Tutorial Letter 201/1/2013

QUANTITATIVE MODELLING

Semester 1

Department of Decision Sciences

This tutorial letter contains the solutions for assignment 01.
Dear student

I hope that by this stage you have worked through chapters 1 and 2 of the textbook and have completed your first assignment. As the assignments contain questions from old examination papers you are already, in a way, preparing for the examination. Practice makes perfect! Try to do as many examples as possible, as the more examples you work through the more you will be able to recognise a problem and know how to solve it.

Remember help is just a phone call or e-mail away. You are welcome to contact me if you need any help with the second assignment. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)

Office: Hazelwood Campus, Room 4-37, Unisa
Tel: +27 12 433 4602
E-mail: mabemgv@unisa.ac.za

Lastly, I wish you everything of the best with your preparation for the second assignment.

Ms Victoria Mabe-Madisa
**Question 1**

We have to simplify the fraction

\[
\frac{(x - 4)(x + 3)}{(x - 4)(x + 5)} = \frac{x + 3}{x + 5}.
\]

The \((x - 4)\) above and below the line cancels out.

[Option 2]

**Question 2**

\[
\begin{align*}
-6 - 3x & \geq 2x \\
-3x - 2x & \geq 6 \\
-5x & \geq 6 \\
-x & \geq \frac{6}{5} \\
x & \leq -\frac{6}{5}
\end{align*}
\]

Multiplying both sides of the inequality by \(-1\). The inequality sign changes.

[Option 2]

**Question 3**

\[
\frac{2}{3} + \frac{5}{6} + \frac{5}{4} - \frac{1}{3} \times 6
\]

\[
= \frac{2 \times 6}{3 \times 6} + \frac{5 \times 4}{6 \times 4} - \frac{4 \times 6}{3 \times 1}
\]

\[
= \frac{12}{15} + \frac{20}{6} - \frac{24}{3}
\]

Because \(\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times c}\).

\[
= \frac{12}{15} + \frac{60}{15} - \frac{120}{15}
\]

Multiply fractions.

\[
= \frac{-48}{15}
\]

Common denominator.

\[
= -\frac{3 \times 3}{15} = -\frac{3 \times 1}{5}
\]

Subtract fractions.

Simplify.

[Option 4]
Question 4

Let the price in 2007 be $x$.

The price in 2009 is 35% lower than in 2007, and is given as R3 315.

Now

\[ \text{price in 2009} = \text{price in 2007} - 35\% \text{ of price in 2007}. \]

Therefore

\[
3315 = x - \left( \frac{35}{100} \times x \right) \\
3315 = x - 0,35x \\
3315 = 1x - 0,35x \\
3315 = x(1 - 0,35) \\
3315 = 0,65x \\
\frac{3315}{0,65} = x \\
x = 5100.
\]

The price of the computer in 2007 was R5 100.

[Option 2]

Question 5

Let the price of the suit be $x$.

The cost of a suit is 800 in 2000. The price in 2001 is 21% higher than in 2000 and, in 2002, it is 25% higher than in 2001.

Therefore, in 2001 the price is

\[
x = 800 + \left( \frac{21}{100} \times 800 \right) \\
= 800 + 168 \\
= 968
\]
In 2002 the price is

\[
x = 968 + \left( \frac{25}{100} \times 968 \right)
= 968 + 242
= 1210
\]

Therefore the price in 2002 is R1 210.

**Question 6**

In general the slope of a line in standard format \( y = mx + c \) has the value \( m \).

To determine the slope of the given line \( 2y - 10x + 5 = 0 \), we first need to change the given equation to the general format of a line, namely \( y = mx + c \).

We change the equation so that \( y \) is the subject of the equation. This means that we write \( y \) on its own on one side of the equation. We start with \( 2y - 10x + 5 = 0 \) as given.

Move 5 to the right-hand side by subtracting 5 from both sides of the equation:

\[
2y - 10x + 5 = 0
\]

\[
2y - 10x + 5 - 5 = 0 - 5
\]

\[
2y - 10x = -5
\]

Next we move 10x to the right-hand side by adding 10x to both sides of the equation:

\[
2y - 10x = -5
\]

\[
2y - 10x + 10x = -5 + 10x
\]

\[
2y = -5 + 10x
\]

Lastly to change the equation to the standard format of \( y = mx + c \), we want \( y \) by itself on one side of the equation. Therefore, we divide both sides of the equation by 2:

\[
2y = -5 + 10x
\]

\[
\frac{2y}{2} = \frac{-5 + 10x}{2}
\]

\[
y = \frac{-5}{2} + 5x
\]

\[
y = \frac{-5}{2} + 5x
\]
The slope of the line $2y - 10x + 5 = 0$ is thus the value of $m$ in the rewritten equation in the form $y = mx + c$ of the given line. Comparing the rewritten formula $y = \frac{-5}{2} + 5x$ of the line with the standard format of a line $y = mx + c$, we conclude that the slope of the line $2y - 10x + 5 = 0$ is equal to $5$.

[Option 3]

**Question 7**

The total cost for a wholesaler to purchase $x$ units is given as $c(x) = 300 + 0.92x$.

The total revenue from selling $x$ products is the price $x$ quantity $= 3.10x$

At breakeven point revenue $= cost$

Therefore $3.10x = 300 + 0.92x$

Solving for $x$ to obtain the number of units: make $x$ the subject of the equation

\[3.10x = 300 + 0.92x\]

\[3.10x - 0.92x = 300\]

\[2.18x = 300\]

\[x = \frac{300}{2.18} = 137.61 \approx 138\]

138 units should be sold in order to break even.

[Option 4]

**Question 8**

Given the line $P = 10 + 0.5Q$, we need two points to draw a line. Select any $P$ or $Q$ value and calculate the value of the point.

Say we choose $Q = 0$, then

\[P = 10 + 0.5Q\]

\[= 10 + 0.5(0)\]

\[= 10\]

Therefore point 1 $= (0; 10)$. 
Choose \( P = 0 \) then

\[
\begin{align*}
0 &= 10 + 0,5Q \\
-10 &= 0,5Q \\
-10 &= Q \\
0,5 &= Q \\
Q &= -20
\end{align*}
\]

Therefore point 2 = (–20; 0).

Please note that you can use any \( P \) and/or \( Q \) value to calculate the two points. Normally \( P = 0 \) and \( Q = 0 \) are used to simplify the calculation.

Next we plot the two calculated points of the line and draw the line. Now as \( P \) is the subject of the equation we draw the \( P \) value on the y-axis of the graph and \( Q \) on the x-axis of the graph.

[Option 3]

**Question 9**

The demand function is \( P = 80 - 2Q \).

Now the price elasticity of demand is

\[
\varepsilon_d = -\frac{1}{b} \cdot \frac{P}{Q}
\]

with \( a \) and \( b \) being the values of the demand function \( P = a - bQ \).

To determine the price elasticity of demand we thus need to determine the values of \( b, Q \) and \( P \). It is given that \( P = 80 - 2Q \) and \( P = P \). By comparing \( P = 80 - 2Q \) with \( P = a - bQ \), we can say that \( a = 80 \) and \( b = 2 \). At this stage \( a, b \) and \( P \) are known and \( Q \) is unknown.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving for \( Q \).

To determine the value of \( Q \) we need to change the equation of the demand function \( P = 80 - 2Q \) so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \). Now

\[
\begin{align*}
P &= 80 - 2Q \\
P - 80 &= -2Q \\
P - 80 &= Q \\
Q &= \frac{P - 80}{-2}.
\end{align*}
\]
As we have determined the values of $b$, $P$ and $Q$ we can now substitute them into the formula for elasticity of demand:

$$
\varepsilon_d = -\frac{1}{2} \times \frac{P}{P-80}
$$

$$
= -\frac{1}{2} \times \frac{P}{P-80} \times \frac{-2}{1}
$$

$$
= \frac{P}{P-80}.
$$

Or alternatively,

you can use the given formula of price elasticity of demand in terms of $P$ of a demand function in the form $P = a - bQ$, which is given in the textbook on page 78, equation 2.14 (2nd edition) and page 89, equation 2.14 (3rd edition),

$$
\varepsilon_d = \frac{P}{P-a}.
$$

Now $a = 80$ (intercept on the y-axis of the demand function)

$$
\varepsilon_d = \frac{P}{P-80}.
$$

[Option 4]

**Question 10**

We need to simplify or write $\sqrt{x^8}$ in a different way. Now

$$
\sqrt{x^8}
$$

$$
= \sqrt{x^{8\times 8}}
$$

because $(a^b)^c = a^{b\times c}$

$$
= \sqrt{x^{64}}
$$

$$
= x^{64 \times \frac{1}{2}}
$$

because $\sqrt{a} = a^{\frac{1}{2}}$

$$
= x^{32}
$$

[Option 3]