1. Examination information
2. Discussion classes: questions and solutions
3. Previous examination paper and solutions
Dear Student

This tutorial letter contains information on the examination, discussion class problems and solutions, as well as a previous examination paper with its model answers. Try to work through all the evaluation exercises, assignments, discussion class problems and the previous examination paper when you prepare for the examination. It is also a good idea to work through the assignments of the first semester. The solutions to the first semester’s assignments can be found under Announcements on myUnisa.

Remember to contact me by e-mail, fax, telephone or a personal appointment if you need help regarding the study material. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)
19:00 to 20:00 (Mondays to Thursdays) (telephone only)

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It is better to sort out your problems before the examination than to repeat the module next semester.

Finally, well done you have made it up to here! We wish you everything of the best with the last hurdle, the examination. I hope that you have enjoyed the module. It was a pleasure assisting you in this module. Best wishes for the future.

Kind regards

Mrs Adèle Immelman
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Lecturer
1. **Examination information**

1. Make sure that you know **WHERE** and **WHEN** you are writing DSC1520. Consult the *my*Unisa website https://my.unisa.ac.za if you are not sure of the date and time of the examination.

2. The examination is a **two-hour** paper. It counts **100** marks in total.

3. The examination paper consists of two sections, Section A and Section B.

4. Section A consists of 20 multiple-choice questions, each worth 3 marks, giving a subtotal of 60 marks. These questions have to be answered on a mark-reading sheet.

5. Section B consists of structured questions, giving a subtotal of 40 marks and has to be answered on the paper itself.

6. Read the questions carefully before answering. See what is asked and then answer the question accordingly. Use the rough work paper supplied for your calculations.

7. Do not panic if you cannot answer a question. Go to the next question and return later.

8. You must take your calculator with you to the examination hall. Make sure it is in working order. A programmable calculator is permitted.

9. Remember to write your student number and module code on the multiple-choice answer sheet.

10. Composition of your final mark for the module:

    **Assignment 1**
    
    | 10% | Semester mark | 10% |
    |-----|---------------|-----|
    | 90% |               |     |

    | Assignment 2 | Final mark : 50% pass |
    |--------------|-----------------------|
    | Examination  |                       |
    | 90%          |                       |

11. **TO PASS, YOU NEED AN AGGREGATED TOTAL OF 50% FROM THE EXAMINATION AND COMPULSORY ASSIGNMENTS.**

12. **YOUR ASSIGNMENT MARKS WILL ONLY BE CONSIDERED IF YOU OBTAIN AT LEAST 40% IN THE EXAMINATION.**
13. Chapters in the textbook and study guide that are relevant for the examination:

**Textbook:**
- Study Unit 1: Mathematical preliminaries
  - Chapter 1: Sections 1.1 – 1.7
- Study Unit 2: Linear functions
  - Chapter 2: Sections 2.1 – 2.4 and 2.6
- Study Unit 3: Linear algebra
  - Chapter 3 and 9: Sections 3.1; 3.2.1; 3.2.5; 3.3 and 9.1
- Study Unit 4: Non-linear functions
  - Chapter 4: Sections 4.1 – 4.4
- Study Unit 5: Introduction to calculus
  - Chapter 6 and 8: Sections 6.1; 6.2.1; 6.3.1; 6.3.2; 6.4 and 8.1; 8.2; 8.5

**Study guide:**
Appendices A, B and C may be studied for additional explanation of sections in the textbook.

14. Try to work through all the assignments (first and second semester), evaluation exercises, the discussion class problems and the previous examination paper when you prepare for the examination. Remember: “Practice makes perfect”.

15. The questions in the examination paper are similar to the problems in the assignments (first and second semester), the discussion classes and the previous examination paper.
2. Discussion classes: Questions and solutions

The slides of my notes of the discussion classes can be found on myUnisa.

**Question 1**

Calculate

\[
\frac{1}{6} \times \frac{5}{6} + \frac{2}{3} \div \frac{1}{4}
\]

**Solution**

\[
\frac{1}{6} \times \frac{5}{6} + \frac{2}{3} \div \frac{1}{4} = \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}
\]

\[
= \frac{1}{6} \times \frac{15}{3} + \frac{3}{12}
\]

multiply fractions

\[
= \frac{2 \times 15 + 3}{12}
\]

common denominator

\[
= \frac{-10}{12}
\]

add and subtract fractions

\[
= \frac{-5}{6}
\]

simplify by dividing nominator and denominator by 2

**Question 2**

Solve for \(x\) in

\[
-2x + \frac{5}{6} + \frac{x}{2} \geq -2x - 4 \left( -\frac{x}{3} - 1 \right)
\]
Solution

\[-2x + \frac{\frac{5}{6} + \frac{x}{2}}{\frac{5}{2}} \geq -2x - 4 \left( \frac{\frac{1}{3} - \frac{1}{4}}{\frac{5}{4}} \right) \]

\[-2x + \frac{\frac{5}{6} + \frac{x}{2}}{\frac{5}{2}} \geq -2x - 4 \left( \frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{4}} \right) \quad \text{change fraction} \]

\[-2x + \frac{\frac{5}{6} + \frac{x}{2}}{\frac{5}{2}} \geq -2x + \left( \frac{4x}{3} + \frac{20}{4} \right) \quad \text{multiply 4 into ( )} \]

\[-2x + \frac{\frac{5}{6} + \frac{x}{2}}{\frac{5}{2}} \geq -2x + \frac{4x}{3} + 5 \quad \text{remember } \frac{20}{4} = 5 \]

\[-2x + 2x + \frac{x}{2} - \frac{4x}{3} \geq 5 - \frac{5}{6} \quad \text{move all similar terms to one side} \]

\[\frac{x}{2} - \frac{4x}{3} \geq 5 - \frac{5}{6} \]

\[3x - 8x \geq 30 - 5 \quad \text{common denominator} \]

\[6 \quad \frac{6}{6} \]

\[-\frac{5x}{6} \geq \frac{25}{6} \quad \text{multiply both sides by 6} \]

\[-\frac{5x}{6} \times \frac{6}{1} \geq \frac{25}{6} \times \frac{6}{1} \]

\[-5x \geq 25 \quad \text{divide both sides by } -5 \]

\[-\frac{5x}{-5} \leq \frac{25}{-5} \quad \text{inequality sign changes because we divide by a negative number} \]

\[x \leq -5 \]

Question 3

(a) Find the equation of the line passing through the points (1; 20) and (5; 60).

(b) Draw the graph of the line \(y = 10x + 10\).

Solution

(a) Let \((x_1 ; y_1) = (1 ; 20)\) and \((x_2 ; y_2) = (5 ; 60)\). We need to determine the slope \(m\) and the \(y\) intercept \(c\) of the line \(y = mx + c\).

Now the slope \(m\) is defined by

\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 20}{5 - 1} = \frac{40}{4} = 10\]

Therefore \(y = 10x + c\).
Now both \((x_1; y_1)\) and \((x_2; y_2)\) lie on the line. We can thus substitute **any one of the points** into the equation of the line to determine \(c\). Let’s choose the point \((1; 20)\). Then
\[
\begin{align*}
y &= 10x + c \\
20 &= 10 \times 1 + c \\
20 &= 10 + c \\
-c &= 10 - 20 \\
-c &= -10 \\
c &= 10
\end{align*}
\]

The equation of the line is \(y = 10x + 10\).

(b) Given the line \(y = 10x + 10\), we need two points to draw a line. Select **any** \(x\) or \(y\) value and calculate the value of the point.

Say we choose \(x = 0\), then \(y = 10(0) + 10 = 10\). Therefore point 1 = \((0; 10)\).

Choose \(y = 0\) then 
\[
\begin{align*}
0 &= 10x + 10 \\
-10x &= 10 \\
x &= 10 / (-10) \\
x &= -1. Therefore point 2 = (-1; 0).
\end{align*}
\]

**Please note that you can use any \(x\) and/or \(y\) value to calculate the two points. Normally \(x = 0\) and \(y = 0\) are used to simplify the calculation.**

Next we plot the two calculated points of the line and draw the line:
Question 4
If the demand function is \( P = 80 - 2Q \), where \( P \) and \( Q \) are the price and quantity, respectively, determine the expression for price elasticity of demand
(a) if the price \( P = 20 \).
(b) in terms of \( P \) only.

Solution
(a) The demand function is given as \( P = 80 - 2Q \).

Now the price elasticity of demand is
\[
\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q}
\]
with \( a \) and \( b \) the values of the standard demand function \( P = a - bQ \).

To determine the price elasticity of demand, we need to determine the values of \( b, Q \) and \( P \).
Given \( P = 80 - 2Q \) and \( P = 20 \). Comparing \( P = 80 - 2Q \) with \( P = a - bQ \), we can conclude that \( a = 80 \) and \( b = 2 \).

Now \( a, b \) and \( P \) are known. All we need to calculate is the value of \( Q \). To determine the value of \( Q \), we substitute \( P = 20 \) into the equation of the demand function and solve for \( Q \):
\[
20 = 80 - 2Q
\]
\[
20 - 80 = -2Q
\]
\[
-60 = -2Q
\]
\[
Q = 30
\]

Now
\[
\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q}
\]
\[
= -\frac{1}{2} \cdot \frac{20}{30}
\]
\[
= -\frac{1}{3}
\]
\[
= -0.33 \quad \text{(rounded to 2 decimal places)}
\]

At \( P = 20 \) a 1% increase (decrease) in price will cause a 0.33% decrease (increase) in the quantity demanded.
(b) The expression for the price elasticity of demand is \( \varepsilon_d = -\frac{1}{b} \times \frac{P}{Q} \) with \( a \) and \( b \) the values of the standard demand function \( P = a - bQ \).

To determine the price elasticity of demand we thus need to determine the values of \( b, Q \) and \( P \). It is given that \( P = 80 - 2Q \) and question asked in terms of \( P \) thus \( P = P \). Comparing \( P = 80 - 2Q \) with \( P = a - bQ \), we can say that \( a = 80 \) and \( b = 2 \). At this stage \( Q \) is unknown.

The demand function now denotes the relationship between the price \( P \) and the demand \( Q \). If given \( P \), we can derive \( Q \) by substituting \( P \) into the demand function and we can then solve for \( Q \).

To determine the value of \( Q \) we need to change the equation of the demand function so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \). Now

\[
P = 80 - 2Q \\
P = 80 - 2Q \\
\frac{P - 80}{-2} = Q.
\]

As we have determined the values of \( b, P \) and \( Q \) we can now substitute them into the formula for the price elasticity of demand:

\[
\varepsilon_d = -\frac{1}{2} \times \frac{P}{P - 80} \\
= -\frac{1}{2} \times \frac{P}{P - 80} \times \frac{-2}{1} \\
= \frac{P}{P - 80}
\]

**Or alternatively**

You can use the given formula of price elasticity of demand in terms of \( P \) of a demand function in the form \( P = a - bQ \) as given in the textbook on page 78, equation 2.14 (Edition 2) and page 89, equation 2.14 (Edition 3).

\[
\varepsilon_d = \frac{P}{P - a}
\]

Now \( a = 80 \) (intercept on the \( y \)-axis of the demand function) \( \varepsilon_d = \frac{P}{P - 80} \).
Question 5

Given the demand function \( P = 60 - 0.2Q \) where \( P \) and \( Q \) are the price and quantity respectively, calculate the arc price elasticity of demand when the price decreases from R50 to R40.

Solution

The arc price elasticity of a demand function \( P = a - bQ \) between two prices \( P_1 \) and \( P_2 \) is

\[
\text{arc elasticity of demand} = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2}
\]

with \( b \) the slope of the demand function and \( P_1, P_2 \) and \( Q_1, Q_2 \) the price and quantity demanded.

Now for the given function \( P = 60 - 0.2Q \), we can derive that \( a = 60 \) and \( b = 0.2 \). It is given that \( P_1 = 50 \) and \( P_2 = 40 \). All we need to determine are \( Q_1 \) and \( Q_2 \). By making \( Q \) the subject of the equation, we can rewrite the equation \( P = 60 - 0.2Q \) as

\[
P = 60 - 0.2Q
\]

\[
0.2Q = 60 - P
\]

\[
Q = 300 - 5P.
\]

We can now determine \( Q_1 \) and \( Q_2 \) by substituting \( P_1 = 50 \) and \( P_2 = 40 \) into the equation.

Therefore if \( P_1 = 50 \) then \( Q_1 = 300 - 5 \times 50 = 50 \)

and if \( P_2 = 40 \) then \( Q_2 = 300 - 5 \times 40 = 100 \).

Therefore

\[
\text{elasticity of demand} = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2}
\]

\[
= -\frac{1}{0.2} \times \frac{50 + 40}{50 + 100}
\]

\[
= -\frac{1}{0.2} \times \frac{90}{150}
\]

\[
= -\frac{90}{30}
\]

\[
= -3
\]
Question 6

Solve the following set of linear equations:

\[ y + 2x = 3 \quad (1) \]
\[ y - x = 2 \quad (2) \]

by using the

(a) elimination method.
(b) substitution method.
(c) graphical method.

Solution

(a) Elimination method:

Step 1: Eliminate one variable, say \( y \), by adding or subtracting one equation or multiple of an equation from another equation:

Subtract equation (2) from equation (1) and solve for \( x \):

\[
\begin{align*}
  y + 2x &= 3 \quad (1) \\
 -(y - x &= 2) \quad (2) \\
  0 + 3x &= 1 \\
  x &= \frac{1}{3}
\end{align*}
\]

Step 2: Solve for \( y \). Substitute value of \( x \) back into any one of equations and solve \( y \).

Substitute the value of \( x \) into say equation (1):

\[
\begin{align*}
  y + 2x &= 3 \quad (1) \\
  y + 2 \left( \frac{1}{3} \right) &= 3 \\
  y &= 3 - \frac{2}{3} \\
  y &= 2 \frac{1}{3}
\end{align*}
\]
(b) **Substitution method:**

Step 1: Change one of the equations so that a variable is the subject of the equation, say \( y \) in equation (1):

\[
y + 2x = 3
\]

\[
y = 3 - 2x \quad (3)
\]

Step 2: Substitute the value of \( y \) (equation (3)) into the unchanged equation (2) and solve for \( x \). Substitute \( y = 3 - 2x \) into \( y - x = 2 \):

\[
y - x = 2
\]

\[
(3 - 2x) - x = 2
\]

\[
-3x = 2 - 3
\]

\[
-3x = -1
\]

\[
x = \frac{1}{3}
\]

Step 3: Substitute the calculated value of the variable into any equation and calculate the value of the other variable. Substitute \( x = \frac{1}{3} \) into equation (1) or equation (2). Let’s say we choose equation (2):

\[
y - x = 2 \quad (2)
\]

\[
y - \frac{1}{3} = 2
\]

\[
y = 2 \frac{1}{3}
\]

(c) **Graphical method:**

To solve the two equations simultaneously by using the graphical method, we first draw the graphs of the two lines and secondly determine the point where the two lines intersect.

To draw a line we need two points:
Equation (1): $y + 2x = 3$

Let’s choose $x = 0$, then $y + 2(0) = 3$ or $y = 3$. Therefore one point is $(0 ; 3)$.

Next we choose $y = 0$. If $y = 0$ then $0 + 2x = 3$ or $x = \frac{3}{2}$. Therefore a second point is $(3/2 ; 0)$.

Equation (2): $y - x = 2$

If $x = 0$ then $y - (0) = 2$ or $y = 2$. Thus the first point is $(0 ; 2)$.

If $y = 0$ then $0 - x = 2$ or $x = -2$. Thus a second point is $(-2 ; 0)$

Please note that you can use any $x$ or $y$ values to calculate the two points. Normally $x = 0$ and $y = 0$ are used to simplify the calculation.

Next we plot the two calculated points of the lines and draw the lines. The solution is the point where the two lines intercept, namely the point $\left(\frac{1}{3}; \frac{2}{3}\right)$. 
Question 7
Solve the following sets of equations:
(a)
\[ \begin{align*}
  x - y + z &= 0 \\
 2y - 2z &= 2 \\
-x + 2y + 2z &= 29 
\end{align*} \]
(b)
\[ \begin{align*}
  x - 2y + 3z &= -11 \\
 2x - z &= 8 \\
3y + z &= 10 
\end{align*} \]

Solution
(a) We need to solve the following system of equations:
\[ \begin{align*}
  x - y + z &= 0 \\
  2y - 2z &= 2 \\
-x + 2y + 2z &= 29 
\end{align*} \]

Step 1: Determine two equations with two unknowns (variables) by adding or subtracting two of the three equations at a time:

Now equation (2) is already an equation with only two variables. To determine another equation with just two variables we add equation (1) and equation (3):
\[ \begin{align*}
  x - y + z &= 0 \\
-x + 2y + 2z &= 29 
\end{align*} \]
\[0 + y + 3z = 29 \]

Step 2: Next we solve two equations with two unknowns, using any method described previously. Let’s use the substitution method:

- Step 1: Make \( y \) the subject of equation (4):
\[ y = 29 - 3z \]
• Step 2: Substitute equation (5) into equation (2) and solve for $z$:

$$2y - 2z = 2 \quad (2)$$

$$2(29 - 3z) - 2z = 2$$

$$58 - 6z - 2z = 2$$

$$-8z = 2 - 58$$

$$-8z = -56$$

$$z = \frac{-56}{-8}$$

$$z = 7$$

• Step 3: Substitute $z = 7$ into equation (2) and solve for $y$:

$$2y - 2z = 2 \quad (2)$$

$$2y - 2(7) = 2$$

$$2y = 2 + 14$$

$$2y = 16$$

$$y = 8$$

• Step 4: Substitute $z = 7$ and $y = 8$ into equation (1) and solve for $x$:

$$x - y + z = 0 \quad (1)$$

$$x - 8 + 7 = 0$$

$$x = 1$$

Therefore the solution of the set of equations is $x = 1$, $y = 8$ and $z = 7$.

(b) We need to solve the following system of equations:

$$x - 2y + 3z = -11 \quad (1)$$

$$2x - z = 8 \quad (2)$$

$$3y + z = 10 \quad (3)$$
Make \( z \) the subject of equation (2) and \( y \) the subject of equation (3):

\[
\begin{align*}
z &= 2x - 8 \quad (4) \\
y &= \frac{10 - z}{3} \quad (5)
\end{align*}
\]

Substitute equation (4) into equation (5):

\[
y = \frac{10 - (2x - 8)}{3} = \frac{18 - 2x}{3} \quad (6)
\]

Substitute equation (2) and equation (6) into equation (1):

\[
\begin{align*}
x - \frac{2}{3} (18 - 2x) + 3(2x - 8) &= -11 \\
x - 12 + \frac{4}{3}x + 6x - 24 &= -11 \\
\frac{3 + 4 + 18}{3}x &= -11 + 12 + 24 \\
\frac{25}{3}x &= 25 \\
x &= 3
\end{align*}
\]

Substitute \( x = 3 \) into equation (4) and equation (6):

\[
z = 2 \times 3 - 8 = -2
\]

and

\[
y = \frac{18 - 2 \times 3}{3} = \frac{18 - 6}{3} = 4
\]

Therefore \( x = 3, y = 4 \) and \( z = -2 \).
Question 8

In a market we have the following:

Demand function: \( Q = 50 - 0,1P \)
Supply function: \( Q = -10 + 0,1P \)

where \( P \) and \( Q \) are the price and quantity respectively.

(a) Calculate the equilibrium price and quantity.
(b) Draw the two functions, and label the equilibrium point.

Solution

(a) Equilibrium is the price and quantity where the demand and supply functions are equal. Therefore determine \( Q = Q \), or

\[
50 - 0,1P = -10 + 0,1P
\]

\[
-0,1P - 0,1P = -10 - 50
\]

\[
-0,2P = -60
\]

\[
P = \frac{-60}{-0,2} = 300
\]

To calculate the quantity at equilibrium, we substitute the value of \( P \) into the demand or supply function and calculate \( Q \). Say we use the demand function, then

\[
Q = 50 - 0,1(300)
\]

\[
Q = 20
\]

The equilibrium price is equal to 300 and the quantity is 20.
Question 9

A company manufactures and sells $x$ hand-held toy radios per week. The weekly costs are given by

$$c(x) = 5000 + 2x$$

What is the company’s break-even point if a radio sells for R202?

Solution

The break-even point of a company occurs when the company does not make a profit or a loss, meaning break-even is where the revenue of the company is equal to the cost. Therefore revenue = cost.

Now revenue = price $\times$ quantity. Therefore

revenue = 202$x$.

Cost is given by

$$c(x) = 5000 + 2x.$$
Therefore break-even is when
\[
\text{revenue} = \text{cost}
\]
\[
202x = 5000 + 2x
\]
\[
202x - 2x = 5000
\]
\[
200x = 5000
\]
\[
x = 25
\]

Or alternatively

At break-even the profit is equal to zero. Therefore
\[
\text{profit} = 0
\]
\[
\text{profit} = \text{revenue} - \text{cost}
\]
\[
\text{profit} = 202x - (5000 + 2x) = 0
\]
\[
200x - 5000 = 0
\]
\[
200x = 5000
\]
\[
x = 25
\]

Thus the company breaks even when they manufacture 25 hand-held toy radios.

**Question 10**

Calculate the consumer surplus for the demand function \( P = 60 - 4Q \) when the market price is \( P = 12 \).

**Solution**

Consumer surplus is the monetary value of the benefit that accrues to consumers from the matching of supply and demand in the market. The consumer surplus is the difference between the amount the consumer is willing to spend for successive units of a product from \( Q = 0 \) to \( Q = Q_0 \) and the amount that the consumer actually spent on \( Q_0 \) units of the product at a market price of \( P_0 \) per unit:

\[
CS = \text{Amount willing to pay} - \text{Amount actually paid}
\]
If you need to determine the demand surplus for a linear demand function of \( P = a - bQ \) then the consumer surplus can be calculated by calculating an area of the triangle \( P_0Q_0a \) which is equal to

\[
\frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times (a - Q_0) \times (P_0 - 0) = \frac{1}{2} \times (a - Q_0) \times (P_0)
\]

with

- \( P_0 \) the value given to you as the market price,
- \( Q_0 \) the value of the demand function if \( P \) equals the given market price (substitute \( P_0 \) into the demand function and calculate \( Q_0 \) ), and
- \( a \) the \( y \)-intercept of the demand function \( P = a - bQ \) also known as the value of \( P \) if \( Q = 0 \), or the point where the demand function intercepts the \( y \)-axis.

In general we can summarise the steps of determining the consumer surplus as follows:

**Method:**

1. Calculate \( Q_0 \) if \( P_0 \) is given.
2. Draw a rough graph of the demand function.
3. Read the value of \( a \) from the demand function – the \( y \)-intercept of the demand function.
4. Calculate the area of \( CS = \frac{1}{2} \times (a - P_0) \times (Q_0) \).
First we need to determine \( Q \) from the demand function if \( P = 12 \). Therefore

\[
P = 60 - 4Q \\
12 = 60 - 4Q \\
4Q = 60 - 12 \\
4Q = 48 \\
Q = 12
\]

Next we draw a rough sketch of the demand function:

Now the consumer surplus is the area of the shaded triangle:

\[
CS = \frac{1}{2} \times \text{base} \times \text{height} \\
= \frac{1}{2} \times 12 \times (60 - 12) \\
= 288
\]

**Question 11**

(a) Graph the lines representing the following constraints:

\[
2x + y \leq 120 \quad (1) \\
x + 2y \leq 140 \quad (2) \\
x + y \leq 80 \quad (3) \\
x_1, x_2 \geq 0 \quad (4)
\]

(b) Show the feasible region.

(c) Determine the maximum value of \( P = 20x + 30y \) subject to the constraints above.
Solution
(a) and (b)

**Step 1:**
To graph a linear inequality we first change the inequality sign (≥ or ≤ or > or <) to an equal sign (=) and draw the graph of the line. But we need two points to draw a line. Choose the two points where the lines cut the x-axis (x-axis intercept, thus y = 0) and y-axis (y-axis intercept, x = 0). Calculate (0 ; y) and (x ; 0) and draw a line through the two points. See the table below for the calculations.

**Step 2:**
Determine the feasible region for each inequality by substituting a point on either side of this line into the equation of the inequality. The inequality region is the area where the selected point makes the inequality true. The calculations are given below:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>y-axis intercept</th>
<th>x-axis intercept</th>
<th>Inequality region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2x + y ≤ 120</strong></td>
<td>2x + y = 120</td>
<td>2x + y = 120</td>
<td>Select the point (0;0) below the line</td>
</tr>
<tr>
<td>(1)</td>
<td>2(0) + y = 120</td>
<td>2(0) + 0 = 120</td>
<td>2(0) + 0 ≤ 120 – True</td>
</tr>
<tr>
<td></td>
<td>y = 120</td>
<td>x = 120</td>
<td>Area below the line</td>
</tr>
<tr>
<td></td>
<td>Point : (0 ; 120)</td>
<td>Point : (60 ; 0)</td>
<td></td>
</tr>
<tr>
<td><strong>x + 2y ≤ 140</strong></td>
<td>x + 2y = 140</td>
<td>x + 2y = 140</td>
<td>Select the point (0;0) to the left of the line</td>
</tr>
<tr>
<td>(2)</td>
<td>0 + 2y = 140</td>
<td>0 + 2(0) = 140</td>
<td>0 + 2(0) ≤ 140 – True</td>
</tr>
<tr>
<td></td>
<td>y = 70</td>
<td>x = 140</td>
<td>Area to the left of the line</td>
</tr>
<tr>
<td></td>
<td>Point (0 ; 70)</td>
<td>Point (140 ; 0)</td>
<td></td>
</tr>
<tr>
<td><strong>x + y ≤ 80</strong></td>
<td>x + y = 80</td>
<td>x + y = 80</td>
<td>Select the point (0;0) below the line</td>
</tr>
<tr>
<td>(3)</td>
<td>0 + y = 80</td>
<td>0 + 0 = 80</td>
<td>0 + 0 ≤ 80 – True</td>
</tr>
<tr>
<td></td>
<td>y = 80</td>
<td>x = 80</td>
<td>Area below the line</td>
</tr>
<tr>
<td></td>
<td>Point (0 ; 80)</td>
<td>Point (80 ; 0)</td>
<td></td>
</tr>
<tr>
<td><strong>x, y ≥ 0</strong></td>
<td></td>
<td></td>
<td>Area above the x-axis and to the right of the y-axis</td>
</tr>
</tbody>
</table>
Step 3:
The feasible region is the one where all the inequalities are true simultaneously.

(c) Determine all the corner points of the feasible region and substitute them into the objective function (function you want to maximise or minimise) and determine the maximum value.

**Step 1:** Determine the coordinates of all corners of the feasible region by solving two equations with two unknowns (substitution) or read them from the graph.

**Step 2:** Substitute the corner points into the objective function.

**Step 3:** Choose the corner point which results in the highest (maximisation) or the lowest (minimisation) objective function value.

<table>
<thead>
<tr>
<th>Corner points of feasible region</th>
<th>Value of $P = 20x + 30y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $x = 60; y = 0$</td>
<td>$P = 20(60) + 30(0) = 1200$</td>
</tr>
<tr>
<td>B: $x = 0; y = 70$</td>
<td>$P = 20(0) + 30(70) = 2100$</td>
</tr>
<tr>
<td>C: $x = 20; y = 60$</td>
<td>$P = 20(20) + 30(60) = 2200 \rightarrow \text{Maximum}$</td>
</tr>
<tr>
<td>D: $x = 40; y = 40$</td>
<td>$P = 20(40) + 30(40) = 2000$</td>
</tr>
<tr>
<td>Origin: $x = 0; y = 0$</td>
<td>$P = 20(0) + 30(0) = 0$</td>
</tr>
</tbody>
</table>

Maximum of $P$ is at point C where $x = 20$, $y = 60$ and $P = 2200$. 
**Question 12**

Giapetto woodcarving manufactures two types of wooden toys: soldiers and trains.
- A soldier sells for R27 and uses R10 worth of raw materials.
- A train sells for R21 and uses R9 worth of raw materials.
- The manufacturer of wooden soldiers and trains requires two types of skilled labour: carpentry and finishing.
- A soldier requires 2 hours of finishing and 1 hour of carpentry.
- A train requires 1 hour of finishing and 1 hour of carpentry.
- Each week, at most 100 finishing hours and 80 carpentry hours are available.
- The demand for trains is unlimited, but at most 40 soldiers are bought each week.
- Giapetto’s has a weekly budget of R10 000 for the raw materials.

If $x$ is the number of toy soldiers made per week and $y$ is the number of trains made per week, formulate the linear constraints that describe Giapetto’s situation and write down the revenue function (objective function) if his objective is to maximise his revenue.

**Solution**

We first define the variables. Let $x$ be the number of toy soldiers manufactured per week and $y$ the number of trains manufactured per week. To help us with the formulation, we summarise the information given in a table with the headings: resources (items on which there are restrictions), the variables ($x$ and $y$) and capacity (amount or number of the resources available).

<table>
<thead>
<tr>
<th>Resource</th>
<th>$x$</th>
<th>$y$</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soldier</td>
<td>1</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>Train</td>
<td>2</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Carpentry</td>
<td></td>
<td></td>
<td>80</td>
</tr>
<tr>
<td>Finishing</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Raw materials budget</td>
<td>10</td>
<td>9</td>
<td>10 000</td>
</tr>
<tr>
<td>Maximum per week</td>
<td>40</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Sales / Revenue</td>
<td>27</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Number of toys</td>
<td></td>
<td></td>
<td>Never negative</td>
</tr>
</tbody>
</table>
Using the table, the following constraints can be defined:

\[
\begin{align*}
    x + y & \leq 80 & \text{Carpentry} \\
    2x + y & \leq 100 & \text{Finishing} \\
    10x + 9y & \leq 10000 & \text{Budgetary constraint on raw materials} \\
    x & \leq 40 & \text{Maximum demand per week} \\
    x, y & \geq 0 & \text{Non-negative}
\end{align*}
\]

As Giapetto would like to maximise his revenue, and as revenue is equal to quantity times the price or sales, the objective function can be written as \(27x + 21y\).

**Question 13**

A manufacturing plant makes two types of inflatable boats: a two-person boat and a four-person boat. Each two-person boat requires 0.9 labour hours from the cutting department and 0.8 labour hours from the assembly department. Each four-person boat requires 1.8 labour hours from the cutting department and 1.2 labour hours from the assembly department. The maximum hours available for the cutting and assembly departments are 864 and 672 respectively. The company makes a profit of R2 500 on a two-person boat and R4 000 on a four-person boat. Let \(x\) and \(y\) be the number of two-person boats and four-person boats made respectively. Formulate the constraints and objective function of the linear programming problem if the company would like to maximise its profit.

**Solution**

First we define the variables. Let \(x\) and \(y\) be the number of two-person boats and four-person boats made respectively. To help us with the formulation, we summarise the information given in a table with the headings: resources (items with restrictions), the variables \((x\ and\ y)\) and capacity (amount or number of the resources available).
Using the table, the following constraints can be defined:

\[ 0.9x + 1.8y \leq 864 \]
\[ 0.8x + 1.2y \leq 672 \]
\[ x, y \geq 0 \]

As the company would like to maximise their profit, the objective function can be written as

\[ 2500x + 4000y. \]

**Question 14**

The demand function for a commodity is \( Q = 6000 - 30P \). Fixed costs are R72 000 and the variable costs are R60 per additional unit produced.

(a) Write down the equation of total revenue and total costs in terms of \( P \).

(b) Determine the profit function in terms of \( P \).

(c) Determine the price at which profit is a maximum, and hence calculate the maximum profit.

(d) What is the maximum quantity produced?

(e) What is the price and quantity at the break-even point(s)?
Solution

(a) The quantity demanded is $Q = 6000 - 30P$, the fixed costs of R72 000 and the variable costs per unit of R60 are given. Now

Total revenue = Price $\times$ Quantity

\[
TR = PQ
\]

\[
TR = P(6000 - 30P)
\]

\[
TR = 6000P - 30P^2
\]

Total cost = Fixed cost + Variable cost

\[
TC = 72000 + 60Q
\]

\[
TC = 72000 + 60(6000 - 30P)
\]

\[
TC = 72000 + 360000 - 1800P
\]

\[
TC = 432000 - 1800P
\]

(b) Profit is total revenue minus total cost. Thus

\[
Profit = TR - TC
\]

\[
= 6000P - 30P^2 - (432000 - 1800P)
\]

\[
= -30P^2 + 7800P - 432000
\]

(c) The profit function derived in (b) is a quadratic function with

\[
a = -30, \ b = 7800 \text{ and } c = -432000.
\]

As $a < 0$ the shape of the function looks like a “sad face” and the function thus has a maximum at the function’s turning point or vertex $(P; Q)$.

The price $P$ at the turning point, or where the profit is a maximum, is

\[
P = \frac{-b}{2a} = \frac{-7800}{2 \times -30} = \frac{-7800}{-60} = 130
\]

and thus the maximum profit

\[
Profit = -30(130)^2 + 7800(130) - 432000 = 75000.
\]
(d) The maximum quantity produced at the maximum price of R130 calculated in (c) is
\[ Q = 6000 - 30(130) = 2100. \]

(e) At break-even the profit is equal to zero. Thus

\[ \text{Profit} = -30P^2 + 7800 - 432000 = 0. \]

As the profit function is a quadratic function we use the quadratic formula with
\[ a = -30 \quad \text{and} \quad b = 7800 \quad \text{and} \quad c = -432000 \]
to solve \( P \). Thus

\[ P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-7800 \pm \sqrt{(7800)^2 - 4(-30)(-432000)}}{2 \times -30} \]

\[ = \frac{-7800 \pm \sqrt{9000000}}{-60} \]

\[ = \frac{-7800 \pm 3000}{-60} \]

\[ = 80 \text{ or } 180 \]

Now if \( P = 80 \) then \( Q = 6000 - 30(80) = 3600 \) and if \( P = 180 \) then \( Q = 6000 - 30(180) = 600 \).

Thus the two break-even points are where the price is R80 and the quantity 3600, and where the price is R180 and the quantity 600.
Question 15
Simplify the following expression:
\[
\left( \frac{4L^2}{L^2} \right)^2
\]

Solution
\[
\left( \frac{4L^2}{L^2} \right)^2 = (4L^2 \times L^2)^2 \quad \text{since} \quad \frac{1}{a^b} = a^{-b}
\]
\[
= (4L^2L^2)^2 \quad \text{since} \quad a^b \times a^c = a^{b+c}
\]
\[
= (4L^4)^2
\]
\[
= 4^2L^{4+2} \quad \text{since} \quad (a^b)^b = a^{ab}
\]
\[
= 16L^6
\]

Question 16
An investment in a bank is said to grow according to the following formula:
\[
P(t) = \frac{6 \, 000}{1 + 29e^{-0.4t}}
\]
where \( t \) is the time in years and \( P \) is the amount (principle plus interest).

(a) What is the initial amount invested?
(b) Determine algebraically the time in years when the amount will be R4 000.

Solution
(a) Initial means \( t = 0 \).
\[
P = \frac{6000}{1 + 29e^{-0.4 \times 0}} = \frac{6000}{30} = 200.
\]
(b) If \( P = 4000 \) then

\[
4000 = \frac{6000}{1 + 29e^{-0.4t}}
\]

\[
1 + 29e^{-0.4t} = \frac{6000}{4000}
\]

\[
1 + 29e^{-0.4t} = \frac{3}{2}
\]

divide nominator and denominator by 2000

\[
29e^{-0.4t} = \frac{3}{2} - 1
\]

\[
29e^{-0.4t} = \frac{1}{2}
\]

\[
e^{-0.4t} = \frac{1}{2} \cdot \frac{29}{1}
\]

\[
e^{-0.4t} = \frac{1}{2} \cdot \frac{1}{29}
\]

\[
e^{-0.4t} = \frac{1}{58}
\]

\[
\ln(e^{-0.4t}) = \ln\left(\frac{1}{58}\right)
\]

take ln on both sides

\[
-0.4t \ln e = \ln\left(\frac{1}{58}\right)
\]

\[
\ln a^b = b \ln a
\]

\[
t = \frac{\ln \left(\frac{1}{58}\right)}{-0.4}
\]

since \( \ln e = 1 \)

\[
t = 10,15110753
\]

using your calculator, rounded to 8 decimal places

\[
t = 10,2\text{ years}
\]

rounded to one decimal place
Question 17

Evaluate \( \frac{\log_3 12.34}{\ln \sqrt{12.34}} \).

Solution

\[
\frac{\log_3 12.34}{\ln \sqrt{12.34}} = \frac{\log_3 12.34}{\ln 3} \times \frac{1}{\ln \sqrt{12.34}} \\
= \frac{1,820}{1} \quad \text{using your calculator, rounded to 3 decimal places}
\]

Question 18

Solve for \( Q \) if \( \log Q - \log \left( \frac{Q}{Q+1} \right) = 0.8 \).

Solution

\[
\log(Q) - \log \left( \frac{Q}{Q+1} \right) = 0.8
\]

\[
\log \left( \frac{Q}{Q+1} \right) = 0.8 \quad \log \frac{a}{b} = \log a - \log b
\]

\[
\log \left( Q \times \frac{Q+1}{Q} \right) = 0.8
\]

\[
\log(Q+1) = 0.8
\]

\[
Q+1=10^{0.8} \quad \log_a b = c \text{ can be written as } a^c = b
\]

\[
Q=10^{0.8} - 1
\]

\[
Q=5.309573445 \quad \text{using your calculator, rounded to 9 decimal places}
\]

\[
Q=5.31 \quad \text{rounded to 2 decimal places}
\]
Question 19

Differentiate the following expression:

\[ \frac{x^3 - 4x^2 + 4x}{x - 2} \]

Solution

First we need to simplify the given expression so that we can use the basic rule of differentiation.

\[ \frac{x^3 - 4x^2 + 4x}{x - 2} = \frac{x(x^2 - 4x + 4)}{x - 2} \]

\[ = \frac{x(x-2)(x-2)}{x - 2} \]

\[ = x(x - 2) \]

\[ = x^2 - 2x \]

Next we can differentiate the new expression using the basic rule \( \frac{d}{dx}x^n = nx^{n-1} \) where \( n \neq 0 \).

Therefore

\[ \frac{d}{dx}\left(\frac{x^3 - 4x^2 + 4x}{x - 2}\right) = \frac{d}{dx}(x^2 - 2x) \]

\[ = 2x^{2-1} - 2x^{1-1} \]

\[ = 2x - 2 \]

Question 20

What is the marginal cost when \( Q = 10 \) if the total cost is given by

\[ TC = 20Q^4 - 30Q^2 + 300Q + 200? \]
Solution
The marginal cost function is the differentiated total cost function. Thus by differentiating the total cost function we can determine the marginal cost function. Now if the total cost function is

\[ TC = 20Q^4 - 30Q^2 + 300Q + 200 \]

then the marginal cost function is

\[ MC = \frac{dTC}{dQ} = 80Q^3 - 60Q + 300. \]

Now the value of the marginal cost function when \( Q \) is equal 10, is

\[ MC(10) = 80(10)^3 - 60(10) + 300 = 80000 - 600 + 300 = 79700. \]

Question 21
Evaluate the following:

\[ \int (x^2 + 2x + 3)\,dx \]

Solution
To integrate the function we make use of the basic rule of integration, namely

\[ \int x^n \,dx = \frac{x^{n+1}}{n+1} + c \text{ when } n \neq -1. \] Therefore

\[ \int (x^2 + 2x + 3)\,dx = \int x^2 \,dx + \int 2x \,dx + \int 3\,dx \]

\[ = \frac{x^{2+1}}{2+1} + \frac{2x^{1+1}}{1+1} + \frac{3x^{0+1}}{0+1} + c \]

\[ = \frac{x^3}{3} + \frac{2x^2}{2} + \frac{3x}{1} + c \]

\[ = \frac{x^3}{3} + x^2 + 3x + c \]
Question 22

Determine \( \int \frac{Q^{1+1}}{\sqrt{Q}} \, dQ \)

Solution

First simplify the function to be integrated:

\[
\frac{Q^{1+1}}{\sqrt{Q}} = \frac{(Q+1)}{Q^{\frac{1}{2}}}
\]

\[= (Q+1)Q^{-\frac{1}{2}} \]

\[= Q^{\frac{1}{2}} + Q^{-\frac{1}{2}} \]

Now we can integrate the function using the basic integration rule \( \int x^n = \frac{x^{n+1}}{n+1} + c \) when \( n \neq -1 \).

\[
\int (Q^{\frac{1}{2}} + Q^{-\frac{1}{2}}) \, dQ = \int Q^{\frac{1}{2}} + \int Q^{-\frac{1}{2}}
\]

\[= \frac{Q^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{Q^{\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \]

\[= \frac{Q^{\frac{3}{2}}}{3} + \frac{Q^{\frac{1}{2}}}{\frac{1}{2}} + c \]

\[= \sqrt{Q^3} \cdot \frac{\frac{3}{2}}{3} + \sqrt{Q} \cdot \frac{\frac{1}{2}}{1} + c \]

\[= \frac{2\sqrt{Q^3}}{3} + 2\sqrt{Q} + c \]
Question 23

Evaluate

\[ \int_{-1}^{1} (z + 1) \, dz \]

**Solution**

First simplify the function to be integrated:

\[ \int_{-1}^{1} (z + 1) \, dz = \int_{-1}^{1} z \, dz + \int_{-1}^{1} 1 \, dz \]

Now we can integrate the function, using the basic rule \( \int x^n = \frac{x^{n+1}}{n+1} + c \) when \( n \neq -1 \), and then substitute the values between which the integral has to be calculated, into the integrated function:

\[ \int_{-1}^{1} z \, dz + \int_{-1}^{1} 1 \, dz = \frac{1}{2} z^{1+1} \bigg|_{-1}^{1} + z \bigg|_{-1}^{1} \]

\[ = \left( \frac{1}{2} z^2 \right) \bigg|_{-1}^{1} + \left( z \right) \bigg|_{-1}^{1} \]

\[ = \left( \frac{1}{2} 1^2 + 1 \right) \left( \frac{1}{2} - 1 \right) \]

\[ = 1 \frac{1}{2} \left( \frac{1}{2} - 1 \right) \]

\[ = 1 \frac{1}{2} \left( -\frac{1}{2} \right) \]

\[ = 1 \frac{1}{2} + \frac{1}{2} \]

\[ = 2 \]
3. Sample examination paper and solution

Examination paper:
The examination paper consists of two sections: Section A and Section B

SECTION A
Answer ALL the questions in this section on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet.
Also pay attention to the following information. Suppose you are asked the following question:

\[ 3 + 2 \times -1 + 4 \div 2 = \]


The correct answer is [3]. Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. If you mark more than one option, you will not receive any marks for the question. If your answer is correct, you will receive 3 MARKS. Marks WILL NOT be subtracted for incorrect answers.

Section A consists of 20 questions and counts 60 marks. Hand in the completed mark-reading sheet with your answers for Section B. DO NOT STAPLE IT!

SECTION B
This section must be completed in the spaces provided below each question. Section B counts 40 marks.
Remember to include your MARK-READING SHEET.
SECTION A

Question 1
Find the slope of the line \(0 = 6 + 3x - 2y\).

[1] \(\frac{2}{3}\)
[2] \(\frac{3}{2}\)
[3] 3
[4] 2
[5] None of the above

Question 2
\(\log_{3} \left( \frac{3}{\sqrt{3}} \right)\), to four decimal places, equals

[1] –0.0795.
[2] 0.0795.
[4] 0.5000.

Question 3
Solve the inequality \(x^2 - 3x \geq 6 - 2\).

[1] \(-2 \leq x \leq 3\)
[2] \(-6 \leq x \leq 1\)
[3] \(x \leq -2; x \geq 3\)
[4] \(x \leq -3; x \geq 2\)
[5] \(-3 \leq x \leq 2\)
Question 4
Find the equation of the straight line passing through the points (4;2) and (2;4).

[1] \( y = -1x + 6 \)
[2] \( y = -1x \)
[3] \( y = 2x + 4 \)
[4] \( y = 1x + 2 \)
[5] None of the above

Question 5
Find the value of quantity \( Q \) for the demand function \( P = 60 - 4Q \) when the market price is \( P = 24 \).

[1] 8
[2] 9
[3] 10
[4] 11
[5] 12

Question 6
Calculate the consumer surplus for the demand function \( P = 60 - 4Q \) when the market price is \( P = 16 \).

[1] 242
[2] 484
[3] 88
[4] 32
[5] 352
**Question 7**

If the demand function is \( P = 90 - 0.05Q \), where \( P \) and \( Q \) are the price and quantity respectively, determine the expression for price elasticity of demand in terms of \( P \).

\[
\begin{align*}
[1] & \quad \frac{P}{P-90} \\
[2] & \quad \frac{P-90}{P} \\
[3] & \quad \frac{P}{P-1800} \\
[4] & \quad \frac{P}{P-1800} \\
[5] & \quad \text{None of the above}
\end{align*}
\]

**Question 8**

The supply and demand functions are given by

\[
\begin{align*}
P &= 50 - 3Q \quad \text{(supply function)} \\
P &= 14 + 1.5Q \quad \text{(demand function)}
\end{align*}
\]

where \( P \) and \( Q \) are the price and quantity respectively. Calculate the level of excess supply if price \( P = 20 \).

\[
\begin{align*}
[1] & \quad 10 \\
[2] & \quad 4 \\
[3] & \quad 14 \\
[4] & \quad 6 \\
[5] & \quad \text{None of the above}
\end{align*}
\]
**Question 9**

What is the value of maximum revenue if total revenue is given by

\[ R(x) = -\frac{1}{5}x^2 + 30x + 81 \]

where \( x \) is the quantity?

[1] 75  
[2] 1206  
[3] 152,65  
[4] 81  
[5] None of the above

**Question 10**

Solve the following system of linear equations:

\[ \begin{align*} 
    x + y + z &= 8 \\
    x - 3y &= 0 \\
    5y - z &= 10 
\end{align*} \]

[1] \( x = 6; y = 2; z = 0 \)  
[2] \( x = 0; y = 6; z = 2 \)  
[3] \( x = 2; y = 0; z = 6 \)  
[4] \( x = -6; y = 2; z = 6 \)  
[5] None of the above
Question 11

Determine the roots of $4x^2 + 3x - 1$.

[1] $x = \frac{1}{4}; x = -1$
[2] $x = \frac{1}{4}; x = 1$
[3] $x = -\frac{1}{4}; x = 1$
[4] $x = -\frac{1}{4}; x = -1$
[5] None of the above

Question 12

If $y = 2^{-x}$, find $x$ if $y = 0.0625$.

[1] $x = -2$
[2] $x = 3$
[3] $x = 4$
[4] $x = 5$
[5] None of the above

Question 13

Evaluate the following definite integral:

$$\int_{-2}^{2} (x^2 - 3)dx$$

[1] $6\frac{2}{3}$
[2] $-6\frac{2}{3}$
[3] $3\frac{1}{3}$
[4] $-3\frac{1}{3}$
[5] None of the above
**Question 14**
Evaluate

\[ \int x^2 \left( 1 + \frac{1}{x^2} \right) \, dx \]

1. \( x^2 + x + c \)
2. \( \frac{1}{3} x^3 + x + c \)
3. \( x^2 + 1 \)
4. \( \frac{1}{2} x^2 + x + c \)
5. None of the above

**Question 15**
Simplify

\[ \frac{d}{dx} \left[ \frac{x - x^2}{\sqrt{x}} \right] \]

1. \( \frac{2}{3} \sqrt{x} + \frac{1}{2\sqrt{x}} \)
2. \( \frac{1}{2\sqrt{x}} - \frac{3}{2} \sqrt{x} \)
3. \( \frac{3}{2} \sqrt{x} - \frac{1}{2\sqrt{x}} \)
4. \( \frac{3}{2} \sqrt{x} + \frac{1}{2\sqrt{x}} \)
5. None of the above
**Question 16**

The demand function of a firm is $Q = 150 - 0.5P$, where $P$ and $Q$ represent the quantity and price respectively. At what value of $Q$ is marginal revenue equal to zero?

[1] 150  
[2] 75  
[3] 113  
[4] 0  
[5] None of the above

**Question 17**

Given the demand function $P = 60 - 0.2Q$. What is the arc price elasticity of demand when the price decreases from R50 to R40?

[1] $-\frac{1}{3}$  
[2] $\frac{1}{3}$  
[3] $-3$  
[4] $3$  
[5] None of the above
Question 18

Consider the market defined by the following functions:

\[
\begin{align*}
\text{demand function:} & \quad P = 60 - 0.6Q \\
\text{supply function:} & \quad P = 20 + 0.2Q
\end{align*}
\]

where \( P \) and \( Q \) are the price and quantity respectively. Calculate the equilibrium price and quantity.

[1] \( P = 300; \ Q = 20 \)
[2] \( P = 200; \ Q = 30 \)
[3] \( P = 20; \ Q = 300 \)
[4] \( P = 30; \ Q = 200 \)
[5] None of the above

Question 19

What is the point of intersection of the following lines?

\[
\begin{align*}
2x + y - 5 &= 0 \\
3x - 2y - 4 &= 0
\end{align*}
\]

[1] \( x = 3; \ y = 1 \)
[2] \( x = 1; \ y = 2 \)
[3] \( x = 2; \ y = 1 \)
[4] \( x = 1; \ y = 3 \)
[5] None of the above
Question 20

The graph of $y = -2x + x^2 - 3$ is represented by

[1]

[2]

[3]

[4]

[5] None of the above
SECTION B

Question 21
The monthly demand for a new line of computers, $t$ months after it has been introduced in the market, is given by

$$D(t) = 2000 - 1500e^{-0.05t} \text{ for } t > 0.$$  

(a) Find demand two years after these computers were introduced.  
(b) Algebraically, determine the number of months after which demand will be 1 000 units.

[5]

Question 22
An electronics company manufactures radios and television sets. The time needed to manufacture a radio is 90 minutes and it takes 5 minutes to test the radio. The time needed to manufacture a television set is 150 minutes and it takes 15 minutes to test a television set. It costs R175 to make a radio and R850 to make a television set. The company has at most 95 hours of manufacturing time and at least 9 hours of testing time available. The production cost must not exceed R13 500.

Write down the inequalities that this production process must satisfy.

[10]

Question 23
ABC intends manufacturing and marketing a new product. It has been determined that the cost of producing the product, as a function of price, is given by

$$C(P) = 432 000 - 1800P.$$  

The revenue generated when units are sold at price $P$ rand each is given by

$$R(P) = 6000P - 30P^2.$$  

Plot the income and cost functions on the same graph. Indicate clearly on the graph, the break-even point(s) and profit area.

[9]
Question 24

(a) Graph the lines representing the following constraints: (4)

\[ \begin{align*}
(1) & \quad 6x + 2y \leq 840 \\
(2) & \quad 2x + y \leq 300 \\
(3) & \quad x + y \leq 250 \\
& \quad x, y \geq 0
\end{align*} \]

(b) Show the feasible region. (1)

(c) Determine the maximum value of \( P = 120x + 95y \), subject to the constraints above. (5)

[10]

Question 25

Let \( f(x) = 3x^2 - x \). Find the equation of the line tangent to the graph \( y = f(x) \) at \( x = 1 \). (6)

[6]

[40]

Total: 100
Solutions

SECTION A

Question 1
In general the line \( y = mx + c \) has a slope of \( m \).

First we need to change the given function to the general format of a line, namely \( y = mx + c \). We need to change the equation so that \( y \) is the subject of the equation, i.e. writing it on its own on one side of the equation. Now we are given the line \( 0 = 6 + 3x - 2y \).

Therefore

\[
0 = 6 + 3x - 2y \\
2y = 6 + 3x \quad \text{add } 2y \text{ on both sides} \\
y = \frac{6 + 3x}{2} \quad \text{divide both sides by } 2 \\
y = \frac{6}{2} + \frac{3x}{2} \\
y = 3 + \frac{3}{2}x
\]

The slope of the line \( 0 = 6 + 3x - 2y \) is equal to \( \frac{3}{2} \).

[Option 2]

Question 2

\[
\log_3\left(\frac{3}{\sqrt{3}}\right) = \frac{\ln\left(\frac{3}{\sqrt{3}}\right)}{\ln 3} \\
\log_a b = \frac{\ln b}{\ln a}
\]

\[
= \frac{0.54931}{1.09861} \quad \text{using your calculator, rounded to 5 decimal places} \\
= 0.5000 \quad \text{rounded to 4 decimal places}
\]

[Option 4]
Question 3
First we write the inequality in the standard quadratic format \( y = ax^2 + bx + c \):

\[
x^2 - 3x \geq 6 - 2x \\
x^2 - 3x - 6 + 2x \geq 0 \\
x^2 - x - 6 \geq 0
\]

To determine the \( x \)-values for which the inequality holds, we use the quadratic formula to determine the solution or roots of the equation \( x^2 - x - 6 = 0 \) using the formula

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

with \( a, b, \) and \( c \) the values of the coefficients in the function \( ax^2 + bx + c = 0 \).

From the given function \( x^2 - x - 6 = 0 \), we can derive that \( a = 1 \), \( b = -1 \) and \( c = -6 \). Substituting \( a, b \) and \( c \) into the formula gives

\[
x = \frac{(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)} \\
x = \frac{+1 \pm \sqrt{1 + 24}}{2} \\
x = \frac{1 \pm 5}{2} \\
x = \frac{1+5}{2} \quad \text{or} \quad \frac{1-5}{2} \\
x = \frac{6}{2} \quad \text{or} \quad \frac{-4}{2} \\
x = 3 \quad \text{or} \quad -2.
\]
Graphing the solution of the equation $x^2 - x - 6 = 0$ on a number line:

![Number line with regions](image)

Now to determine the area where the inequality $x^2 - x - 6 \geq 0$ is true, we substitute a point in the region smaller than $-2$, the region between $-2$ and $3$ and a value in the region greater than $3$, into the inequality. The inequality region is then the area in which the selected point makes the inequality true. You can choose any values in the different regions.

First we choose a value smaller than $-2$, for example $x = -3$. Now the left-hand side of the inequality $x^2 - x - 6 \geq 0$ is as follows:

\[
\text{LHS} = x^2 - x - 6 \\
= (-3)^2 - (-3) - 6 \\
= 9 + 3 - 6 \\
= 6
\]

The right-hand side: \( \text{RHS} = 0 \).
We need the \( \text{LHS} \geq \text{RHS} \) and the \( \text{LHS} \geq \text{RHS} \). The inequality $x^2 - x - 6 \geq 0$ is therefore true for values smaller than $-2$.

Next we choose a value between $-2$ and $3$ for example $x = 0$. Now the left-hand side of the inequality $x^2 - x - 6 \geq 0$ is

\[
\text{LHS} = x^2 - x - 6 \\
= (0)^2 - (0) - 6 \\
= -6.
\]

The right-hand side is \( \text{RHS} = 0 \).
We need the LHS $\geq$ RHS but the LHS $\leq$ RHS. The inequality $x^2 - x - 6 \geq 0$ is therefore not true for values between $-2$ and 3.

Finally, we choose a value greater than 3, for example $x = 4$. Now the left-hand side of the inequality $x^2 - x - 6 \geq 0$ is

$$LHS = x^2 - x - 6$$
$$= (4)^2 - (4) - 6$$
$$= 16 + 4 - 6$$
$$= 14$$

The right-hand side is

$$RHS = 0$$

We need the LHS $\geq$ RHS, and the LHS $\geq$ RHS. The inequality $x^2 - x - 6 \geq 0$ is therefore true for values greater than 3.

The area of the inequality is therefore true if $x \leq -2$ and $x \geq 3$.

[Option 3]

**Question 4**

Let $(x_1 ; y_1) = (4 ; 2)$ and $(x_2 ; y_2) = (2 ; 4)$

We need to determine the slope $m$ and $y$-intercept $c$ of the line $y = mx + c$. Now the slope $m$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{2 - 4} = \frac{-2}{-2} = 1$$

Therefore $y = -1x + c$ or $y = -x + c$.

Now both $(x_1 ; y_1)$ and $(x_2 ; y_2)$ lie on the line. We can thus substitute any one of the points into the equation of the line to determine $c$. Let’s choose the point $(4 ; 2)$ then
\[
\begin{align*}
y &= -1x + c \\
2 &= -1 \times 4 + c \\
2 &= -4 + c \\
2 + 4 &= c \\
c &= 6
\end{align*}
\]

The equation of the line is \( y = -1x + 6 \).

[Option 1]

**Question 5**

We need to determine the value \( Q \) if \( P \) is equal to 24. Thus we substitute the value of \( P \) into the function and solve for \( Q \).

Therefore

\[
\begin{align*}
P &= 60 - 4Q \\
24 &= 60 - 4Q \\
4Q &= 60 - 24 \\
4Q &= 36 \\
Q &= 9
\end{align*}
\]

[Option 2]

**Question 6**

If you need to determine the demand surplus for a demand function of \( P = a - bQ \) then the consumer surplus can be calculated by calculating an area of the triangle \( P_0Q_0a \) which is equal to

\[
\frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times (a - P_0) \times (Q_0 - 0) = \frac{1}{2} \times (a - P_0) \times (Q_0)
\]

with

- \( P_0 \) the value given as the market price,
- \( Q_0 \) the value of the demand function if \( P \) equals the given market price (Substitute \( P_0 \) into the demand function and calculate \( Q_0 \)), and
- \( a \) the \( y \)-intercept of the demand function \( P = a - bQ \), also known as the value of \( P \) if \( Q = 0 \), or the point where the demand function intercepts the \( y \)-axis.
In general we can summarise the steps of determining the consumer surplus as follows:

**Method:**

1. Calculate $Q_0$ if $P_0$ is given.
2. Draw a rough graph of the demand function.
3. Read the value of $a$ from the demand function – the $y$-intercept of the demand function.
4. Calculate the area of $CS = \frac{1}{2} \times (a - P_0) \times (Q_0)$.

First we calculate $Q$ if $P = 16$. Therefore

\[
P = 60 - 4Q \]
\[
16 = 60 - 4Q \]
\[
4Q = 60 - 16 \]
\[
4Q = 44 \]
\[
Q = 11
\]

The consumer surplus is the area of the shaded triangle on the right:

\[
CS = \frac{1}{2} \times \text{base} \times \text{height} \\
= \frac{1}{2} \times 11 \times (60 - 16) \\
= \frac{1}{2} \times 11 \times 44 \\
= \frac{482}{2} \\
= 242
\]
Question 7

The demand function is given as \( P = 90 - 0.05Q \). Now the price elasticity of demand is

\[
\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q}
\]

with \( a \) and \( b \) the values of the standard demand function \( P = a - bQ \).

To determine the elasticity of demand, we need to determine the values of \( b, Q \) and \( P \). It is given that \( P = 90 - 0.05Q \) and question asked in terms of \( P \) thus \( P = P \). Comparing \( P = 90 - 0.05Q \) with \( P = a - bQ \) we can say that \( a = 90 \) and \( b = 0.05 \). Now \( a, b \) and \( P \) are known and \( Q \) is unknown at this stage.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving for \( Q \).

\( P = 90 - 0.05Q \). We need to change the equation so that \( Q \) is the subject of the equation.

That means we write \( Q \) in terms of \( P \) as asked. Now

\[
P = 90 - 0.05Q
\]

\[
P - 90 = -0.05Q
\]

\[
\frac{P - 90}{-0.05} = Q
\]

\[
Q = \frac{P - 90}{-0.05}.
\]

As we have determined the values of \( b, P \) and \( Q \) we can now substitute them into the formula for the elasticity of demand:

\[
\varepsilon_d = -\frac{1}{0.05} \cdot \frac{P}{P - 90}
\]

\[
= -\frac{1}{0.05} \cdot \frac{P}{P - 90} \cdot -0.05
\]

\[
= \frac{P}{P - 90}
\]
Or alternatively,

you can use the given formula of elasticity of demand in terms of $P$ as given in the textbook on page 78, equation 2.14 (Edition 2) and page 89, equation 2.14 (Edition 3).

\[ \varepsilon_d = \frac{P}{P - a} \]

Now $a = 90$ (intercept on the $y$-axis of the demand function)

\[ \varepsilon_d = \frac{P}{P - 90} \]

[Option 1]

**Question 8**

We need to determine the difference between the quantity supplied and the quantity demanded if the price is equal to 20. First we determine what the quantity supplied is if the price is 20. Thus we substitute the value $P = 20$ into the supply function and solve for the quantity supplied. Therefore

\[
\begin{align*}
P &= 50 - 3Q \\
20 &= 50 - 3Q \\
3Q &= 50 - 20 \\
3Q &= 30 \\
Q &= \frac{30}{3} \\
Q &= 10
\end{align*}
\]
Now we determine what the quantity demanded, is if the price is 20. Thus we substitute the value \( P = 20 \) into the demand function and solve for \( Q \). Therefore

\[
P = 14 + 1.5Q
\]

\[
20 = 14 + 1.5Q
\]

\[
-1.5Q = 14 - 20
\]

\[
-1.5Q = -6
\]

\[
Q = \frac{-6}{-1.5}
\]

\[
Q = 4
\]

The supplier supplied 10 units and only 4 units were demanded. Thus the supplier supplied \( 10 - 4 = 6 \) units more than were demanded.

**Question 9**
The given total revenue function is a quadratic function. The minimum or maximum value of a quadratic function is where the quadratic function changes direction or turns, also called the vertex. The vertex is given by the coordinate pair \((x; y)\). There exists different methods to determine the maximum or minimum value of a quadratic function.

**Method 1:**
The \( x \)-coordinate of the vertex can be calculated by using the formula

\[
x = \frac{-b}{2a}
\]

with \( a \), \( b \), and \( c \) the coefficients in the standard quadratic function \( y = ax^2 + bx + c \).

It is given that \( R(x) = -\frac{1}{5}x^2 + 30x + 81 \). Comparing it with the standard form of the quadratic function \( y = ax^2 + bx + c \), we can see that \( a = -\frac{1}{5} \), \( b = 30 \) and \( c = 81 \) for the given function. As the \( a \)-value is negative we can say that the graph of the function is in the form of a sad
face, thus a maximum extreme point exists for the function. Therefore the $x$-value of the extreme point or vertex is:

\[
x = \frac{-30}{2 \times -\frac{1}{5}}
= \frac{-30}{-\frac{10}{5}}
= \frac{-30 \times -2}{5}
= \frac{60}{5}
= 12
\]

To calculate the $y$-value of the vertex we substitute the calculated $x$-value into the given equation $R(x) = -\frac{1}{5}x^2 + 30x + 81$ and calculate $R(x)$. Therefore

\[
R(x) = -\frac{1}{5}x^2 + 30x + 81
= -\frac{1}{5}(75)^2 + 30(75) + 81
= 1206
\]

The function has a maximum value in the point $(75; 1206)$ or where the revenue is equal to 1206.

**Method 2:**
You can also make use of the method of differentiation, as discussed in Chapter 6, to determine the maximum or minimum value or vertex of a function.

The maximum or minimum value of a function is the point where the differentiated function is equal to zero or $\frac{dy}{dx} = 0$. The function $R(x) = -\frac{1}{5}x^2 + 30x + 81$ was given.

Differentiating the function $R(x)$, using the basic rule of differentiation namely $\frac{d}{dx} x^n = nx^{n-1}$ with $n \neq 0$, we get
\[ R(x) = -\frac{1}{5} x^2 + 30x + 81 \]

\[ \frac{d}{dx} R(x) = -\frac{1}{5} (2)x^{2-1} + 30x^{1-1} + 0 \quad \text{because } \frac{d}{dx} a = 0 \text{ if } a \text{ is a constant} \]

\[ \frac{d}{dx} R(x) = -\frac{2}{5} x^1 + 30x^0 \quad \text{but } x^0 = 1 \]

\[ \frac{d}{dx} R(x) = -\frac{2}{5} x + 30 \]

But the maximum or minimum occurs when \( \frac{dy}{dx} = 0 \). Therefore

\[ \frac{d}{dx} R(x) = 0 \]

\[ -\frac{2}{5} x + 30 = 0 \]

\[ -\frac{2}{5} x = -30 \]

\[ x = -30 \times -\frac{5}{2} \]

\[ x = \frac{150}{2} \]

\[ x = 75 \]

To calculate the \( y \)-value of the extreme point or vertex we substitute the calculated \( x \)-value into the given equation \( R(x) = -\frac{1}{5} x^2 + 30x + 81 \). Therefore

\[ R(x) = -\frac{1}{5} x^2 + 30x + 81 \]

\[ = -\frac{1}{5} (75)^2 + 30(75) + 81 \]

\[ = 1206. \]

The function has a maximum value in the point \( (75; 1206) \), that is where the revenue is equal to 1206.
Method 3:
You can determine the minimum or maximum value of a quadratic function by using the symmetry of the quadratic function. Because the quadratic function is symmetrical, the vertex \((x; y)\) occurs halfway between the two roots of the quadratic function. Therefore you can determine the \(x\)-value of the vertex by calculate the roots and then determine the halfway mark \((\text{root 1} + \text{root 2})/2\) and then substitute the \(x\)-value of the vertex into the quadratic function to determine the \(y\)-value of the vertex.

Question 10

We need to solve the following system of equations:

\[
\begin{align*}
\text{(1)} & \quad x + y + z = 8 \\
\text{(2)} & \quad x - 3y = 0 \\
\text{(3)} & \quad 5y - z = 10
\end{align*}
\]

Make \(x\) the subject of equation (2) and \(z\) the subject of equation (3):

\[
\begin{align*}
\text{(4)} & \quad x = 3y \\
\text{(5)} & \quad z = -10 + 5y
\end{align*}
\]

Substitute equation (4) and equation (5) into equation (1):

\[
x + y + z = 8 \\
(3y) + y + (-10 + 5y) = 8 \\
9y = 8 + 10 \\
9y = 18 \\
y = \frac{18}{9} \\
y = 2
\]

Substitute \(y = 2\) into equation (4) and equation (5):

\[
x = 3y = 3 \times 2 = 6
\]

and

\[
z = -10 + 5y = -10 + 5(2) = -10 + 10 = 0
\]

Therefore \(x = 6; y = 2\) and \(z = 0\).
Question 11

The roots or solutions of a function can be found where the function, if drawn, intersects the \(x\)-axis. We therefore need to determine the value of \(x\) at the point(s) where the graph of the function intersects the \(x\)-axis, in other words where the function value is zero:

\[
y = 0 \quad \text{or} \quad 4x^2 + 3x - 1 = 0.
\]

We make use of the quadratic formula

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

with \(a, b,\) and \(c\) the values of the coefficients in the equation \(0 = ax^2 + bx + c\) to determine the roots.

Comparing the given equation \(4x^2 + 3x - 1 = 0\) with the general format of \(0 = ax^2 + bx + c\), we derive that \(a = 4;\ b = 3\) and \(c = -1\). Substituting \(a, b\) and \(c\) into the formula gives

\[
x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(4)} = \frac{-3 \pm \sqrt{9 + 16}}{8} = \frac{-3 \pm \sqrt{25}}{8} = \frac{-3 \pm 5}{8}.
\]

\[
x = \frac{-3 + 5}{8} \quad \text{or} \quad \frac{-3 - 5}{8} = \frac{2}{8} \quad \text{or} \quad \frac{-8}{8} = \frac{1}{4} \quad \text{or} \quad -1.
\]

The roots of the function \(4x^2 + 3x - 1 = 0\) are \(-1\) and \(\frac{1}{4}\). [Option 1]
Question 12

To solve for \( x \) if \( y = 2^{-x} \) and \( y = 0,0625 \) we substitute \( y = 0,0625 \) into the equation and solve for \( x \). Therefore

\[ 0,0625 = 2^{-x} \]

Taking log or \( \ln \) on both sides of the equation yields

\[ \log 0,0625 = \log (2^{-x}) \quad \text{or} \quad \ln 0,0625 = \ln (2^{-x}) \]

Apply rule 3 of logarithms ( \( \ln a^b = b \ln a \)):

\[ \frac{\log 0,0625}{\log 2} = -x \quad \text{or} \quad \frac{\ln 0,0625}{\ln 2} = -x \]

\[ -4,00 = -x \quad \text{or} \quad -4,00 = -x \]

\[ x = 4,00 \quad \text{or} \quad x = 4,00 \quad \text{rounded to two decimals} \]

The value of \( x \), if \( y = 0,0625 \), is equal to 4,00 (rounded to 2 decimal places). Actually it happens to be exactly 4 but one cannot say that for sure if a calculator was used.

[Option 3]
Question 13

To determine the definite integral we first use the basic rule, namely
\[ \int x^n = \frac{x^{n+1}}{n+1} + c \] when \( n \neq -1 \), to integrate the function and secondly we substitute the given values between which we must integrate (2 and –2 in this case) into the integrated function.

Therefore
\[
\int_{-2}^{2} (x^2 - 3) \, dx = \int_{-2}^{2} x^2 \, dx - \int_{-2}^{2} 3 \, dx
\]
\[
= \left[ \frac{x^3}{3} \right]_{-2}^{2} - \left[ \frac{3x}{1} \right]_{-2}^{2}
\]
\[
= \left[ \frac{2^3}{3} - \frac{(-2)^3}{3} \right] - \left[ 3(2) - 3(-2) \right]
\]
\[
= \left[ \frac{8}{3} + \frac{8}{3} \right] - [6 + 6]
\]
\[
= \frac{16}{3} - 12
\]
\[
= \frac{16 - 36}{3}
\]
\[
= -\frac{20}{3}
\]
\[
= -6\frac{2}{3}
\]

[Option 2]

Question 14

First we simplify the expression before we integrate. Therefore
\[
x^2 \left( 1 + \frac{1}{x^2} \right) = x^2 + \frac{x^2}{x^2}
\]
\[
= x^2 + 1
\]

To determine the integral we use the basic rule, namely \( \int x^n = \frac{x^{n+1}}{n+1} + c \) when \( n \neq -1 \), to integrate the function
\[
\int x^2 (1 + \frac{1}{x^2}) \, dx = \int (x^2 + 1) \, dx
\]
\[
= \frac{x^{2+1}}{2+1} + x + c
\]
\[
= \frac{x^3}{3} + x + c
\]

[Option 2]

**Question 15**

First we simplify the expression. We can write \( \sqrt{x} \) as \( x^{\frac{1}{2}} \) when changing from surd (square root) form to exponential form. Therefore

\[
\frac{x - \frac{x^2}{\sqrt{x}}}{\sqrt{x}} = \frac{x - \frac{x^2}{x^{\frac{1}{2}}}}{x^{\frac{1}{2}}}
\]
\[
= \frac{x^{\frac{1}{2}} - \frac{x^2}{x^{\frac{1}{2}}}}{x^{\frac{1}{2}}}
\]
\[
= x^{\frac{1}{2}} - x^{\frac{3}{2}}
\]

Next we differentiate the simplified expression using the basic rule of differentiation, namely \( \frac{d}{dx} x^n = nx^{n-1} \) when \( n \neq 0 \):
\[
\frac{d}{dx} \left[ \frac{x - x^2}{\sqrt{x}} \right] = \frac{d}{dx} \left( x^{\frac{1}{2}} - x^{\frac{3}{2}} \right)
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
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\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
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\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
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= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
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= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
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= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
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= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]
\[
= \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{-\frac{3}{2}}
\]

**Question 16**

Revenue is price \( \times \) demand or \( R = P \times Q \)

It is given that price is \( P \) and demand is \( Q = 150 - 0.5P \).

Therefore substitute \( Q = 150 - 0.5P \) into the formula for \( R \):

\[
R = P \times (150 - 0.5P)
\]

\[
R = 150P - 0.5P^2
\]

To determine the marginal revenue we need to differentiate the ordinary revenue function. Thus the marginal revenue function \((MR)\) is

\[
MR = \frac{dR}{dP} = \frac{d}{dP} (150P - 0.5P^2)
\]

\[
MR = 150 - (2 \times 0.5)P
\]

\[
MR = 150 - P
\]
You are asked at what value of $Q$ is $MR$ equal to 0. But $MR$ contains only $P$. We first need to solve for $P$ and then substitute its value into the demand function to solve for $Q$. (Remember the demand function denotes the relationship between $P$ and $Q$.) Therefore if

$$MR = 0$$

Then

$$0 = 150 - P$$
$$P = 150.$$ 

But we need to determine $Q$. Now $P$ and $Q$ are related as $Q = 150 - 0,5P$. Therefore

$$Q = 150 - 0,5 \times (150)$$
$$Q = 150 - 75$$
$$Q = 75.$$ 

The value of $Q$ is equal to 75 if the marginal revenue is equal to 0.

[Option 2]

Question 17

The arc price elasticity of a demand function $P = a - bQ$ between two prices $P_1$ and $P_2$ is

$$\text{arc elasticity of demand} = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

with $b$ the slope of the demand function and $P_1$, $P_2$ and $Q_1$, $Q_2$ the price and quantity demanded.

For the given function $P = 60 - 0,2Q$, we see that $a = 60$ and $b = 0,2$ and it is given that $P_1 = 50$ and $P_2 = 40$. We only need to determine $Q_1$ and $Q_2$. We can rewrite the equation $P = 60 - 0,2Q$, by making $Q$ the subject of the equation, as
\[ P = 60 - 0.2Q \]
\[ 0.2Q = 60 - P \]
\[ Q = 300 - 5P. \]

We can now determine \( Q_1 \) and \( Q_2 \) by substituting \( P_1 = 50 \) and \( P_2 = 40 \) into the equation:

If \( P_1 = 50 \) then \( Q_1 = 300 - 5 \times 50 = 50 \)
and if \( P_2 = 40 \) then \( Q_2 = 300 - 5 \times 40 = 100. \)

Therefore

\[
\text{arc elasticity of demand} = - \frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2}
\]
\[
= - \frac{1}{0.2} \times \frac{50 + 40}{50 + 100}
\]
\[
= - \frac{1}{0.2} \times \frac{90}{150}
\]
\[
= -90 \div 30
\]
\[
= -3.
\]

[Option 3]

Question 18

Equilibrium is the price and quantity where the demand and supply functions are equal. Thus \( P_d = P_s \) or

\[ 60 - 0.6Q = 20 + 0.2Q \]
\[ -0.6Q - 0.2Q = 20 - 60 \]
\[ -0.8Q = -40 \]
\[ Q = \frac{-40}{-0.8} \]
\[ Q = 50 \]
To calculate the quantity at equilibrium, we substitute the value of $Q$ into the demand or supply function and calculate $P$. Say we use the demand function, then

$$P = 60 - 0.6Q$$
$$P = 60 - 0.6(50)$$
$$P = 60 - 30$$
$$P = 30$$

The equilibrium price is equal to 50 and the quantity to 30.

**Question 19**

To determine the point of intersection of two lines we need to determine a point ($x; y$) so that the $x$ and $y$ values satisfy both equations of the lines. Thus we need to solve the two equations simultaneously.

Let  
$$2x + y - 5 = 0$$
$$3x - 2y - 4 = 0$$

and  
$$2x + y = 5$$
$$3x - 2y = 4$$

$2 \times$ equation (1):
$$4x + 2y = 10$$

Equation (2) + equation (3):
$$3x - 2y = 4$$
$$+ (4x + 2y = 10)$$
$$7x = 14$$

Now solve for $x$:
$$x = \frac{14}{7}$$
$$x = 2$$

Substitute $x = 2$ into equation (1) or equation (2) and solve for $y$. If we, for example, substitute $x = 2$ into equation (1) we get

$$2(2) + y = 5$$
$$4 + y = 5$$
$$y = 5 - 4$$
$$y = 1$$
The two lines intersect in the point \((x, y) = (2, 1)\).

**Question 20**

Given the function \(y = -2x + x^2 - 3\) or written in the format \(y = ax^2 + bx + c\), the function \(y = x^2 - 2x - 3\) with \(a = 1\), \(b = -2\) and \(c = -3\). To graph the function we need to determine the vertex, roots and \(y\)-intercept of the function:

The function has the shape of a “smiling face” as \(a > 0\).

The \(y\)-intercept is the value \(c\) or the value \(-3\).

The vertex is the point

\[
x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1
\]

\[
y = (1^2 - 2(1) - 3) = (1 - 2 - 3) = -4.
\]

The roots, \(a = 1\), \(b = -2\) and \(c = 3\), enable us to determine the value of the quadratic formula. Therefore

\[
x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}
\]

\[
x = \frac{2 \pm \sqrt{4 + 12}}{2}
\]

\[
x = \frac{2 \pm \sqrt{16}}{2}
\]

\[
x = \frac{2 \pm 4}{2}
\]

\[
x = \frac{2 + 4}{2} \quad \text{or} \quad \frac{2 - 4}{2}
\]

\[
x = \frac{6}{2} \quad \text{or} \quad \frac{-2}{2}
\]

\[
x = 3 \quad \text{or} \quad -1.
\]
Hence the graph of the function \( y = x^2 - 2x - 3 \) is

\[
\begin{align*}
\text{(a)} & \quad \text{We need to find the demand } D(t) \text{ after 2 years. But } t \text{ is defined in months. We therefore need to change 2 years to months. Now 2 years is equal to } 2 \times 12 = 24 \text{ months. If } t = 2 \text{ years} = 24 \text{ months then} \\
D &= 2000 - 1500e^{-0.05 \times 24} \\
&= 1548, \text{ using your calculator, rounded to 6 decimal places} \\
&= 1548 \text{ computers. rounded to an integer}
\end{align*}
\]

\[
\begin{align*}
\text{(b)} & \quad \text{We need to determine the value of } t \text{ in months } D(t) = 1000. \text{ If } D = 1000, \text{ then}
\end{align*}
\]
\[
1 000 = 2 000 - 1 500e^{-0.05t}
\]
\[
1 500^{-0.05t} = 2 000 - 1 000 = 1 000
\]
\[
e^{-0.05t} = \frac{1 000}{1 500} = \frac{2}{3}
\]
\[
\ln e^{-0.05t} = \ln \left( \frac{2}{3} \right)
\]
\[
-0.05t \ln e = \ln \left( \frac{2}{3} \right)
\]
\[
t = 8,109302162
\]
\[
t = 8,11 \text{ months}
\]

The demand will be equal to 100 units after 8,11 months (rounded to 2 decimal places).

**Question 22**

First we define the variables. Let \(x\) be the number of radios manufactured per week and \(y\) the number of television sets manufactured per week. To help us with the formulation we summarise the information given in a table with the headings: resources (items with restrictions), the variables (\(x\) and \(y\)) and capacity (amount or number of the resources available).

<table>
<thead>
<tr>
<th>Resources</th>
<th>Radio</th>
<th>Television</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing time (min)</td>
<td>90</td>
<td>150</td>
<td>(95 \times 60 = 540) minutes</td>
</tr>
<tr>
<td>Testing time (min)</td>
<td>5</td>
<td>15</td>
<td>(9 \times 60 = 540) minutes</td>
</tr>
<tr>
<td>Production cost</td>
<td>175</td>
<td>850</td>
<td>13 500</td>
</tr>
</tbody>
</table>

Using the table, we can formulate our linear program as follows:

Let \(x\) = number of radios

\(y\) = number of television sets
then

\[ 90x + 150y \leq 5700 \] manufacturing time

\[ 5x + 15y \geq 540 \] testing time

\[ 175x + 850y \leq 13500 \] production cost

\[ x, y \geq 0 \] non-negative

**Question 23**

The cost function is a linear function. We need two points to draw the line of the cost function. Choose the two points where the lines cut the \( x \)-axis (\( x \)-axis intercept, thus \( y = 0 \)) and \( y \)-axis (\( y \)-axis intercept, \( x = 0 \)). Calculate \((0 ; y)\) and \((x ; 0)\) and draw a line through the two points. Therefore if \( P = 0 \) then

\[ C(P) = 432000 - 1800(0) \]

\[ = 432000. \]

This gives the point \((0 ; 432000)\).

If \( C(P) = 0 \) then

\[ 0 = 432000 - 1800(P) \]

\[ 1800P = 432000 \]

\[ P = \frac{432000}{1800} \]

\[ P = 240. \]

This gives the point \((240 ; 0)\).

The revenue function is a quadratic function. We need to determine the vertex, roots and \( y \)-intercept of the function to draw it. It is given that the function \( R(P) = 6000P - 30P^2 \) with the coefficients equal to \( a = -30 \), \( b = 6000 \) and \( c = 0 \).

The function has the shape of a “sad face” as \( a < 0 \).

The \( y \)-intercept is the value \( c \) which is 0.
The vertex is the point \[ x = \frac{-b}{2a} = \frac{-(6000)}{2(-30)} = 100 \]

\[ y = 6000(100) - (30)(100)^2 = 300000. \]

The roots, \( a = -30 \), \( b = 6000 \) and \( c = 0 \), enable us to determine the value of the quadratic formula. Therefore

\[ x = \frac{-(6000) \pm \sqrt{(6000)^2 - 4(0)(-30)}}{2(-30)} \]
\[ x = \frac{-6000 \pm \sqrt{(6000)^2}}{-60} \]
\[ x = \frac{-6000 \pm 6000}{-60} \]
\[ x = \frac{-6000 + 6000}{-60} \text{ or } \frac{-6000 - 6000}{-60} \]
\[ x = 0 \text{ or } -120 \]
\[ x = 0 \text{ or } 200. \]

The graphs of the two functions are shown in the figure below:
The break-even points are the points where the cost function is equal to the revenue function. These are the points A and B. Profit is revenue minus cost, therefore the profit area is where the revenue is greater than the cost (the revenue function lies above the cost function) and the loss is where the cost is greater than the revenue (the cost function lies above the revenue function).

Question 24

Step 1:
To graph a linear inequality, we first change the inequality sign (≥ or ≤ or > or <) to an equal sign (=) and draw the graph of the line. But we need two points to draw a line. Choose the two points where the lines cut the x-axis (x-axis intercept, thus y = 0) and y-axis (y-axis intercept, x = 0). Calculate (0 ; y) and (x ; 0) and draw a line through the two points. See the table below for the calculations.

Step 2:
Determine the feasible region for each inequality by substituting a point on either side of this line into the equation of the inequality. The inequality region is the area in which the selected point makes the inequality true. The calculations are given below:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>y-axis intercept</th>
<th>x-axis intercept</th>
<th>Inequality region</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 6x + 2y \leq 840 )</td>
<td>( 6x + 2y = 840 )</td>
<td>Select point ( (0;0) ) below the line ( 6(0) + 2(0) \leq 840 ) – True</td>
</tr>
<tr>
<td></td>
<td>Point ( (x ; y) ) if ( x = 0 )</td>
<td>Point ( (x ; y) ) if ( y = 0 )</td>
<td>( 6x + 2(0) = 840 )</td>
</tr>
<tr>
<td></td>
<td>( 6(0) + 2y = 840 )</td>
<td>( 6x + 2y = 840 )</td>
<td>( 6(0) + 2(0) \leq 840 ) – True</td>
</tr>
<tr>
<td></td>
<td>( y = 420 )</td>
<td>( x = 140 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{Point} : (0 ; 420) )</td>
<td>( \text{Point} : (140 ; 0) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 2x + y \leq 300 )</td>
<td>( 2x + y = 300 )</td>
<td>Select ( (0;0) ) to the left of line ( 2(0) + (0) \leq 300 ) – True</td>
</tr>
<tr>
<td></td>
<td>( 2(0) + y = 300 )</td>
<td>( 2x + y = 300 )</td>
<td>( 2(0) + (0) \leq 300 ) – True</td>
</tr>
<tr>
<td></td>
<td>( y = 300 )</td>
<td>( x + (0) = 300 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{Point} : (0 ; 300) )</td>
<td>( \text{Point} : (300 ; 0) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 2x + y = 300 )</td>
<td>( 2x + y = 300 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x + (0) = 300 )</td>
<td>( x + (0) = 300 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x = 300 )</td>
<td>( x = 300 )</td>
<td></td>
</tr>
</tbody>
</table>
Step 3:
The feasible region is the area where all the inequalities are true simultaneously.

Step 4:
Determine all the corner points of the feasible region and substitute them into the objective function (function you want to maximise or minimise) and determine the maximum value.

Step 1: Determine the coordinates of all the corners of the feasible region by solving two equations with two unknowns (substitution) or read the values from the graph.

Step 2: Substitute the corner points into the objective function.

Step 3: Choose the corner point that results in the highest (maximisation) or lowest (minimisation) objective function value.
Corner points of feasible region & Value of $P = 120x + 95y$ \\
A: $x = 0; y = 250$ & $P = 120(0) + 95(250) = 23750$ \\
B: $x = 50; y = 195$ & $P = 120(50) + 95(195) = 24525$ (maximum) \\
C: $x = 120; y = 60$ & $P = 120(120) + 95(60) = 20100$ \\
D: $x = 140; y = 0$ & $P = 120(140) + 95(0) = 16800$ \\
Origin: $x = 0; y = 0$ & $P = 120(0) + 95(0) = 0$

The maximum of $P$ is at the point B where $x = 50$, $y = 195$ and $P = 24525$.

**Question 25**

We need to determine the tangent line $y = mx + c$ to the graph of $f(x)$, where $x = 1$.

If $x = 1$ then the function value $f(x) = f(1) = 3(1)^2 - 1 = 2$. We therefore need to determine the tangent line at the point $(x ; y) = (1 ; 2)$.

The slope $m$ of the tangent line can be determined by differentiation and is equal to

$$\frac{df}{dx} = \frac{d(3x^2 - x)}{dx} = 6x - 1.$$

The value of the slope in the point where $x = 1$, is therefore $m = 6x - 1 = 6(1) - 1 = 5$. Therefore the tangent line is equal to $y = 6x + c$.

As the tangent line runs through the point $(1; 2)$ we can use it to determine the value of $c$, namely

$$y = 5x + c$$
$$2 = 5(1) + c$$
$$2 - 5 = c$$
$$c = -3.$$

The equation of the tangent line is $y = 5x - 3$. 