Tutorial Letter 201/2/2013

QUANTITATIVE MODELLING

Semester 2

Department of Decision Sciences

This tutorial letter contains the solutions for assignment 01.
Dear Student

I hope that by this stage you have worked through chapters 1 and 2 of the textbook and have completed your first assignment. As the assignments contain questions from old examination papers you are already in a way preparing for the examination. Practice makes perfect! Try and do as many examples as possible. The more examples you work through the more you will be able to recognise a problem and know how to solve it.

Remember, help is just a phone call or e-mail away. Please contact me if you need any help with the second assignment. My contact details and contact hours are as follows:

08:00 to 13:30 (Mondays to Fridays) (appointments and telephone)
13:30 to 16:00 (Mondays to Thursdays) (telephone only)

Office: Hazelwood Campus, Room 4-37, Unisa
Tel : +27 12 433 4602
E-mail: mabemgv@unisa.ac.za

Lastly, I wish you everything of the best with your preparation for the second assignment.

Victoria Mabe-Madisa
ASSIGNMENT 01: SOLUTIONS

Question 1

Let the original price of a pair of boots be $x$.

The price after the discount is given as R375 and the discount rate as 40% of the original price. Thus, the original price – 40% of the original price = the discounted price.

Therefore,

$$x - (40\% \text{ of } x) = 375$$

$$x - \left(\frac{40}{100} \times x\right) = 375$$

$$x - 0.40x = 375$$

$$x(1 - 0.40) = 375$$

Taking $x$ out as a common factor.

$$x(0.60) = 375$$

$$x = \frac{375}{0.60}$$

$$x = 625.$$ 

The original price of the boots was R625.

[Option 3]

Question 2

In general, the line $y = mx + c$ has a slope of $m$.

Firstly, we need to change the given function $0 = 6 + 3x - 2y$ to the general format of a line, namely, $y = mx + c$. We need to change the equation so that $y$ is the subject of the equation; this means writing it on its own on one side of the equation. Now, given the line $0 = 6 + 3x - 2y$.

Move $2y$ to the left side by adding $2y$ to both sides of the equation:

$$0 = 6 + 3x - 2y$$

$$0 + 2y = 6 + 3x - 2y + 2y$$

$$2y = 6 + 3x$$
We want $y$ on one side of the equation on its own. Divide by 2 on both sides of the equation:

\[
\frac{2y}{2} = \frac{6 + 3x}{2}
\]

\[
y = \frac{6}{2} + \frac{3x}{2}
\]

\[
y = 3 + \frac{3}{2}x
\]

The slope of the line $0 = 6 + 3x - 2y$ is, therefore, the value of $m$ in the rewritten equation in the form $y = mx + c$ of the given line. As the equation has been rewritten as $y = 3 + \frac{3}{2}x$, the value of $m$ or the slope is therefore equal to $\frac{3}{2}$.

[Option 2]

Question 3

Find the equation of the line passing through the points $(2 ; 1)$ and $(\frac{4}{3} ; 3)$.

Solution

\[y = mx + c.\]

Let $(x_1 ; y_1) = (2 ; 1)$ and $(x_2 ; y_2) = (\frac{4}{3} ; 3)$

The slope $m$ is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{\frac{4}{3} - 2} = \frac{2}{-\frac{2}{3}} = -3
\]

Therefore $y = -3x + c$.

Substitute any one of the points into the equation of the line to determine $c$.

Let’s choose the point $(2 ; 1)$. Then

\[
y = -3x + c
\]

\[
1 = -3 \times 2 + c
\]

\[
1 = -6 + c
\]

\[
1 + 6 = c
\]

\[
c = 7
\]

The equation of the line is $y = -3x + 7$.

[Option 1]
Question 4
Let the price of a flat in 2008 be \( x \).

The price in 2010 is 25% higher than in 2008, and given as R634 000.

Now the price in 2010 = the price in 2008 + 25% of the price in 2008.

Therefore

\[
634000 = x + (25\% \text{ of } x)
\]

\[
634000 = x + \left( \frac{25}{100} \times x \right)
\]

\[
634000 = x + 0,25x
\]

Taking \( x \) out as a common factor in both terms on the right-hand side we get

\[
634000 = x(1 + 0,25)
\]

\[
634000 = x(1,25)
\]

\[
x = \frac{634000}{1,25}
\]

\[
x = R507 200
\]

In 2008 the price of the flat was R507 200.

[Option 2]

Question 5
Bozo receives 5 parts of a total of \( 7 + 5 + 4 = 16 \) parts of the R240 000.

Thus

\[
\text{Bozo's share} = \frac{5}{16} \times 240000
\]

\[
= R75000
\]

[Option 3]
Question 6

The demand function is \( P = 70 - 0.5Q \).

Now the price elasticity of demand is \( \varepsilon_d = \frac{1}{b} \cdot \frac{P}{Q} \) with \( a \) and \( b \) the values of the demand function \( P = a - bQ \).

To determine the price elasticity of demand we thus need to determine the values of \( b, Q \) and \( P \). It is given that \( P = 70 - 0.5Q \) and a question asked in terms of \( P \) will therefore indicate that \( P = P \).

Comparing \( P = 70 - 0.5Q \) with \( P = a - bQ \), we can say that \( a = 70 \) and \( b = 0.5 \). At this stage \( a, b \) and \( P \) are known and \( Q \) is unknown.

The demand function denotes the relationship between the price \( P \) and the demand \( Q \). Therefore, if \( P \) is given, we can derive \( Q \) by substituting \( P \) into the demand function and solving \( Q \).

To determine the value of \( Q \) we need to change the equation of the demand function

\[
P = 70 - 0.5Q
\]

so that \( Q \) is the subject of the equation. That means we write \( Q \) in terms of \( P \). Now

\[
\begin{align*}
P &= 70 - 0.5Q \\
P - 70 &= -0.5Q \\
\frac{P - 70}{-0.5} &= Q \\
Q &= \frac{P - 70}{-0.5}.
\end{align*}
\]

As we have determined the values of \( b, P \) and \( Q \), we can now substitute them into the formula for elasticity of demand:

\[
\varepsilon_d = -\frac{1}{0.5} \times \frac{P}{P - 70} = -2 \times \frac{P}{P - 70}
\]

\[
\begin{align*}
\varepsilon_d &= -\frac{1}{0.5} \times \frac{P}{P - 70} \\
&= -\frac{1}{0.5} \times \frac{P}{P - 70} \times \frac{-0.5}{1} \\
&= \frac{P}{P - 70}.
\end{align*}
\]

Or alternatively,

you can use the given formula of price elasticity of demand in terms of \( P \) of a demand function in the form \( P = a - bQ \), given in the textbook on page 78, equation 2.14 (2nd edition) and page 89, equation 2.14 (3rd edition).

\[
\varepsilon_d = \frac{P}{P - a}
\]
Now $a = 70$ (intercept on the $y$-axis of the demand function)

$$
\varepsilon_d = \frac{P}{P - 70}.
$$

[Option 4]

**Question 7**

$$
\frac{3}{4} + 2 \left( \frac{5}{6} - \frac{1}{2} \right) + \frac{3}{2} \times \frac{5}{2}
$$

change the mixed fraction to an improper fraction

and add the terms in the brackets

$$
= \frac{3}{4} + 2 \left( \frac{11}{6} - \frac{1}{2} \right) + \frac{3}{2} \times \frac{5}{2}
$$

write terms in brackets as a common denominator

$$
= \frac{3}{4} + 2 \left( \frac{11 - 3}{6} \right) + \frac{3}{2} \times \frac{5}{2}
$$

$$
= \frac{3}{4} + 2 \left( \frac{8}{6} \right) + \frac{3}{2} \times \frac{5}{2}
$$

$$
= \frac{3}{4} \times \left( \frac{8}{3} \right) + \frac{3}{2} \times \frac{5}{2}
$$

$$
= \frac{3}{4} \times \frac{8}{3} + \frac{3}{2} \times \frac{5}{2}
$$

$$
= \frac{3}{4} \times \frac{15}{4}
$$

write as common denominator

$$
= \frac{9}{32} + \frac{15}{4}
$$

$$
= \frac{9 + 120}{32}
$$

$$
= \frac{129}{32}
$$

simplify

$$
= 4 \frac{1}{32}
$$

[Option 1]
Question 8

\[-3(x + 1) + 6 \left( x + \frac{1}{3} \right) \leq 4 \left( x - \frac{1}{2} \right)\]

multiply the value outside the bracket with values inside bracket

\[-3x - 3 + 6x + \frac{6}{3} \leq 4x - \frac{4}{2}\]

add the like terms

\[-3x + 6x - 4x \leq 3 - \frac{6}{3} - \frac{4}{2}\]

\[-x \leq 3 - 2 - 2\]

\[-x \leq -1\]

\[x \geq +1\]

multiplying by \(-1\) sign changes \(\leq\) to \(\geq\)

[Option 3]

Question 9

It is given that the cost \(c\) is linear related to the output \(Q\) or \(cost = f(Q)\). You are asked to determine the cost if 35 items were produced or \(cost = f(35)\).

Firstly, we thus need to determine the linear cost function \(cost = f(Q)\). We need two points to determine the equation of the cost function \((x_1 ; y_1)\) and \((x_2 ; y_2)\) with \(x\) the quantity and \(y\) the cost. It is given that the cost of manufacturing 10 units is R40. Thus \((x_1 ; y_1) = (10 ; 40)\). Secondly, it is given that the cost of manufacturing 20 units is R70. Thus \((x_2 ; y_2) = (20;70)\).

The cost function \(y = mx + c\) or in terms of cost and quantity \(cost = mQ + c\). We need to determine the slope of the \(m\) and \(y\)-intercept \(c\) of the line. Now the slope \(m\) is calculated as

\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 40}{20 - 10} = \frac{30}{10} = 3\]

Therefore \(cost = 3x + c\).

Now both \((x_1 ; y_1)\) and \((x_2 ; y_2)\) lie on the line. We can thus substitute any one of the points into the equation of the line to determine the \(y\)-intercept \(c\). Let’s choose the point \((10 ; 40)\) then

\[y = 3x + c\]

\[40 = 3(10) + c\]

\[40 = 30 + c\]

\[40 - 30 = c\]

\[c = 10\]

The equation of the cost function is \(cost = 3Q + 10\).
We need to determine cost (35):

\[
\text{cost} = 3Q + 10 = 3(35) + 10 = 105 + 10 = 115
\]

The cost of manufacturing 35 items is R115,00.

Question 10

\[
-3x \geq -2x + 6
\]

\[
-3x + 2x \geq 6
\]

\[
-x \geq 6
\]

\[
x \leq -6
\]

Multiplying both sides of the inequality by \(-1\). The inequality sign changes.

[Option 1]

[Option 3]