000.\end{aligned}$$

3. The profit function derived in (2) is a quadratic function with

$$a = -30,\ b = 7\,800 \text{ and } c = -432\,000.$$

The price P at the turning point, or where the profit is a maximum, is

$$P = -\frac{b}{2a} = -\frac{7\,800}{2\times -30} = \frac{-7\,800}{-60} = 130$$

and thus the maximum profit

$$\text{Profit} = -30(130)^2 + 7\,800(130) - 432\,000 = 75\,000.$$

4. The maximum quantity produced at the maximum price of R130 calculated in (3) is

$$Q = 6\ 000 - 30(130) = 2\ 100.$$

5. At break-even the profit is equal to zero. Thus

$$\text{Profit} = -30P^2 + 7\ 800P - 432\ 000 = 0.$$

As the profit function is a quadratic function we use the quadratic formula with $a = -30$ and $b = 7\,800$ and $c = -432\ 000$ to solve P. Thus

$$\begin{aligned} P &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7\,800 \pm \sqrt{(7\ 800)^2 - 4(-30)(-432\ 000)}}{2 \times -30} \\ &= \frac{-7\,800 \pm 3\,000}{-60} \\ &= 80 \ \text{ or } 180. \end{aligned}$$

Now if $P = 80$ then $Q = 6\ 000 - 30(80) = 3\,600$ and if $P = 180$ then $Q = 6\ 000 - 30(180) = 600$.

Thus the two break-even points are where the price is R80 and the quantity 3 600, and where the price is R180 and the quantity 600.

Question 3

At break-even $P(x) = 0$, therefore, $P(100) = 0$ and $P(300) = 0$.

The factors of the function $P(x)$ are $(x - 100)$ and $(x - 300)$ and a few quadratic profit functions can be drawn from these roots.

In general we write $P(x)$ with factors $(x - 100)$ and $(x - 300)$ as

$P(x) = A(x - 100)(x - 300)$ with A a constant.

The price P when the profit is a maximum $= [(\text{root } 1 + \text{root } 2)/2] = (100 + 300)/2 = 200$,

and the maximum profit is given as 40 000.

$P(200) = 40\,000$ or $A(200 - 100)(200 - 300) = 40\,000$.

Solving for A as gives

$$\begin{aligned} A\,(200 - 100)\,(200 - 300) &= 40\,000 \\ A\,(100)\,(-100) &= 40\,000 \\ A\,(-10\,000) &= 40\,000 \\ A = -4 \end{aligned}$$

The quadratic profit function is $P(x) = -4(x - 100)(x - 300)$.

Question 4

1. $-x^2 + 6x + 9$.

   For roots we make use of the quadratic formula

$$\begin{aligned} x &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 - \sqrt{(6)^2 - 4(-1)(9)}}{2(-1)} \\ &= \frac{-6 \pm 8{,}49}{-2} \\ &= -1{,}25 \text{ or } 7{,}25. \end{aligned}$$

2. $$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-(6)}{2 \times -1} \\ &= \frac{-6}{-2} \\ &= 3 \text{ units.} \end{aligned}$$

Question 5

1. $$y = x^2 - 2x - 3$$

   Turning point:

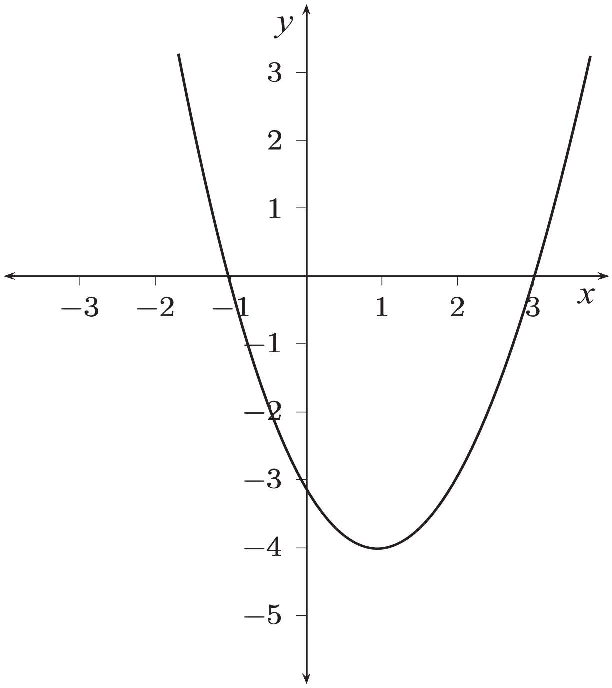
$$\begin{aligned} x &= \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1 \\ y &= (1^2 - 2(1) - 3) = (1 - 2 - 3) = -4 \end{aligned}$$

   The roots:

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4^2}}{2} \\ &= \frac{2 \pm 4}{2} && \text{or} \quad (x-3)(x+1) = 0 \\ &= 3 \text{ or } -1 && \phantom{\text{or}} \quad x = 3 \text{ or } x = -1 \end{aligned}$$

   $a > 0$, the y-intercept is -3.

2. The graph of the function $y = x^2 - 2x - 3$ is:



Question 6

1.

$$\begin{aligned} 5^{3x+8} &= 5^5 \\ 3x+8 &= 5 \\ 3x &= -3 \\ x &= \tfrac{-3}{3} = -1. \end{aligned}$$

Or

$$\begin{aligned} 5^{3x+8} &= 3125 \\ \ln 5^{3x+8} &= \ln 3125 \\ (3x+8)\ln 5 &= \ln 3125 \\ 3x &= \frac{\ln 3125}{\ln 5} - 8 \\ 3x &= -3 \\ x &= \frac{-3}{3} = -1. \end{aligned}$$

The value of x if given the equation $5^{3x+8} = 3125$ is equal to -1.

2. $0{,}0625 = 2^{-x}$

$$\begin{array}{rcl} \log 0{,}0625 &=& \log(2^{-x}) \\ \log 0{,}0625 &=& -x\log(2) \\ \dfrac{\log 0{,}0625}{\log 2} &=& -x \\ -4{,}00 &=& -x \\ x &=& 4{,}00 \end{array} \quad \begin{array}{c} \text{or} \\ \text{or} \\ \\ \\ \text{or} \end{array} \quad \begin{array}{rcl} \ln 0{,}0625 &=& ln(2^{-x}) \\ \ln 0{,}0625 &=& -x\ln(2) \\ \dfrac{\log 0{,}0625}{\ln 2} &=& -x \\ -4{,}00 &=& -x \\ x &=& 4{,}00 \end{array}$$

Question 7

1.

$$\begin{aligned} \log_5\left(\frac{15}{0{,}45}\right) &= \frac{\ln\left(\frac{15}{0{,}45}\right)}{\ln 5} \\ &= \frac{3{,}50656}{1{,}60944} \\ &= 2{,}179 \text{ rounded to 3 decimal places.} \end{aligned}$$

Or

$$\begin{aligned} \log_5\left(\frac{15}{0{,}45}\right) &= \frac{\ln\left(\frac{15}{0{,}45}\right)}{\ln 5} \\ &= \frac{\ln 15 - \ln 0{,}45}{\ln 5} \\ &= \frac{2{,}70805 - (-0{,}79851)}{1{,}60944} \\ &= \frac{3{,}50656}{1{,}60944} = 2{,}179 \quad \text{rounded to 3 decimal places.} \end{aligned}$$

The value of $\log_5\left(\frac{15}{0{,}45}\right)$ to 3 decimal places is equal to 2,179.

2.

$$\begin{aligned} 2\ln(3x^2) - 3\ln x &= 3 \\ \ln(3x^2)^2 - \ln x^3 &= 3 \\ \ln\left(\tfrac{9x^4}{x^3}\right) &= \ln\ 9x = 3 \\ 9x &= e^3 \\ x &= \frac{e^3}{9} = \frac{20{,}0855}{9} = 2{,}2317. \end{aligned}$$

Question 8

1.

$$\begin{aligned} S(10) &= 1\,800 + 1\,500e^{-0{,}3t+1{,}5} \\ S(10) &= 1\,800 + 1\,500e^{-0{,}3(10)+1{,}5} \\ S(10) &= 1\,800 + 1\,500e^{-1{,}5} = 2\,135. \end{aligned}$$

2.

$$\begin{aligned}
2\,010 &= 1\,800 + 1\,500e^{-0{,}3t+1{,}5} \\
\frac{210}{1\,500} &= e^{-0{,}3t+1{,}5} \\
\ln\left(\tfrac{210}{1\,500}\right) &= (-0{,}3t + 1{,}5)\ln e \\
\frac{\ln\left(\frac{210}{1500}\right) - 1{,}5}{-0{,}3} &= t = 11{,}5 \approx 12.
\end{aligned}$$

Question 9

$$\begin{aligned}
N &= 125{,}5e^{0{,}12t} \\
200 &= 125{,}5e^{0{,}12t} \\
\frac{200}{125{,}5} &= e^{0{,}12t} \\
\ln\left(\frac{200}{125{,}5}\right) &= \ln e^{0{,}12t} = 0{,}12t\ln e \\
t &= \frac{\ln\left(\frac{200}{125{,}5}\right)}{0{,}12} = 3{,}88343 \ \approx \ 4 \text{ days.}
\end{aligned}$$

Question 10

1. Equilibrium when demand and supply are equal, that is $P_d = P_s$ or

$$\begin{aligned}
P_s &= P_d \\
16 + 2Q &= \frac{500}{Q+1} \\
(16 + 2Q)(Q + 1) &= 500 \\
2Q^2 + 18Q - 484 &= 0
\end{aligned}$$

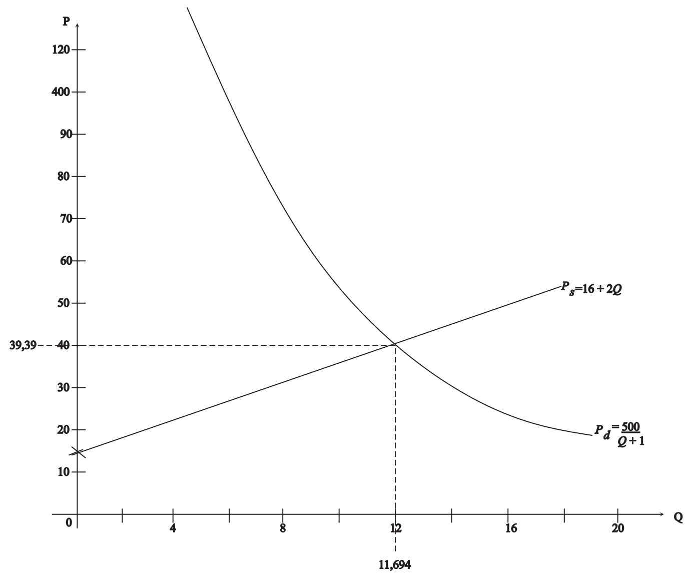
Solve to find

$$Q_e = \frac{-18 \pm \sqrt{18^2 - 4(2)(-484)}}{2(2)} = \frac{-18 \pm \sqrt{4\,196}}{16} = 11{,}694 \text{ or } -20{,}694.$$

2. Substitute into P_s or P_d to find

$$P_e = 16 + 2 \times 11{,}694 = 39{,}39.$$

3.


P
120
400
90
80
70
60
50
39,39
40
30
20
10
0
4
8
12
16
20
Q
11,694
$P_s = 16 + 2Q$
$P_d = \frac{500}{Q+1}$