Line passing through \((1, 3), (3, 0)\): \(y = -\frac{3}{2}x + \frac{9}{2}\)

**Steps**

Find the line \(y = mx + b\) passing through \((1, 3), (3, 0)\)

**Compute the slope \((1, 3), (3, 0)\):** \(m = -\frac{3}{2}\)

Slope between two points: \(\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}\)

\((x_1, y_1) = (1, 3), (x_2, y_2) = (3, 0)\)

\[ m = \frac{0 - 3}{3 - 1} = \frac{-3}{2} \]

Refine

\(m = -\frac{3}{2}\)

**Compute the \(y\) intercept:** \(b = \frac{9}{2}\)

Plug the slope \(-\frac{3}{2}\) into \(y = mx + b\)

\[ y = \left(-\frac{3}{2}\right)x + b \]

Plug in \((1, 3)\): \(x = 1, y = 3\)

\[ 3 = \left(-\frac{3}{2}\right) \cdot 1 + b \]

Isolate \(b\)

\[ 3 = \left(-\frac{3}{2}\right) \cdot 1 + b \Rightarrow b = \frac{9}{2} \]

Construct the line equation \(y = mx + b\) where \(m = -\frac{3}{2}\) and \(b = \frac{9}{2}\)

\[ y = -\frac{3}{2}x + \frac{9}{2} \]
\[
\frac{a^{5} \sqrt{a^{5}}}{2a^{0.3}} = 0.5a^{7.2}
\]

**Steps**

\[
\frac{a^{5} \sqrt{a^{5}}}{2a^{0.3}}
\]

\[
a^{5} \sqrt{a^{5}} = a^{15/2}
\]

\[
\frac{a^{5} \sqrt{a^{5}}}{2a^{0.3}} = \frac{a^{15/2}}{2a^{0.3}}
\]

Apply exponent rule: \(a^{b} \cdot a^{c} = a^{b+c}\)

\[
a^{15/2} = a^{5 + 5/2} = a^{15/2}
\]

\[
\frac{a^{15/2}}{2a^{0.3}}
\]

Apply exponent rule: \(\frac{x^{a}}{x^{b}} = x^{a-b}\)

\[
\frac{a^{15/2}}{a^{0.3}} = a^{15/2 - 0.3} = a^{7.2}
\]

\[
\frac{a^{7.2}}{2}
\]

Convert element to a decimal form

\[
\frac{1}{2} = 0.5
\]

\[
= 0.5a^{7.2}
\]
3. 3
At breakeven TR = TC

\[ TR = 2 \times Q \]

Therefore

Solution

\[ 2Q = \sqrt{4 + 6Q} \quad : \quad Q = 2 \]

Steps

\[ 2Q = \sqrt{4 + 6Q} \]

Square both sides

\[ (2Q)^2 = (\sqrt{4 + 6Q})^2 \]

Expand \((2Q)^2\):

\[ 4Q^2 \]

Expand \((\sqrt{4 + 6Q})^2\):

\[ 4 + 6Q \]

\[ 4Q^2 = 4 + 6Q \]

Solve \(4Q^2 = 4 + 6Q\):

\[ Q = 2, \quad Q = -\frac{1}{2} \]

\[ Q = 2, \quad Q = -\frac{1}{2} \]

Verifying Solutions: \( Q = 2 \) True, \( Q = -\frac{1}{2} \) False

Therefore, the final solution for \( 2Q = \sqrt{4 + 6Q} \) is \( Q = 2 \)
4. 3

Solution

\[ \log_2(32x) - \log_2(8x) = 2 \]

**Steps**

\[ \log_2(32x) - \log_2(8x) \]

Apply log rule: \[ \log_c(a) - \log_c(b) = \log_c\left(\frac{a}{b}\right) \]

\[ \log_2(32x) - \log_2(8x) = \log_2\left(\frac{32x}{8x}\right) \]

\[ = \log_2\left(\frac{32x}{8x}\right) \]

\[ \frac{32x}{8x} = 4 \]

\[ = \log_2(4) \]

Rewrite 4 in power – base form: \[ 4 = 2^2 \]

\[ = \log_2(2^2) \]

Apply log rule: \[ \log_\alpha(a^b) = b \cdot \log_\alpha(a) \]

\[ \log_2(2^2) = 2 \log_2(2) \]

\[ = 2 \log_2(2) \]

Apply log rule: \[ \log_\alpha(a) = 1 \]

\[ \log_2(2) = 1 \]

\[ = 2 \]
5. 2

\[ 50 - 0.6Q = 20 + 0.4Q \]

\[ Q = 30 \]

\[ P = 50 - 0.6(30) = 32 \]

6. To calculate arc elasticity of demand we take the midpoint in between.

Formula for Average of ‘midpoint’ elasticity of demand

\[
\frac{\text{change in } Q}{\text{average } Q}\div\frac{\text{change in } P}{\text{average } P}
\]

7. 2

**Price elasticity of demand** (PED or \( E_d \)) is a measure used in economics to show the responsiveness, or **elasticity**, of the quantity demanded of a good or service to a change in its **price**, ceteris paribus. ... Revenue is maximized when **price** is set so that the PED is exactly one.
9. 2

(2.5, 4.5)

10. 2

(1, 0)

(4, 0)
At equilibrium

\[ \frac{50}{Q + 2} = 10 + 2Q \]

Solution

\[ \frac{50}{Q + 2} = 10 + 2Q \quad : \quad Q = \frac{\sqrt{109} - 7}{2}, \ Q = -\frac{7 + \sqrt{109}}{2} \quad \text{(Decimal: } \ Q = 1.72015\ldots, \ Q = -8.72) \]

Steps

\[ \frac{50}{Q + 2} = 10 + 2Q \]

Multiply both sides by \( Q + 2 \)

\[ \frac{50}{Q + 2} (Q + 2) = 10(Q + 2) + 2Q(Q + 2) \]

Refine

\[ 50 = 10(Q + 2) + 2Q(Q + 2) \]

Solve \[ 50 = 10(Q + 2) + 2Q(Q + 2) \quad : \quad \begin{align*} Q &= \frac{\sqrt{109} - 7}{2}, \quad Q = -\frac{7 + \sqrt{109}}{2} \end{align*} \]

Verifying Solutions

Find undefined (singularity) points: \( Q = -2 \)

Combine undefined points with solutions:

\[ Q = \frac{\sqrt{109} - 7}{2}, \ Q = -\frac{7 + \sqrt{109}}{2} \]

\[ P = 10 + 2 = 14 \]
Solution

\[ x^3 - 3x^2 - 4x = 0 \quad : \quad x = 0, x = -1, x = 4 \]

**Steps**

\[ x^3 - 3x^2 - 4x = 0 \]

Solve by factoring

**Factor** \[ x^3 - 3x^2 - 4x \]
\[ x(x + 1)(x - 4) \]

Factor out common term \( x \)

\[ x(x^2 - 3x - 4) \]

**Factor** \[ x^2 - 3x - 4 \]
\[ (x + 1)(x - 4) \]

\[ x(x + 1)(x - 4) = 0 \]

Using the Zero Factor Principle:

\[ x = 0 \]

**Solve** \[ x + 1 = 0 \]
\[ x = -1 \]

**Solve** \[ x - 4 = 0 \]
\[ x = 4 \]

The final solutions to the equation are:

\[ x = 0, x = -1, x = 4 \]

12. 3

\[-4y + 1 = -2x^2 + x\] can be expressed as \[ y = 0.5x^2 - 0.25x + 0.25\]
Solution

**Extreme Points of** \(0.5x^2 - 0.25x + 0.25\): Minimum \((0.25, 0.21875)\)

**Steps**

First Derivative Test definition

Suppose that \(x = c\) is a critical point of \(f(x)\) then,

- If \(f''(x) > 0\) to the left of \(x = c\) and \(f''(x) < 0\) to the right of \(x = c\) then \(x = c\) is a local maximum.
- If \(f''(x) < 0\) to the left of \(x = c\) and \(f''(x) > 0\) to the right of \(x = c\) then \(x = c\) is a local minimum.
- If \(f''(x)\) is the same sign on both sides of \(x = c\) then \(x = c\) is neither a local maximum nor a local minimum.

Find the critical points: \(x = 0.25\)

**Domain of** \(0.5x^2 - 0.25x + 0.25\): \(-\infty < x < \infty\)

Combine the critical point(s): \(x = 0.25\) with the domain

The function monotone intervals are:

\(-\infty < x < 0.25, 0.25 < x < \infty\)

Check the sign of \(f''(x) = x - 0.25\) at each monotone interval

- Check the sign of \(x - 0.25\) at \(-\infty < x < 0.25\): Negative
- Check the sign of \(x - 0.25\) at \(0.25 < x < \infty\): Positive

**Summary of the monotone intervals behavior**

<table>
<thead>
<tr>
<th>Sign</th>
<th>(-\infty &lt; x &lt; 0.25)</th>
<th>(x = 0.25)</th>
<th>(0.25 &lt; x &lt; \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behavior</td>
<td>Decreasing</td>
<td>Minimum</td>
<td>Increasing</td>
</tr>
</tbody>
</table>

Plug the extreme point \(x = 0.25\) into \(0.5x^2 - 0.25x + 0.25\) ⇒ \(y = 0.21875\)

Minimum \((0.25, 0.21875)\)

13. **2**

Profit = TR - TC

\[ TR = P \times Q = 40P - 0.2P^2 \]
$\text{TC} = \text{FC} + \text{VC} = 1000 + 15Q = 1000 + 15(40 - 0.2P)$

Profit = $40P - 0.2P^2 - (1000 + 15(40 - 0.2P)) = -0.2P^2 + 43P - 1600$

14. 3
When $p = 80$ $Q = 10$
Consumer surplus = $0.5(10) \times (120 - 80) = 200$

15. 4
Solution

\[ x + 2y - z = 5, \quad 2x - y + z = 2, \quad y + z = 2 \quad : \quad z = \frac{1}{4}, \quad x = \frac{7}{4}, \quad y = \frac{7}{4} \]

Steps

This System of Equations can be solved in the following ways

**Solve by substitution**

\[
\begin{align*}
\begin{bmatrix}
x + 2y - z = 5 \\
2x - y + z = 2 \\
y + z = 2
\end{bmatrix}
\end{align*}
\]

**Isolate \( y \) for \( y + z = 2 \): \quad y = 2 - z**

Substitute \( y = 2 - z \)

\[
\begin{align*}
\begin{bmatrix}
x + 2(2 - z) - z = 5 \\
2x - (2 - z) + z = 2
\end{bmatrix}
\end{align*}
\]

**Isolate \( x \) for \( x + 2(2 - z) - z = 5 \): \quad x = 3z + 1**

Substitute \( x = 3z + 1 \)

\[
\begin{align*}
\begin{bmatrix}
2(3z + 1) - (2 - z) + z = 2
\end{bmatrix}
\end{align*}
\]

**Isolate \( z \) for \( 2(3z + 1) - (2 - z) + z = 2 \): \quad z = \frac{1}{4}**

For \( x = 3z + 1 \)

Substitute \( z = \frac{1}{4} \)

\[
\begin{align*}
x = 3 \cdot \frac{1}{4} + 1 & \Rightarrow x = \frac{7}{4}
\end{align*}
\]

\[
x = \frac{7}{4}
\]

For \( y = 2 - z \)

Substitute \( z = \frac{1}{4} \)

\[
\begin{align*}
y = 2 - \frac{1}{4} & \Rightarrow y = \frac{7}{4}
\end{align*}
\]

\[
y = \frac{7}{4}
\]
The solutions to the system of equations are:

\[ z = \frac{1}{4}, \quad x = \frac{7}{4}, \quad y = \frac{7}{4} \]

Solve by elimination

\[ z = \frac{1}{4}, \quad x = \frac{7}{4}, \quad y = \frac{7}{4} \]
Solution

**Extreme Points of** \( x^3 - 12x - 6 \): Maximum \((-2, 10)\), Minimum \((2, -22)\)

**Steps**

*First Derivative Test definition*

Suppose that \( x = c \) is a critical point of \( f(x) \) then,

- If \( f''(x) > 0 \) to the left of \( x = c \) and \( f''(x) < 0 \) to the right of \( x = c \) then \( x = c \) is a local maximum.
- If \( f''(x) < 0 \) to the left of \( x = c \) and \( f''(x) > 0 \) to the right of \( x = c \) then \( x = c \) is a local minimum.
- If \( f''(x) \) is the same sign on both sides of \( x = c \) then \( x = c \) is neither a local maximum nor a local minimum.

**Find the critical points:** \( x = -2, x = 2 \)

**Domain of** \( x^3 - 12x - 6 \): \(-\infty < x < \infty\)

Combine the critical point(s): \( x = -2, x = 2 \) with the domain

The function monotone intervals are:

\(-\infty < x < -2, -2 < x < 2, 2 < x < \infty\)

Check the sign of \( f''(x) = 3x^2 - 12 \) at each monotone interval

- **Check the sign of** \( 3x^2 - 12 \) at \(-\infty < x < -2\): Positive
- **Check the sign of** \( 3x^2 - 12 \) at \(-2 < x < 2\): Negative
- **Check the sign of** \( 3x^2 - 12 \) at \(2 < x < \infty\): Positive

**Summary of the monotone intervals behavior**

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\infty &lt; x &lt; -2)</th>
<th>( x = -2)</th>
<th>(-2 &lt; x &lt; 2)</th>
<th>( x = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Behavior</td>
<td>Increasing</td>
<td>Maximum</td>
<td>Decreasing</td>
<td>Minimum</td>
</tr>
</tbody>
</table>

Plug the extreme point \( x = -2 \) into \( x^3 - 12x - 6 \) \( \Rightarrow y = 10 \)

Maximum \((-2, 10)\)

Plug the extreme point \( x = 2 \) into \( x^3 - 12x - 6 \) \( \Rightarrow y = -22 \)

Minimum \((2, -22)\)
When $t = 15$,

$$Q(t) = \frac{5000}{2 + 1249e^{-0.33 \cdot 15}} = 460.949393$$
Solution

\[ \text{solve for } t, \quad 1000 = \frac{5000}{2 + 1249e^{-0.33t}} \quad : \quad t = -\frac{\ln\left(\frac{3}{1249}\right)}{0.33} \]

**Steps**

1000 = \frac{5000}{2 + 1249e^{-0.33t}}

Multiply both sides by \(2 + 1249e^{-0.33t}\)

\[ 1000\left(2 + 1249e^{-0.33t}\right) = \frac{5000}{2 + 1249e^{-0.33t}} \left(2 + 1249e^{-0.33t}\right) \]

Simplify

\[ 1000\left(2 + 1249e^{-0.33t}\right) = 5000 \]

Divide both sides by 1000

\[ \frac{1000\left(2 + 1249e^{-0.33t}\right)}{1000} = \frac{5000}{1000} \]

Simplify

\[ 2 + 1249e^{-0.33t} = 5 \]

Subtract 2 from both sides

\[ 2 + 1249e^{-0.33t} - 2 = 5 - 2 \]

Simplify

\[ 1249e^{-0.33t} = 3 \]

Divide both sides by 1249

\[ \frac{1249e^{-0.33t}}{1249} = \frac{3}{1249} \]

Simplify

\[ e^{-0.33t} = \frac{3}{1249} \]

If \(f(x) = g(x)\), then \(\ln(f(x)) = \ln(g(x))\)

\[ \ln\left(e^{-0.33t}\right) = \ln\left(\frac{3}{1249}\right) \]
Apply log rule: \( \log_a(b^x) = b \cdot \log_a(x) \)

\[ \ln(e^{-0.33t}) = (-0.33t) \ln(e) \]

\[ (-0.33t) \ln(e) = \ln\left(\frac{3}{1249}\right) \]

Simplify \((-0.33t) \ln(e)\): \(-0.33t\)

\[ -0.33t = \ln\left(\frac{3}{1249}\right) \]

Solve \(-0.33t = \ln\left(\frac{3}{1249}\right)\): \(t = -\frac{100\ln\left(\frac{3}{1249}\right)}{33}\)

\[ t = -\frac{100\ln\left(\frac{3}{1249}\right)}{33} \]

19. 4

Assembly hours available is at least 150

Finishing hours are at most 100 hours
Equation is \((x - 3)(x + 2) = 0\)

Solution

**Steps**

First Derivative Test definition

Suppose that \(x = c\) is a critical point of \(f(x)\) then,
- If \(f''(x) > 0\) to the left of \(x = c\) and \(f''(x) < 0\) to the right of \(x = c\) then \(x = c\) is a local maximum.
- If \(f''(x) < 0\) to the left of \(x = c\) and \(f''(x) > 0\) to the right of \(x = c\) then \(x = c\) is a local minimum.
- If \(f''(x)\) is the same sign on both sides of \(x = c\) then \(x = c\) is neither a local maximum nor a local minimum.

Find the critical points: \(x = \frac{1}{2}\)

Domain of \((x - 3)(x + 2)\): \(-\infty < x < \infty\)

Combine the critical point(s): \(x = \frac{1}{2}\) with the domain

The function monotone intervals are:

\(-\infty < x < \frac{1}{2}\); \(\frac{1}{2} < x < \infty\)

Check the sign of \(f'(x) = 2x - 1\) at each monotone interval

Check the sign of \(2x - 1\) at \(-\infty < x < \frac{1}{2}\): Negative

Check the sign of \(2x - 1\) at \(\frac{1}{2} < x < \infty\): Positive

**Summary of the monotone intervals behavior**

<table>
<thead>
<tr>
<th>(-\infty &lt; x &lt; \frac{1}{2})</th>
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<th>(\frac{1}{2} &lt; x &lt; \infty)</th>
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</tr>
<tr>
<td>Behavior</td>
<td>Decreasing</td>
<td>Minimum</td>
</tr>
</tbody>
</table>

Plug the extreme point \(x = \frac{1}{2}\) into \((x - 3)(x + 2)\) \(\Rightarrow\) \(y = -\frac{25}{4}\)

Minimum \(\left(\frac{1}{2}, -\frac{25}{4}\right)\)
21. 2
Total revenue = P \times Q = 90P - 1.5P^2

Find the maximum point of the function

**Solution**

**Extreme Points of** 90x - 1.5x^2: Maximum (30, 1350)

**Steps**

First Derivative Test definition

Suppose that \( x = c \) is a critical point of \( f(x) \) then,

If \( f''(x) > 0 \) to the left of \( x = c \) and \( f''(x) < 0 \) to the right of \( x = c \) then \( x = c \) is a local maximum.

If \( f''(x) < 0 \) to the left of \( x = c \) and \( f''(x) > 0 \) to the right of \( x = c \) then \( x = c \) is a local minimum.

If \( f''(x) \) is the same sign on both sides of \( x = c \) then \( x = c \) is neither a local maximum nor a local minimum.

Find the critical points: \( x = 30 \)

Domain of 90x - 1.5x^2: \( -\infty < x < \infty \)

Combine the critical point(s): \( x = 30 \) with the domain

The function monotone intervals are:

\( -\infty < x < 30, 30 < x < \infty \)

Check the sign of \( f''(x) = 90 - 3x \) at each monotone interval

Check the sign of 90 - 3x at \( -\infty < x < 30 \): Positive

Check the sign of 90 - 3x at \( 30 < x < \infty \): Negative

Summary of the monotone intervals behavior

<table>
<thead>
<tr>
<th></th>
<th>(-\infty &lt; x &lt; 30)</th>
<th>( x = 30 )</th>
<th>( 30 &lt; x &lt; \infty )</th>
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<tbody>
<tr>
<td><strong>Sign</strong></td>
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<td>0</td>
<td>( - )</td>
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<tr>
<td><strong>Behavior</strong></td>
<td>Increasing</td>
<td>Maximum</td>
<td>Decreasing</td>
</tr>
</tbody>
</table>

Plug the extreme point \( x = 30 \) into \( 90x - 1.5x^2 \) \( \Rightarrow \) \( y = 1350 \)

Maximum (30, 1350)
22. 3
When \( P = 90 \) \( Q = 20 \)

Producer surplus = \( 0.5(20)(90 - 50) = 400 \)

23. 1

\[
\frac{d}{dQ} \left( 15 - Qe^Q + \frac{Q^2}{2} \right) = Q - e^Q - e^Q Q
\]

Steps

\[
\frac{d}{dQ} \left( 15 - Qe^Q + \frac{Q^2}{2} \right)
\]

Apply the Sum/Difference Rule: \((f \pm g)' = f' \pm g'\)

\[
= \frac{d}{dQ} (15) - \frac{d}{dQ} (Qe^Q) + \frac{d}{dQ} \left( \frac{Q^2}{2} \right)
\]

\[
\frac{d}{dQ} (15) = 0
\]

\[
\frac{d}{dQ} (Qe^Q) = e^Q + Qe^Q
\]

Apply the Product Rule: \((f \cdot g)' = f' \cdot g + f \cdot g'\)

\[
f = Q, \quad g = e^Q
\]

\[
= \frac{d}{dQ} (Q)e^Q + \frac{d}{dQ} (e^Q)Q
\]

\[
\frac{d}{dQ} (Q) = 1
\]

\[
\frac{d}{dQ} (e^Q) = e^Q
\]

\[
= e^Q + e^Q Q
\]

Simplify

\[
= e^Q + e^Q Q
\]

\[
\frac{d}{dQ} \left( \frac{Q^2}{2} \right) = Q
\]

Show Steps
\[ = 0 - (e^Q + e^Q) + Q \]

Simplify
\[ = Q - e^Q - e^Q \]

24. 1

\[ \frac{d}{dx} \left(5x^{-1} + 3\sqrt{x^5} \right) = -\frac{5}{x^2} + \frac{15x^4}{2\sqrt{x^5}} \]

Steps
\[ \frac{d}{dx} \left(5x^{-1} + 3\sqrt{x^5} \right) \]

Apply the Sum/Difference Rule: \((f \pm g)' = f' \pm g'\)
\[ = \frac{d}{dx} \left(5x^{-1} \right) + \frac{d}{dx} \left(3\sqrt{x^5} \right) \]

\[ \frac{d}{dx} \left(5x^{-1} \right) = -\frac{5}{x^2} \]

\[ \frac{d}{dx} \left(3\sqrt{x^5} \right) = \frac{15x^4}{2\sqrt{x^5}} \]

Take the constant out: \(a \cdot f' = a \cdot f'\)
\[ = 3 \cdot \frac{d}{dx} \left(\sqrt{x^5} \right) \]

Apply the chain rule: \(\frac{df(u)}{du} = \frac{df}{du} \cdot \frac{du}{dx}\)
Let \(x^5 = u\)
\[ = 3 \cdot \frac{d}{du} \left(\sqrt{u} \right) \cdot \frac{d}{dx} \left(x^5 \right) \]

\[ \frac{d}{du} \left(\sqrt{u} \right) = \frac{1}{2\sqrt{u}} \]

\[ \frac{d}{dx} \left(x^5 \right) = 5x^4 \]

\[ = 3 \cdot \frac{1}{2\sqrt{u}} \cdot 5x^4 \]
25. 2

\[ TC = 2Q^3 - Q^2 + 80Q + 150 \]

Marginal cost = \[ \frac{d}{dQ} = 6Q^2 - 2Q + 80 \]

When \( Q = 10 \) therefore;

Marginal cost = 660
26. 3

Solution

\[ \frac{d}{dx} \left( \ln \left( 3x^5 \right) \right) = \frac{5}{x} \]

**Steps**

\[ \frac{d}{dx} \left( \ln \left( 3x^5 \right) \right) \]

Apply the chain rule:  
\[ \frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx} \]

Let \( 3x^5 = u \)

\[ = \frac{d}{du} \left( \ln (u) \right) \frac{d}{dx} \left( 3x^5 \right) \]

\[ \frac{d}{du} \left( \ln (u) \right) = \frac{1}{u} \]

\[ \frac{d}{dx} \left( 3x^5 \right) = 15x^4 \]

\[ = \frac{1}{u} \cdot 15x^4 \]

Substitute back \( u = 3x^5 \)

\[ = \frac{1}{3x^5} \cdot 15x^4 \]

**Simplify** \( \frac{1}{3x^5} \cdot 15x^4 \cdot \frac{5}{x} \)

\[ = \frac{5}{x} \]
\[
\int_0^1 3x^3 \sqrt{1 + 2x^2} \, dx = \frac{1}{2} (3\sqrt{3} - 1) \quad (\text{Decimal: } 2.09808) \]

**Steps**

\[
\int_0^1 3x^3 \sqrt{1 + 2x^2} \, dx
\]

Compute the indefinite integral: \[
\int 3x^3 \sqrt{1 + 2x^2} \, dx = \frac{1}{2} \left(1 + 2x^2\right)^{3/2} + C
\]

- Take the constant out: \[
\int a \cdot f(x) \, dx = a \cdot \int f(x) \, dx
\]
  \[
= 3 \cdot \int x^3 \sqrt{1 + 2x^2} \, dx
\]

**Apply u-substitution:** \( u = 1 + 2x^2 \)

\[
= 3 \cdot \int \frac{1}{4} \sqrt{u} \, du
\]

- Take the constant out: \[
\int a \cdot f(x) \, dx = a \cdot \int f(x) \, dx
\]
  \[
= \frac{3}{4} \cdot \int \sqrt{u} \, du
\]

Apply the Power Rule: \[
\int x^a \, dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1
\]

\[
= \frac{3}{4} \cdot \frac{u^{3/2} + 1}{1/2 + 1}
\]

Substitute back \( u = 1 + 2x^2 \)

\[
= \frac{3}{4} \cdot \frac{(1 + 2x^2)^{3/2} + 1}{1/2 + 1}
\]

Simplify

\[
= \frac{1}{2} \left(1 + 2x^2\right)^{3/2}
\]
\[
= \frac{1}{2} \left( 1 + 2x^2 \right)^{\frac{3}{2}} + C
\]

Compute the boundaries: \[
\int_0^1 3x\sqrt{1 + 2x^2} \, dx = \frac{3\sqrt{3}}{2} - \frac{1}{2}
\]

Simplify

\[
= \frac{1}{2} (3\sqrt{3} - 1)
\]
28. \hspace{1cm} 2

Make Q the subject and find the area

Solution

\[ \int_{10}^{13.333} \frac{40}{p} - 3dp = 1.50729 \]

**Steps**

\[ \int_{10}^{13.333} \frac{40}{p} - 3dp \]

Compute the indefinite integral: \[ \int \frac{40}{p} - 3dp = 40 \ln |p| - 3p + C \]

\[ \int \frac{40}{p} - 3dp \]

Apply the Sum Rule: \[ \int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx \]

\[ = \int \frac{40}{p}dp - \int 3dp \]

\[ \int \frac{40}{p}dp = 40 \ln |p| \]

\[ \int 3dp = 3p \]

\[ = 40 \ln |p| - 3p \]

Add a constant to the solution

\[ = 40 \ln |p| - 3p + C \]

Compute the boundaries: \[ \int_{10}^{13.333} \frac{40}{p} - 3dp = 63.61069\ldots - (40 \ln(10) - 30) \]

\[ = 63.61069\ldots - (40 \ln(10) - 30) \]

Simplify

\[ = 1.50729 \]
29. Solution

\[ \int_0^7 3 + 4LdL = 119 \]

**Steps**

\[ \int_0^7 3 + 4LdL \]

Compute the indefinite integral: \( \int 3 + 4LdL = 3L + 2L^2 + C \)

\[ \int 3 + 4LdL \]

Apply the Sum Rule: \( \int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx \)

\[ = \int 3dL + \int 4LdL \]

\[ \int 3dL = 3L \]

\[ \int 4LdL = 2L^2 \]

\[ = 3L + 2L^2 \]

Add a constant to the solution

\[ = 3L + 2L^2 + C \]

Compute the boundaries: \( \int_0^7 3 + 4LdL = 119 - 0 \)

\[ = 119 - 0 \]

Simplify

\[ = 119 \]
30. 2

Maximum value for \( x_1 = 6 \)

Maximum value for \( x_2 \) for which \( x_1 \) can be 6 is 16 (solve using equation 1)

Therefore \( P = 6(6) + 20(16) = 356 \)