Tutorial letter 203/1/2017

Quantitative Modelling 1
DSC1520

Semesters 1

Department of Decision Sciences

Solutions to Assignment 3
Dear Student

Here are the solutions to the third compulsory assignment. Please contact me if you have any questions. My contact details are as follows:

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Kind regards

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Question 1
\[ f(x) = x^{-2} - 4x^{-1}, \text{ therefore } f'(x) = -\frac{2}{x^3} + \frac{4}{x^2}. \] [Option 3]

Question 2
\[ f'(x) = 12e^{6x} + 12xe^{6x}.6 = 12e^{6x}(1 + 6x). \] [Option 3]

Question 3
\[ f'(x) = \frac{12x^2 + 16x}{4x^3 + 8x^2} = \frac{3x^2 + 4x}{x^3 + 2x^2} \] [Option 4]

Question 4
\[ f'(x) = 3e^{4x^3 - 3x}(8x - 3) + 8x - 3 = (8x - 3) \left( 3e^{4x^3 - 3x} + 1 \right). \] [Option 2]

Question 5
\[ \int \frac{x + 4}{2} \, dx = \ln x - \frac{4}{x} + c. \] [Option 3]

Question 6
\[ \int \frac{4}{(3x + 1)^3} \, dx = \frac{4(3x + 1)^{-2}}{-2} = -\frac{2}{3(3x + 1)^2} + c \] [Option 2]

Question 7
\[ \int_{0}^{1} 18e^{3x + 1} \, dx = 18 \left. e^{3x+1} \right|_{0}^{1} = 6(e^{4} - e^{1}) = 311. \] [Option 1]

Question 8
\[ MR = \frac{d}{dx} \left( 2x^3 - \frac{x^2}{2} + 10x + 15 \right) = 6x^2 - x + 10. \text{ When } x = 5, MR = 155. \] [Option 2]

Question 9
\[ \frac{d}{dL} \left( 6L^2 - 0,6L^3 \right) = 12L - 0,6L^2 = L(12 - 0,6L) = 0. \text{ Therefore } L = 0 \text{ or } L = 20. \] [Option 2]

When \( L = 0, Q = 0 \cdot, \text{ minimum output. Hence maximum output is achieved when } L = 20. \)

Question 10
\[ f'(x) = 6x^2 - 6x - 12 = 0 \text{ gives } (x - 2)(x + 1) = 0 \text{ or } x = 2 \text{ and } x = -1. \] \( f''(x) = 12x - 6 \text{ giving } f''(2) = 18 > 0 \text{ and } f''(-1) = -18 < 0. \) At \( x = 2, f \text{ is a minimum and at } x = -1 \text{ it is a maximum.} \] [Option 4]
Question 11

If $Q = 10$, then $P = 92$.

\[
PS = PQ - \int_0^{10} \left( Q^2 - 2Q + 12 \right) dQ
\]
\[
= 10 \times 92 - \left( \frac{10^3}{3} - 10^2 + 120 \right) \bigg|_0^{10}
\]
\[
= 920 - \left( \frac{1000}{2} - 100 + 120 \right)
\]
\[
= 920 - 353.33
\]
\[
= 566.67.
\]

[Option 2]

Question 12

At equilibrium $6Q + 10 = 74 - Q^2$ which gives $Q = \frac{-6 \pm \sqrt{6^2 + 256}}{2} = 5,544$, $Q = 5,544 \approx 6$ and $P = 43,26$ (work with rounded $Q$). Now, $CS = \int_0^6 74 - Q^2 dQ - (6 \times 43,26) = 372 - 259,56 = 112,44$. [Option 2]

Question 13

$TR = PQ = 120Q - 5Q^2$ and profit $\pi = TR - TC = -\frac{1}{3}Q^3 - 3Q^2 + 102Q - 60$. [Option 3]

Question 14

\[
\frac{d\pi}{dQ} = -0.4Q^2 - 6Q + 102 = 0 \text{ or } 0.2Q^2 + 3Q - 51 = 0
\]
giving $Q = \frac{-3\pm\sqrt{3^2-4(0.2)(-51)}}{2(0.2)} = 10.14 \text{ or } -25$. [Option 2]

Question 15

Given $P = 44e^{-0.7Q}$ from which we get $\ln P = -0.7Q$ or $Q = \frac{\ln P}{-0.7} - \frac{\ln 44}{-0.7} = 5,41$ giving $\frac{dQ}{dP} = -\frac{1}{0.7}$. 
\[
\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{0.7} \cdot \frac{P}{Q} = -\frac{1}{0.7} \cdot \frac{1}{1} = -0.714.
\]
Since $|\varepsilon_d| = 0.714 < 1$, demand is inelastic. [Option 2]