Study Unit 1 : Mathematical preliminaries

Chapter 1 : Sections 1.1 – 1.6

1. Basics

- Numbers: different type of numbers – Natural, Real, etc.
  Also called constants

- Basic operations
  - + (add); 2 + 3 = 5
  - – (subtract); 3 – 2 = 1
  - x (multiply); also •; 3 x 2 = 3•2 = 6
  - ÷ (division) also / or fraction ($\frac{1}{2}$ = 1 divide by 2);

$$6 \div 3 = 6/3 = \frac{6}{3} = 2$$

Remember:

1 x anything = anything

1 x 8 = 8

0 x anything = 0

0 x 4 = 0

1 + anything = one more than anything

1 + 345 = 346

0 + anything = anything

0 + 34 = 34

anything ÷ 0 = not allowed

12 ÷ 0 = not allowed

0 ÷ anything = 0

0 ÷ 7 = 0

- Brackets ( ) : group operations together

  (3 + 4) – 3

  = 7 – 3

  = 4

- Order of operation: **BODMAS**

  Brackets; **O**f; **D**ivide; **M**ultiply; **A**dd; **S**ubtract

  40 – 4 x (5 + 8) + 20

  = 40 – 4 x (13) + 20

  = 40 – 52 + 20

  = 8

- Variables: used for **unknown** or **generalisation** of things: place holder: use alphabetic characters for example *X* or *A* or *Y*. Can take on different values

  3x + 2y +7g + x

  3x is known as a term with coefficient 3 and variable x

  Remember : the last term x has a coefficient value of 1 in front of it namely 1x

  - Operations on variables or unknown:
    - + and – : only if same variable, then + or – coefficients and variable stays the same

      3x + 4x + 3 = (3 + 4)x + 3 = 7x + 3

      5x – x – 6 = (5 – 1)x – 6 = 4x –6

- x and ÷ : only if same variable, then x and ÷ coefficient and unknowns

  $3a \times 4a = (3 \times 4)(a \times a) = 12a^2$

  $2x^2 \div x = (2 \div 1)\frac{x^2}{x} = 2\frac{xx}{x} = 2x$

- Laws of operations
  - Commutative law : order
    - $a + b = b + a$ $\quad 3 + 4 = 4 + 3 = 7$
    - $a \times b = b \times a$ $\quad 3 \times 4 = 4 \times 3 = 12$
    - $a - b \neq b - a$ $\quad 4 - 3 = 1 \neq 3 - 4 = -1$
    - $a \div b \neq b \div a$ $\quad 4 \div 2 = 2 \neq 2 \div 4 = 0{,}5$

  - Associative law: ( )
    - $(a + b) + c = a + (b + c)$

      $(3 + 4) + 2 = 7 + 2 = 9$

      $3 + (4 + 2) = 3 + 6 = 9$
    - $(a \times b) \times c = a \times (b \times c)$

      $(3 \times 4) \times 2 = 12 \times 2 = 24$

      $3 \times (4 \times 2) = 3 \times 8 = 24$
    - $(a - b) - c \neq a - (b - c)$

      $(3 - 4) - 2 = -1 - 2 = -3$

      $3 - (4 - 2) = 3 - 2 = 1$
    - $(a \div b) \div c \neq a \div (b \div c)$

      $(12 \div 2) \div 2 = 6 \div 2 = 3$

      $12 \div (2 \div 2) = 12 \div 1 = 12$

- Distributive law (addition):
  - a x (b + c) = ab + ac

    $3 \times (4 + 2) = 3 \times 6 = 18$

    $(3 \times 4) + (3 \times 2) = 12 + 6 = 18$

- Exponent or Power: (something)^{power}: short way of writing something multiplied over and over with itself.

  The **bottom** number: **base**

  The **top** number: **exponent** or **power**

  $3 \times 3 = 3^2$ base = 3 power = 2

  $15 \times 15 \times 15 \times 15 \times 15 = 15^5$

  $Y^3 = Y \times Y \times Y$

  - Rules of exponents

  Let A and B be any two bases and *x* and *y* any two powers then

1. $A^x \times A^y = A^{x+y}$ $2^2 \times 2^3 = 2^{2+3} = 2^5$

   $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

2. $A^x \div A^y = A^{x-y}$ $2^4 \div 2^3 = 2^{4-3} = 2^1$

3. $(A \times B)^x = A^x \times B^x$ $(2 \times 3)^3 = 2^3\ 3^3$

4. $(A/B)^x = A^x / B^x$ $(2 / 3)^3 = 2^3 / 3^3$

5. $(A^x)^y = A^{x \times y}$ $(2^3)^3 = 2^{3 \times 3} = 2^9$

Remember : **If a is any number**

1. $(a)^0 = 1$ but $0^0 = 0$ $\quad 4^0 = 1$

2. $(a)^1 = a$

3. $\frac{1}{a^n} = a^{-n}$ $\quad 2/x^4 = 2x^{-4}$

4. $\sqrt[n]{a} = a^{\frac{1}{n}}$ $\quad \sqrt{24} = 24^{\frac{1}{2}}$

5. $a^{2x+4} = a^{15}$ (base the same) **then 2*x*** + 4 = 15

- Roots : Is the reverse of the power statement.
  $\sqrt{25}$ - what number must I multiply 2 times with itself to get an answer of 25 => 5 because $5^2 = 25$
  $\sqrt[3]{8}$ - what number must I multiply 3 times with itself to get an answer of 8 => 2 because $2^3 = 8$ etc.

- Simplify: write it another way

- Solve for x: Determine an answer for x

- Remember : When multiplying positive and negative numbers
  - – × – = +
  - – × + = –
  - + × – = –
  - + × + = +

2. Fractions

- Fraction is a part of a whole : like a slice of a pizza

$$\text{fraction} = \frac{\text{number of slices}}{\text{number of slices in whole pizza}}$$

$$= \frac{\text{numerator}}{\text{denominator (name of fraction)}}$$

For example $\frac{1}{4}$ is one slice of a pizza consisting of 4 pieces.

- Can only add and subtract "same pizzas" if not convert to "same pizzas" – common denominator

$\frac{1}{4}+\frac{2}{4}=$ $+$ $=$ or $\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4}=\frac{3}{4}$

$$\frac{10}{20}+\frac{15}{20}=\frac{10+15}{20}=\frac{25}{20}=25\div 20=1\frac{5}{20}$$

$$\frac{2}{5}+\frac{1}{10}=\frac{20}{50}+\frac{5}{50}=\frac{25}{50}=\frac{1}{2} \text{ or } \frac{2}{5}+\frac{1}{10}=\frac{4}{10}+\frac{1}{10}=\frac{5}{10}=\frac{1}{2}$$

$$\frac{1}{5}+\frac{1}{2}+\frac{2}{3}=\frac{6}{30}+\frac{15}{30}+\frac{20}{30}=\frac{41}{30}=1\frac{11}{30}$$

- If add or subtract whole and slices of pizzas : change whole pizzas to slices : $a\frac{b}{c} = a\frac{b}{c} = \frac{(a \times c) + b}{c}$

$$\frac{1}{4} + 1\frac{1}{2} = \frac{1}{4} + \frac{3}{2} = \frac{2}{8} + \frac{12}{8} = \frac{14}{8} = 1\frac{6}{8}$$

- Multiply : multiply the numbers across the top lines and multiply the numbers across the bottom lines

$$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12}$$

- Divide by = multiply by inverse of fraction

$$\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$$

No 1 of discussion class

Question 1

Simplify

$$\frac{1}{6}-\frac{5}{6}\div\frac{2}{3}+\frac{1}{3}\times\frac{3}{4}$$

Solution

$$\frac{1}{6}-\frac{5}{6}\div\frac{2}{3}+\frac{1}{3}\times\frac{3}{4}$$

$$=\frac{1}{6}-\frac{5}{6}\times\frac{3}{2}+\frac{1}{3}\times\frac{3}{4} \qquad \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

$$=\frac{1}{6}-\frac{15}{12}+\frac{3}{12}$$ Multiply fractions

$$=\frac{2}{12}-\frac{15}{12}+\frac{3}{12}=\frac{2-15+3}{12}$$ Common denominator

$$=-\frac{10}{12}$$ Add and subtract fractions

$$=-\frac{5}{6}$$ Simplify by dividing nominator and denominator by 2

3. Solve equations in 1 variable

- Move values so that unknown is on its own on one side of equation by +, - x or ÷ both sides with same values

$$4x + 7 = 14 - 3x + 5$$
$$4x + 7 + 3x = 14 - 3x + 3x + 5$$
$$4x + 3x + 7 - 7 = 14 + 5 - 7$$
$$7x = 12$$
$$7x/7 = 12/7$$
$$x = 12/7$$

4. Simple inequalities

- Equation if something = something
- Inequality something > or < or ≥or ≤
- Use number line to demonstrate

$$3x + 20 > 14 - 16x$$
$$3x + 16x + 20 > 14 - 16x + 16x$$
$$19x + 20 - 20 > 14 - 20$$
$$19x > -6$$
$$x > -6/19$$

- When solving for *x* remember > and < change if multiply or divide by (–) value.

$$-2x > 4x + 4$$
$$-2x - 4x > 4x - 4x + 4$$
$$-6x > 4$$
$$-6x/-6 < 4/-6$$
$$x < -4/6$$

No 2 of discussion class

Question 2

Solve for *x* in

$$-2x+\frac{5}{6}+\frac{x}{2}\geq -2x-4\left(-\frac{x}{3}-1\frac{1}{4}\right)$$

Solution

$$-2x+\frac{5}{6}+\frac{x}{2}\geq -2x-4\left(-\frac{x}{3}-1\frac{1}{4}\right)$$

$$-2x+\frac{5}{6}+\frac{x}{2}\geq -2x-4\left(-\frac{x}{3}-\frac{5}{4}\right) \text{ Change fraction}$$

$$-2x+\frac{5}{6}+\frac{x}{2}\geq -2x+\left(\frac{4x}{3}+\frac{20}{4}\right) \text{ Multiply 4 into ( )}$$

$$-2x+\frac{5}{6}+\frac{x}{2}\geq -2x+\frac{4x}{3}+5 \quad \text{Remember} \frac{20}{4}=5$$

$$-2x+2x+\frac{x}{2}-\frac{4x}{3}\geq 5-\frac{5}{6} \quad \text{Move all the same terms to one side}$$

$$\frac{x}{2}-\frac{4x}{3}\geq \frac{5}{1}-\frac{5}{6}$$

$$\frac{3x-8x}{6}\geq \frac{30-5}{6} \quad \text{Common denominator}$$

$$-\frac{5x}{6}\geq \frac{25}{6} \quad \text{Multiply both sides by 6}$$

$$-\frac{5x}{6}\times\frac{6}{1}\geq \frac{25}{6}\times\frac{6}{1}$$

$$-5x\geq 25 \quad \text{Divide both sides by } -5$$

$$\frac{-5x}{-5}\leq \frac{25}{-5} \quad \text{Inequality sign changes because we divide by a negative number}$$

$$x\leq -5$$

5. Calculating percentages

- Pizza with 100 slices
- % = fraction = $\frac{something}{100}$ For example 70% = $\frac{70}{100}$
- % always **of** something : 25% of 75 : of means multiply

15% of students at a university are male. How many male students are there in a total of 500 students?

15% of 500

$\frac{15}{100}$ x 500 = 75

The university expects a 10% increase in the number of students for the next year. How many students do they expect in total?

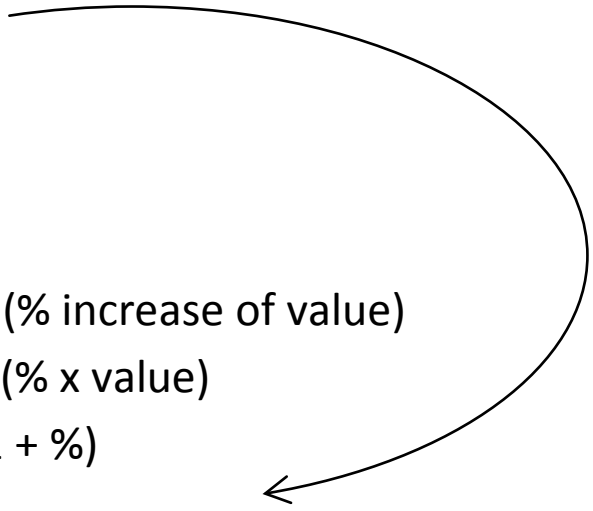
New total = previous number + 10% of previous number

= 500 + (10% x 500)

= 500 + ($\frac{10}{100}$ x 500)

= 500 + (50)

= 550



% increase or decrease : value + (% increase of value)

value + (% x value)

value (1 + %)

= 500 (0,1 + 1) = 500 (1,1) = 550