Tutorial letter 203/1/2018

Quantitative Modelling 1

DSC1520

Semesters 1

Department of Decision Sciences

Solutions to Assignment 3

Bar code

Define tomorrow.

UNISA university of south africa

Dear Student

Here are the solutions to the third compulsory assignment. Please contact me if you have any questions.

Kind regards

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Question 1

$$\begin{aligned} G(x) &= x(x^2 - 4\sqrt{x} + 4) \\ &= x(x^2 - 4x^{\frac{1}{2}} + 4) \\ &= x^3 - 4x^{\frac{3}{2}} + 4x. \end{aligned}$$

$$\begin{aligned} G'(x) &= \frac{d}{dx}\left(x^3 - 4x^{\frac{3}{2}} + 4x\right) \\ &= 3x^2 - 6x^{\frac{1}{2}} + 4 \\ &= 3x^2 - 6\sqrt{x} + 4. \end{aligned}$$

[Option 4]

Question 2

$$\begin{aligned} f(x) &= \frac{x^2+6}{2x+5} \\ &= (x^2+6)(2x+5)^{-1} \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{(2x+5)\cdot 2x - (x^2+6)\cdot 2}{(2x+5)^2} \\ &= \frac{2(x^2+5x-6)}{(2x+5)^2} \\ &= \frac{2(x+6)(x-1)}{(2x+5)^2}. \end{aligned}$$

OR

$$\begin{aligned} f'(x) &= 2x(2x+5)^{-1} + (x^2+6)(-1)(2x+5)^{-2}(2) \\ &= \frac{2x(2x+5) - 2x^2 - 12}{(2x+5)^2} \\ &= \frac{2(x+6)(x-1)}{(2x+5)^2}. \end{aligned}$$

[Option 2]

Question 3

$$\begin{aligned} P(x) &= x^5e^{3x} + \frac{x+1}{x} \\ P'(x) &= 3x^5e^{3x} + 5x^4e^{3x} - \tfrac{1}{x^2} \\ &= e^{3x}\left(3x^5 + 5x^4\right) - \tfrac{1}{x^2} \end{aligned}$$

[Option 1]

Question 4

$$\frac{d}{dx}\ln x + 4x^{-2} = \frac{1}{x} - 8x^{-3} = \frac{1}{x} - \frac{8}{x^3}.$$

[Option 5]

Question 5

We first need to simplify the expression $x^2\left(1+\frac{1}{x^2}\right)$ before we can use the rule.
Therefore

$$\begin{aligned}
\int x^2\left(1+\frac{1}{x^2}\right)\mathrm{d}x &= \int (x^2+\frac{x^2}{x^2})\mathrm{d}x \\
&= \int (x^2+1)\mathrm{d}x \\
&= \frac{x^3}{3}+\frac{x}{1}+c \\
&= \frac{1}{3}x^3+x+c.
\end{aligned}$$

[Option 2]

Question 6

$$\begin{aligned}
\int_{-2}^{2}(x^2-3)\mathrm{d}x &= \int_{-2}^{2}x^2\mathrm{d}x - \int_{-2}^{2}3\mathrm{d}x \\
&= \tfrac{x^3}{3}\Big|_{-2}^{2} - \tfrac{3x}{1}\Big|_{-2}^{2} \\
&= \left[\tfrac{2^3}{3}-\tfrac{-2^3}{3}\right]-[3(2)-3(-2)] \\
&= \tfrac{16}{3}-12 \\
&= -6\tfrac{2}{3}.
\end{aligned}$$

[Option 2]

Question 7

$$\begin{aligned}
&\int\sqrt{9x-5}dx \\
u &= 9x-5,\, du=9dx;\ dx=\tfrac{du}{9} \\
\int\sqrt{u}dx &= \tfrac{1}{9}\int u^{\frac{1}{2}}du = \tfrac{1}{9}\times\tfrac{2}{3}u^{\frac{3}{2}}+c \\
&= \tfrac{2}{27}(9x-5)^{\frac{3}{2}}+c \\
&= \tfrac{2}{27}\sqrt{(9x-5)^3}+c.
\end{aligned}$$

[Option 4**]**

Question 8

$$\int \frac{x^2+4}{x^3}dx = \int \frac{1}{x} + 4x^{-3} = \ln x - 2x^{-2} + c.$$

[Option 2]

Question 9

$$\begin{aligned} R(x) &= -\tfrac{1}{5}x^2 + 30x + 81 \\ \frac{\mathrm{d}}{\mathrm{d}x}R(x) &= -\tfrac{2}{5}x + 30 \end{aligned}$$

The maximum occurs when

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}R(x) &= 0 \\ -\tfrac{2}{5}x + 30 &= 0 \\ x &= \tfrac{150}{2} \\ &= 75 \end{aligned}$$

Calculate the y-value of the extreme point or vertex

$$\begin{aligned} R(x) &= -\tfrac{1}{5}x^2 + 30x + 81 \\ &= -\tfrac{1}{5}(75)^2 + 30(75) + 81 \\ &= 1\,206. \end{aligned}$$

The function has a maximum value in the point $(75;\ 1\,206)$, that is where the revenue is equal to $1\,206$.

[Option 2]

Question 10

$$\begin{aligned} MR &= \tfrac{dTR}{dx} \\ &= 10x^4 - x + 10 \\ MR(5) &= 10(5)^4 - 5 + 10 \\ &= 6\,255. \end{aligned}$$

[Option 3]

Question 11

The marginal cost function is the differentiated total cost function. Thus by differentiating the total cost function we can determine the marginal cost function. Now if the total cost function is

$$TC = 2Q^3 - Q^2 + 80Q + 150,$$

then the marginal cost function is

$$\begin{aligned} MC &= \frac{\mathrm{d}TC}{\mathrm{d}Q} \\ &= \frac{\mathrm{d}}{\mathrm{d}Q}(2Q^3 - Q^2 + 80Q + 150) \\ &= 6Q^2 - 2Q + 80 \end{aligned}$$

Now the marginal cost function's value when Q is equal 10 is

$$\begin{aligned} MC(10) &= 6(10)^2 - 2(10) + 80 \\ &= 600 - 20 + 80 = 660. \end{aligned}$$

[Option 2]

Question 12

The rate of change in revenue is given by

$$f'(t) = \frac{8{,}59}{t}.$$

After 15 years, the rate of change is

$$\begin{aligned} f'(15) &= \frac{8{,}59}{15} \\ &= 0{,}572667 \text{ million rand, that is R572 667.} \end{aligned}$$

[Option 3]

Question 13

When $P = 6$, $Q = 192 - 6^2 = 156$. Therefore, elasticity of demand is

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q} = -2P \cdot \frac{P}{Q} = -\frac{2P^2}{Q} = -\frac{2 \times 36}{156} = -0{,}46.$$

Demand is inelastic since $|\varepsilon_d| < 1$, meaning that a 1% price increase will result in 0,46% less seats to be sold.

[Option 1]

Question 14

$$\begin{aligned} P &= \frac{40}{Q+3} \quad P = 10 \\ 10 &= \frac{40}{Q+3} \\ Q &= 1. \end{aligned}$$

Area under the curve – area of the rectangle

$$\begin{aligned} &= \int_0^1 \frac{40}{Q+3} - P_0Q_0 \\ &= 40[\ln 4 - \ln 3] - 10 = 1{,}5. \end{aligned}$$

[Option 2]

Question 15

$$TC = \int 2Q^2 - 1 \; dx = \frac{2Q^3}{3} - Q + 300, \text{ since FC=300.}$$

[Option 1**]**