Adapted from Liebenberg and Vlok 2000, The interpretation of maps, aerial photographs and satellite images.

**Determining height on aerial photographs**

The height of a phenomenon or object on an aerial photograph, such as a building, chimney, tower or tall tree, can be obtained in three ways. The first is by measuring the degree of image displacement, the second is by measuring the parallax difference between the base and top of the object in the stereo model, and the third is by measuring the length of the shadow of the object.

Note that the height that we are calculating here is not the height of the phenomenon above sea level; it is merely the height of the object itself. The object's height above sea level is not relevant here.

**Image displacement**

All objects that have height exhibit radial displacement from the principal point of the photograph. We refer to this phenomenon as image displacement and one can measure this phenomenon on a photograph.

If you examine figure 7 closely again, you will remember that the distance marked aa' indicates the amount of image displacement that has taken place. We can therefore measure this distance. It is also apparent from the same figure that aa' is a measure of the height of A, because the taller the building the longer aa' is. On the other hand, the length of aa' is also related to the distance between A and the principal point of the photograph P.
Figure 7: Horizontal image displacement on a vertical aerial photograph

If we take all these variables into account, we can write down the following formula:

\[ d = \frac{hr}{H} \]

where \( d \) = the horizontal image displacement of the object
\( h \) = the height of the object, that is the distance between the highest and lowest points of the object
\( r \) = the radial distance between the summit of the object and the principal point of the photograph
\( H \) = the height of the aircraft above the terrain

In order to calculate the height of an object on an aerial photograph, we modify this formula as follows:

\[ h = \frac{Hd}{r} \]

To ensure that you understand all the variables in this formula, we shall explain them with reference to an example. See that you keep up with each successive step so that you will be able to do the measurements yourself and calculate the height.

Suppose you have a vertical aerial photograph of an industrial area showing several factory chimneys. You select one of the chimneys and take the following measurements:
the distance between the highest and lowest points of the chimney on the photograph
= 2 mm
the distance from the principal point to the summit of the chimney on the photograph
= 105.4 mm

The altimeter reading on the photograph is 5 630 m and the area lies at a mean altitude of 1
250 m above sea level. The focal length of the camera lens is 152.36 mm. What is the actual
height of the chimney in metres?

1 The first step is to calculate the scale of the photograph.

\[
\frac{1}{S} = \frac{f}{H - h}
\]

\[
= \frac{0.15236}{5630 - 1250}
\]

\[
= \frac{1}{28747}
\]

Rounded off to the nearest thousand the scale is 1:29 000.

2 Once the scale is known, the actual distances of d and r can be calculated as follows:

\[
d = 2 \times 29000 \text{ mm}
\]

\[
= 58000 \text{ mm}
\]

\[
= 58 \text{ m}
\]

\[
r = 105.4 \times 29000 \text{ mm}
\]

\[
= 3056600 \text{ mm}
\]

\[
= 3056.6 \text{ m}
\]

3 If you substitute the relevant values into the formula, you can calculate the height of
the chimney as follows:
\[ h = \frac{Hd}{r} \]

\[ = \frac{(5630 - 1250) \times 58.0}{3056.6} \]

\[ = \frac{4380 \times 58}{3056.6} \]

\[ = 83.11 \text{ m (rounded off to 2 decimal places)} \]

The actual height of the chimney is 83.11 m.

Parallax difference

If we move from one photograph to another in a stereo pair, we find that an object is not only observed from a camera position obliquely above it, but that it also appears on the adjacent photograph as observed from a completely different camera position. You will get a good idea of this if you extend your right arm, point with your forefinger and close your left eye. Now look at your finger with your right eye and look at the wall at the same time. Keep your arm in the same position but close your right eye and look at your finger and the wall with your left eye. What do you see? Your finger appears to move towards the right in relation to the wall, where in actual fact you kept your finger still. The reason for this apparent movement is that your left and right eyes are about 65 mm apart. In the stereo model the different camera positions represent the position of your eyes and this apparent movement is known as parallax. Operationally defined, it is the apparent displacement in the position of a point or an object on two adjacent vertical aerial photographs as a result of a change in the position of the camera.

What other anomalies did you observe when you looked at the position of your finger? It should have been clear to you that this apparent movement is always parallel to the eye base (the line joining the perspective centres of the two eyes). In order to verify this, bend your head and repeat the experiment. You will notice that the apparent movement is now no longer from left to right but from top to the bottom. In the case of a stereo pair, this phenomenon is demonstrated by the fact that the parallax of a point on the photograph is always measured parallel to the line of flight.

Parallax is therefore not merely a theoretical concept, but also a straight line distance that can
be measured on a stereo pair. It is used to determine the height of objects because it is sometimes impossible to measure the height of an object on a single aerial photograph with the aid of image displacement. The base of the object may not be visible, or else the image displacement may be too small to measure conveniently. The instrument used to measure parallax is known as a parallax bar or stereometer. The interpreter of the photograph uses this instrument to measure the difference in parallax between the base and the top of an object (eg a high tower) in the stereo model while he or she is looking through a stereoscope. This value is then used in a formula for calculating the height of the object in question.

Because you do not have a stereometer at your disposal, we shall not go into the relevant method of measuring parallax with a stereometer in this course. However, it is important that you should know what it is and that difference in parallax is fairly easy to measure.

Length of shadow of an object

In certain circumstances height can be calculated by measuring the length of the shadow of an object. The first requirement is of course that the object should be perfectly vertical, and secondly the shadow should fall on an open, level surface where it can easily be measured. However, measuring height by means of shadow length is a technique which depends largely for its success on the scale of the photograph; the larger the scale the more accurate the measurement.

Because the sun is so far removed from the earth, we can accept that the sun's rays fall parallel to each other at all points in the aerial photograph. This means that the lengths of the shadows of objects in the photograph are in direct proportion at any given stage to the height of the objects. In figure 8 you can clearly see, for example, that if the angle of elevation of the sun remains the same the length of shadow L will be longer if the building is taller, and shorter if the building is lower.
Figure 8: The length of shadow of an object is in direct proportion to the height of the object.

In practice there are two ways of using length of shadow to determine height.

1 When an object of known height occurs on the photograph

Suppose an object of known height (eg a building or tower) occurs on the photograph and you measure the shadow length. By using the following formula you can calculate the height of any other object by measuring its length of shadow and expressing the unknown height as a ratio of the known height and the length of shadow:

\[
h_1 = \frac{h_2 s_1}{s_2 x}
\]

where
- \(h_1\) = height of object 1
- \(s_1\) = length of shadow of object 1 on photograph
- \(h_2\) = height of object 2
- \(s_2\) = length of shadow of object 2 on photograph

Let us take an example. Suppose you have an aerial photograph which shows a radio antenna which is 125 m tall. The length of shadow of the antenna on the photograph is 1,68 cm. How tall would a chimney on the same photograph be if the length of shadow of the chimney is 1,25 cm?
\[
\begin{align*}
h_1 &= \frac{125 \text{ m} \times 1.25 \text{ cm}}{1.68} \\
&= \frac{156.25}{1.68} \\
&= 93 \text{ m}
\end{align*}
\]

The chimney is 93 m tall.

2 No object of known height occurs on the photograph.
In this case we use the following formula:

\[h = L \times \tan x^\circ\]

The various components of this formula are clearly apparent from figure 8, where \( h \) equals the height of the tower, \( L \) the length of shadow and \( x^\circ \) represents the angle of elevation of the sun. When we calculated slope, \( AC \) (the vertical interval) and \( AB \) (the map distance between two contours) were known and we wanted to determine the size of the slope angle \( x^\circ \). In figure 8, \( AB \) (the length of shadow) is known, and we want to calculate the length of \( AC \) (the height of the object). How do we go about this?

You will surely agree that we cannot achieve much with the above formula unless we know the size of angle \( x^\circ \) (the angle of elevation of the sun at the place where the photograph was taken). It is relatively easy to measure the length of the shadow of an object on an aerial photograph (ie \( L \)). However, the angle of elevation of the sun is determined by means of a fairly complicated calculation which is based on the following formula:

\[
\sin x^\circ = (\cos a)(\cos b)(\cos c) - (\sin a)(\sin b)
\]

where \( a \) = the sun's declination on the day on which the photograph was taken
where \( b \) = the latitude of the photographed place
where \( c \) = the difference in longitude between the position of the sun and the photographed place.

The three angles, \( a \), \( b \) and \( c \), are illustrated in figure 9. To assist you, we shall explain how
each of these variables is obtained.

Figure 9: The various angles involved in calculating the angle of elevation of the sun

The declination of the sun is the same as the latitude where the sun shines perpendicularly on a specific day. This angular value is calculated for each day of the year and can be obtained from astronomical or nautical tables. Note that the sign between the two parts of the formula is a plus if the sun is on the same side of the equator as the area photographed. If the sun is on the other side (ie during the South African winter) the sign is a minus.

The latitude of the photographed place can be obtained from a large-scale topographical map. In practice it is customary to use a single mean latitude value for all shadow calculations on photographs taken by the same aircraft on the same day in the same area.

The difference in longitude, also called the hour angle, is obtained by first calculating the Greenwich Mean Time (GMT) for the exact time at which the photograph was taken. In South Africa the standard time is two hours behind GMT, in other words if the photograph was taken at 12:28 SA Standard Time, the GMT was 10:28. Since the sun shines perpendicularly on the Greenwich meridian (ie 0° longitude) at 12:00 GMT, and the earth rotates through 18 every four minutes, the GMT helps us to
determine the longitude on which the sun is shining perpendicularly at the moment when the photograph was taken. If the GMT = 10:28, the sun is shining perpendicularly on a place 238 east of Greenwich (see fig 10). If the longitude of the photographed area is 338, the value of c is therefore 108.

![Diagram](image)

Figure 10: Calculating the difference in longitude

As soon as the values for a, b and c are available, they can be substituted into the formula and you can calculate the size of angle x. Once x is known, the height of the object can be calculated.

Because the calculation of the angle of elevation of the sun x is a fairly complicated procedure, you will not be expected to know the relevant formula by heart and use it in the examination. It is important, however, that you should understand the principles according to which it is calculated, and that you should know that it is part of the method of calculating the height of an object on an aerial photograph.