# Tutorial Letter 202/1/2018

**Basic Numeracy** 

**BNU1501** 

Semester 1

Department of Decision Sciences

 $\begin{array}{c} {\rm Important\ information:} \\ {\bf Solutions\ to\ Assignment\ 02} \end{array}$ 

Bar code



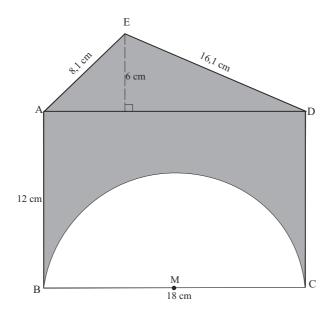
# **ASSIGNMENT 02: Solutions**

## Answers

1.	[3]	6.	[3]	11.	[4]	16.	[2]
2.	[2]	7.	[2]	12.	[1]	17.	[3]
3.	[4]	8.	[1]	13.	[1]	18.	[2]
4.	[1]	9.	[3]	14.	[2]	19.	[1]
5.	[4]	10.	[2]	15.	[3]	20.	[4]

## Explanations

## Question 1



The perimeter of the shaded figure is

$$\left(12 + 8.1 + 16.1 + 12 + \frac{2\pi r}{2}\right) \text{ cm}$$

$$= (48,200 + \pi(9)) \text{ cm}$$

$$= 76,4743 \text{ cm}$$

$$\approx 76,47 \text{ cm}$$

The area of the shaded part of the figure in question 1, is

area of triangle EAD + area of rectangle ABCD – area of semi-circle 
$$= \left(\frac{\text{base} \times \text{height}}{2}\right) + \left(\text{length} \times \text{width}\right) - \frac{\pi r^2}{2}$$

$$= \left(\frac{18 \times 6}{2}\right) + \left(18 \times 12\right) - \left(\frac{\pi \cdot 9^2}{2}\right)$$

$$= 54 + 216 - \frac{254,4690}{2}$$

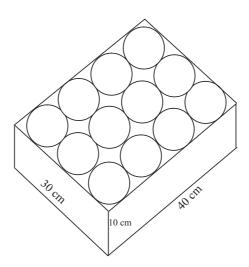
$$= 142,7655$$

$$\approx 142,77$$

Thus the shaded area is  $142,77 \text{ cm}^2$ .

The correct option is [2].

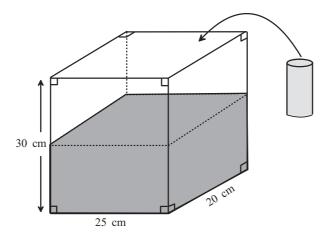
## Question 3



The volume of the rectangular box = 
$$\ell \times b \times h$$
 =  $(40 \times 30 \times 10) \text{ cm}^3$  =  $12\,000 \text{ cm}^3$   
The volume of one tin =  $\pi r^2 h$  =  $(\pi \times 5 \times 5 \times 10) \text{ cm}^3$  =  $785,3982 \text{ cm}^3$ 

The amount of sand necessary is therefore

$$(12\,000 - 785,3982) \,\mathrm{cm}^3$$
  
=  $2\,575,22 \,\mathrm{cm}^3$   
=  $2,58 \,\mathrm{litres}$  because  $1\ell = 1\,000 \,\mathrm{cm}^3$ 



The volume of the cylindric piece of iron  $= \pi r^2 h$   $= \pi (3)^2 10 \, \mathrm{cm}^3$  $= 282,7433 \, \mathrm{cm}^3$  (Calculator is set up to 4 decimal digits)

Therefore, the volume of water that the piece of iron replaces is 282,7433 cm<sup>3</sup>.

Since the water is in a rectangular glass tank, the volume of the replaced water  $= \ell \times b \times h$ .

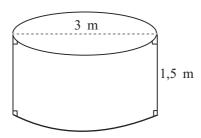
Thus,

$$\begin{array}{rcl} \ell \times b \times h &=& 282{,}7433 \\ \text{i.e.} & 25 \times 20 \times h &=& 282{,}7433 \\ \text{i.e.} & h &=& \frac{282{,}7433}{25 \times 20} \\ &=& 0.5655 \end{array}$$

Therefore, the water in the tank will rise with 0,57 cm.

The correct option is [1].

#### Question 5



The inside area of the fish pond is:

the area of circular floor 
$$+$$
 the area of sides 
$$= \pi r^2 + (2\pi r \times h)$$
$$= \left[\pi \cdot \left(\frac{3}{2}\right)^2 + 2\pi \left(\frac{3}{2}\right) \times \frac{3}{2}\right] \text{ m}^2$$
$$= 21,2058 \text{ m}^2$$

Now, 1 litre seals 3 m<sup>2</sup> (given).

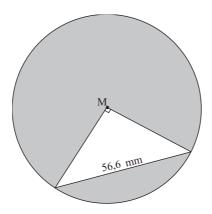
Therefore,  $21,2058 \,\mathrm{m}^2$  will be sealed by

$$\frac{21,2058}{3} \text{ litres}$$
=  $7,068 \text{ litres}$ 
=  $\frac{7,0686}{2} \text{ tins}$ 
=  $3,5343 \text{ tins}$ .

That means that 3,5 tins will be used, but, since we cannot buy parts of tins, 4 two-litre tins have to be purchased.

The correct option is [4].

## Question 6



The area of the circle is:

$$\pi r^2$$
=  $\pi (40)^2$ 
=  $5026,5482 \text{ mm}^2$ 

(Round off at the final answer.)

The area of the triangle is:

$$\frac{l \times b}{2}$$

$$= \frac{40 \times 40}{2} \text{ mm}^2$$

$$= 800 \text{ mm}^2$$

Therefore, the area of the shaded region in the diagram

= area of whole circle – area of triangle  
= 
$$(5\,026,5482-800)$$
 mm<sup>2</sup>  
=  $4\,226,5482$  mm<sup>2</sup>  
=  $42,3$  cm<sup>2</sup>  $(1$  cm<sup>2</sup> =  $10$  mm ×  $10$  mm =  $100$  mm<sup>2</sup>)

$$2(2y-1)-3y = 4-y$$
i.e.  $4y-2-3y = 4-y$  (Remove the brackets by multipliplying.)
i.e.  $4y-3y-2 = 4-y$  (Re-arrange terms for simplification)
i.e.  $y-2 = 4-y$  (Simplify left-hand side.)
i.e.  $y-2+y = 4-y+y$  (Add  $y$  on both sides to get rid of  $y$  on the right-hand side.)
i.e.  $2y-2 = 4$  (Simplify both sides.)
i.e.  $2y = 6$  (Simplify both sides.)
i.e.  $\frac{2y}{2} = \frac{6}{2}$  (Divide by 2 on both sides.)

The correct option is [2].

#### Question 8

$$\frac{2x}{5} - \frac{1}{2} = \frac{x}{5}$$
i.e.  $\frac{2x}{5} \times \frac{10}{1} - \frac{1}{2} \times \frac{10}{1} = \frac{x}{5} \times \frac{10}{1}$  (Multiply with the LCM of the denominators on both sides of the equation to get rid of the fraction.)
i.e.  $2x \times 2 - 1 \times 5 = x \times 2$  (Simplify both sides of the equation.)
i.e.  $4x - 5 = 2x$ 
i.e.  $4x - 5 + 5 = 2x + 5$  (Add 5 to both sides of the equation.)
i.e.  $4x = 2x + 5$  (Simplify where possible.)
i.e.  $4x - 2x = 2x - 2x + 5$  (Subtract  $2x$  on both sides.)
i.e.  $2x = 5$  (Simplify both sides.)
i.e.  $2x = 5$  (Divide by 2 on both sides.)
i.e.  $x = \frac{5}{2}$  (Divide by 2 on both sides.)
i.e.  $x = 2\frac{5}{2}$  (Simplify where possible.)
i.e.  $x = 2\frac{5}{2}$  (Simplify where possible.)

The correct option is [1].

#### Question 9

i.e. 
$$\frac{9C}{5} + 32 = F \qquad \text{(Swop the left- and right- hand sides of the equation around.)}$$
 i.e. 
$$\frac{9C}{5} \times \frac{5}{1} + 32 \times 5 = F \times 5 \qquad \text{(Multiply both sides of the equation by 5.)}$$
 i.e. 
$$9C + 160 = 5F \qquad \text{(Simplify both sides.)}$$
 i.e. 
$$9C + 160 - 160 \qquad = 5F - 160 \qquad \text{(Subtract 160 on both sides of the equation.)}$$
 i.e. 
$$9C = 5F - 160 \qquad \text{(Simplify where possible.)}$$
 i.e. 
$$\frac{9C}{9} = \frac{5F}{9} - \frac{160}{9} \qquad \text{(Divide both sides of the equation by 9.)}$$
 i.e. 
$$C = \frac{5F}{9} - \frac{160}{9} \qquad \text{(Simplified on both sides.)}$$

$$\begin{array}{rcl} V & = & \ell \times b \times h \\ \text{i.e.} & \ell \times b \times h & = & V \\ \text{i.e.} & \frac{\ell \times b \times h}{\ell} & = & \frac{V}{\ell} \\ \text{i.e.} & b \times h & = & \frac{V}{\ell} \\ \text{i.e.} & b \times h & = & \frac{V}{\ell} \\ \text{i.e.} & \frac{b \times h}{b} & = & \frac{V}{\ell \times b} \\ \text{i.e.} & h & = & \frac{V}{\ell \times b} \\ \text{i.e.} & h & = & \frac{V}{\ell \times b} \\ \end{array} \qquad \begin{array}{rcl} \text{(Simplify both sides.)} \\ \text{(Simplify both sides.)} \\ \text{(Simplify both sides.)} \\ \end{array}$$

The correct option is [2].

#### Question 11

We substitute the co-ordinates of the two points (3; -2) and (5; -6), which lie on the straight line, into the following formula:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \qquad \text{where } (x_1;y_1) = (3;-2) \text{ and } (x_2;y_2) = (5;-6)$$
 i.e. 
$$\frac{y-(-2)}{x-3} = \frac{-6-(-2)}{5-3}$$
 i.e. 
$$\frac{y+2}{x-3} = \frac{-6+2}{5-3}$$
 i.e. 
$$\frac{y+2}{x-3} = \frac{-2}{x}$$
 i.e. 
$$\frac{y+2}{x-3} \times \frac{x-3}{1} = -\frac{2}{1} \times \frac{x-3}{1}$$
 i.e. 
$$y+2 = -2x+6$$
 i.e. 
$$y+2-2 = -2x+6-2$$
 i.e. 
$$y = -2x+4$$

The correct option is [4].

#### Question 12

The formula for **simple interest** investments is

$$S = P\left(1 + rt\right)$$

where S is the accumulated amount, P is the principal amount that has been invested, r is the simple interest rate per year and t is the time of the investment.

We want r as the subject of the formula in this case.

Therefore

$$S = P(1+rt)$$
i.e.  $P(1+rt) = S$ 
i.e.  $\frac{P(1+rt)}{P} = \frac{S}{P}$ 
i.e.  $1+rt = \frac{S}{P}$ 
i.e.  $1+rt-1 = \frac{S}{P}-1$ 
i.e.  $rt = \frac{S}{P}-1$ 
i.e.  $\frac{rt}{t} = (\frac{S}{P}-1) \div t$ 
i.e.  $r = (\frac{12859}{8350}-1) \div 6$ 
i.e.  $r = 0,0900$  (See calculator steps below. Calculator is set to 4 decimal digits.)

The simple interest rate is thus 9% per year.

## Calculator steps:

$$12\,859 \div 8\,350 - 1 = \div 6 = \text{ or}$$

$$(12\,859 \div 8\,350 - ) \div 6 =$$

The correct option is [1].

## Question 13

Since we deal with simple interest again in this question, we have to use the **Simple Interest** formula and change the subject of the formula to P. Therefore, we do the following:

$$\begin{array}{rclcrcl} S & = & P\left(1+rt\right) \\ \text{i.e.} & P\left(1+rt\right) & = & S \\ \text{i.e.} & P & = & \frac{S}{1+rt} \\ \text{i.e.} & P & = & \frac{10\,000}{1+0.0975\times\frac{8}{12}} & \left(9,75\% = \frac{9,75}{100} = 0,0975\right) \\ \text{i.e.} & P & = & 9389,6714\ldots & (\text{See calculator steps below.}) \end{array}$$

The principal amount that has to be invested, is R9 389,67.

The correct option is [1].

## Calculator steps:

$$10\,000$$
  $\div$  ( 1 + 0,0975  $\times$  8  $\div$  12 )

**Note:** In this question we deal with a **compound interest investment**. That means that interest is calculated on interest as well. We can find the answer to a compound interest problem in two ways, either by using the financial keys on the financial calculator or by substituting into the relevant formula.

#### Method 1: By using the financial keys on the calculator.

Firstly we investigate the question very carefully.

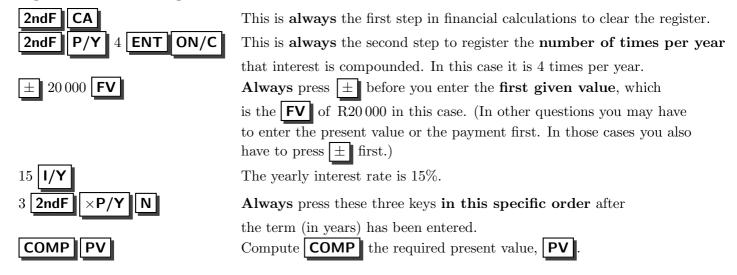
#### What is given?

- (i) The future value, **FV**, is R20 000.
- (ii) The interest rate per year, **I/Y**, is 15%.
- (iii) Interest is compounded every 3 months, that is 4 times per year.
- (iv) The term (time) of the investment is 3 years.

## What is required?

The present value, **PV**, of the principle amount that has to be invested.

#### Steps to follow on the prescribed SHARP financial calculator:



12857,9796 appears on the screen.

Thus, Helen should deposit at least R12857,98  $\approx$  R12858.

## Method 2: By substituting into the correct formula.

Since we deal with a **compound interest investment**, we use the following formula:

$$S = P \left( 1 + i \right)^n$$

We have to make P the subject of the formula since the principle to be invested is required, i.e.

i.e. 
$$P(1+i)^n = S$$
  
i.e.  $P = S \div (1+i)^n$   
 $= S \div \left(1 + \frac{0,15}{4}\right)^{4t}$   
 $= 20\,000 \div \left(1 + \frac{0,15}{4}\right)^{4 \times 3}$   
 $= 12\,857,9796$  (Calculator is set to 4 decimal digits.)  
 $\approx 12\,858$ 

#### Ordinary calculator steps:

$$20\,000$$
  $\div$  ( 1 + 0,15  $\div$  4 )  $y^x$  12  $=$ 

The correct option is [2].

## Question 15

#### Note:

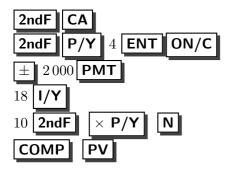
In this question we deal with the **amortisation of a loan**, where an amount has to be paid off in equal payments at equal time intervals (quarters in this case).

Here again, we can use either of two methods.

## Method 1: By using the financial keys on the prescribed SHARP calculator.

#### Note:

The prescribed financial calculator is preprogrammed to assist you in performing financial calculations where **compound interest** is concerned. Make sure you are familiar with the operation of the financial calculator.



 $36\,803,1689$  appears on the screen.

Lerato will have to borrow R36 803 from the bank.

## Method 2: By substituting into the relevant formula.

The **present value** formula is

$$P = R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= 2000 \left[ \frac{\left(1 + \frac{0.18}{4}\right)^{4 \times 10} - 1}{\frac{0.18}{4} \left(1 + \frac{0.18}{4}\right)^{4 \times 10}} \right]$$

$$= 36\,803\,1688$$

(Calculator is set to 4 decimal digits - see steps below.)

## Ordinary calculator steps:

$$2\,000 \ \times \ (\ (\ (\ 1\ +\ 0.18\ \div\ 4\ )\ 2ndF\ y^x\ 40\ -\ 1\ )\ \div\ (\ 0.18\ \div\ 4\ \times\ (\ 1\ +\ 0.18\ \div\ 4\ )\ 2ndF\ y^x\ 40\ )\ )\ \equiv$$

The correct option is [3].

#### Question 16

#### Note:

In this question we deal with **Compound Interest** again. Therefore we have the option to use the financial keys on the financial calculator to solve the problem, or we can substitute into the relevant formula to find the answer.

#### Method 1: By using the financial keys on the calculator.

Firstly we investigate the question carefully.

## What is given?

(i) The present value,  $\boxed{\mbox{\bf PV}}$ , of the loan is.

$$125\,000 - 15\%$$
 of  $12\,500$ 

$$= 125\,000 - \frac{15}{100} \times 12\,500$$

$$= 125\,000 - 18\,750$$

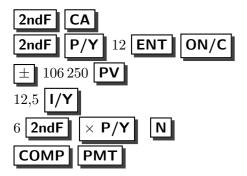
$$= 106250$$

- (ii) The yearly interest rate,  $\boxed{\textbf{I/Y}}$ , is 12,5%.
- (iii) The time/period of the loan is 6 years.
- (iv) Interest is compounded monthly, that is 12 times per year.

#### What is required?

The minimum monthly payment, PMT

Calculator steps on the prescribed SHARP financial calculator:



2 104,9377 appears on the screen.

Thus, Sam's minimum monthly payment is R2 104,94.

The correct option is thus [2].

Method 2: By substituting into the relevant formula.

The **present value** formula is:

$$P = R\left[\frac{(1+i)^n - 1}{i(+i)^n}\right]$$

We have to make the payment, R, the subject of the formula:

$$R = P \div \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= 106 250 \div \left[ \frac{\left(1 + \frac{0.125}{12}\right)^{12 \times 6} - 1}{\frac{0.125}{12}\left(1 + \frac{0.125}{12}\right)^{72}} \right]$$

$$= 2 104,9377 (Calculator is set to 4 decimal digits – see steps below.)$$

Sam's minimum payment is thus R2 104,94.

## Ordinary calculator steps:

2 104,9377 appears on the screen.

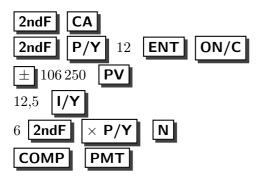
The correct option is thus [2].

#### Note:

The only way to deal with **amortisation** questions in BNU1501 is to use the financial keys on the calculator. We do not have a formula in this module to find the amount outstanding (the balance) on the loan after 3 years' payments have been made, that is after  $3 \times 12 = 36$  payments have been made. It is not necessary to draw up a long amortisation table to find the outstanding amount at the end of year 3. The financial calculator makes it easy to find it in a few steps.

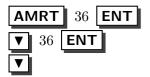
#### Financial calculator steps:

Firstly, we have to determine the monthly payment s in question 16 above, and then we can proceed to find the outstanding balance.



The minimum monthly payment of R2104,9377 appears on the screen. **Keep it on the calculator unrounded.** 

Subsequent steps to find the outstanding balance after 36 payments (made in 3 years) have been done:



A balance of 62 921,0663 appears on the screen.

Therefore, after 3 years' payments have been made in full, that is after 36 payments of  $R2\,104,94$  have been made, the outstanding balance is still  $R62\,921,07$ . (Ignore the – sign on the calculator.)

Question 18

Below is the amortisation table for the loan in question 16 above:

Number of payments	Minimum	Outstanding	Principal	Interest
made (months)	payment	balance	paid off	paid off
		106250		
1	$2104,\!94$	$105251,\!83$	998,16	1 106,77
2	$2104,\!94$	$104243,\!27$	$1008,\!56$	1 096,37
3	$2104,\!94$	103 224,20	$1019,\!07$	1 085,87
4	$2104,\!94$	102 194,51	1 029,69	$1075,\!25$
5	$2104,\!94$	101 154,10	$1040,\!41$	$1064,\!53$
6	2 104,94	100 102,85	$1051,\!25$	1 053,69
7	$2104,\!94$	$99040,\!65$	$1062,\!20$	1042,74
:	:	÷	:	:
10	2 104,94	95 787,20	1 095,74	1 009,20
<u>:</u>	:	÷	÷	:
12				
(1 year)	2 104,94	93 561,36	1 118,69	986,25
:	:	:	:	÷
24				
(2 years)	$2104,\!94$	$79192{,}55$	$1266,\!82$	838,12
:	:	:	:	:
36				
(3 year)	$2104,\!94$	62 921,07	$1434,\!57$	670,37
:	:	:	:	:
42				
(3.5  years)	2 104,94	53 994,35	$1526,\!59$	578,34
:	:	÷	:	÷
48				
(4 years)	$2104,\!94$	44 494,98	$1624,\!53$	480,41
60				
(5 years)	2 104,94	23 628,99	1 839,40	265,30
<u>:</u>	:	:	:	<u>:</u>
72				
(6 years)	$2104,\!94$	00043	$2083,\!24$	21,70

**Note:** In the 7th month it is the first time that the principal - paid-off part (1062,20) of the payment is more than the interest-paid-off part (1042,74).

**Note:** In this question we deal with a loan that is paid of by **equal payments** made into the loan account at **equal time intervals**.

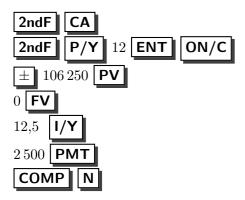
#### What is given?

- (i) The size of the loan, which is the present value  $\boxed{\text{PV}}$  of the loan, is  $R(480\,000-150\,000)=R330\,000$ .
- (ii) The monthly payment **PMT**, is R8 000.
- (iii) The interest rate  $\boxed{\textbf{I/Y}}$  is 24% per year.
- (iv) The interest is compounded monthly, that is 12 times per year.
- (v) The size of the loan when it is paid off is the future value **FV** of R0,00.

#### What is required?

The time  $\boxed{\mathbf{N}}$  it will take to pay the loan off.

## Method 1: By using the financial keys on the SHARP EL738FB calculator.



56,4198 appears on the screen.

Therefore, Justin will pay the R106 250 loan off in 56,4198 months, that is  $\frac{56,4198}{12} = 4,7$  years.

The correct option is [1].

**NB!** By increasing his monthly payment from R2104,94 to R2500, that is with R395,06, Justin pays the loan off in 4,7 years instead of in 6 years (and saves a lot of money/interest).

#### Method 2: By substituting into the relevant formula.

The relevant formula to use in this case is the **present value** formula

$$P = R \left[ \frac{(1+i)^n - 1}{(1+i)^n \cdot i} \right]$$

Since the time, n, is required in this question, we make n the subject of the formula. Therefore, from the formula above we can write

$$\frac{(1+i)^n - 1}{(1+i)^n \cdot i} = \frac{P}{R}$$
i.e.  $(1+i)^n - 1 = \frac{P}{R}(1+i)^n \cdot i$ 
i.e.  $(1+i)^n - \frac{P}{R}(1+i)^n i = 1$ 
i.e.  $(1+i)^n \left[1 - \frac{P}{R}i\right] = 1$ 
i.e.  $(1+i)^n = \frac{1}{1 - \frac{P}{R}i}$ 
i.e.  $n = \frac{\log \left[\frac{1}{1 - \frac{P}{R}i}\right]}{\log (1+i)}$  (See the important note below.)
$$= \frac{\log \left(\frac{1}{1 - \frac{106250}{2500} \times \frac{0.125}{12}}\right)}{\log \left(1 + \frac{0.125}{12}\right)}$$

$$= 56,41981598 \text{ months} \qquad \text{(See calculator step below.)}$$

$$= \frac{56,42}{12} \text{ years}$$

$$= 4.7 \text{ years}$$

The correct option is [1].

Ordinary calculator keys to use on a scientific calculator:

56,42 appears on the screen, and that is 56,42 months or  $\frac{56,42}{12}$  years = 4,7 years.

#### Important Note:

It is clear that **knowledge of logarithms** is needed to find the value of the unknown exponent, n, in this case. Logarithms are not explained in the BNU1501 Study Guide because they are not part of the syllabus of this module. Therefore, you are advised to use the **financial keys** on the financial calculator here and in all other exercises where the **time period** of an investment or annuity has to be calculated.

#### Question 20

Note: In this question we deal with an **annuity** where **equal payments** are made into a certain account at **equal time intervals**.

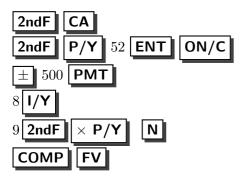
## What is given?

- (i) The payment, **PMT**, is R500.
- (ii) The annual (yearly) interest rate, **I/Y**, is 8%.
- (iii) The time period,  $\mathbf{N}$ , is (29-20) years = 9 years.
- (iv) Interest is compounded weekly, that is 52 times per year.

## What is required?

The future value,  $\boxed{\mathsf{FV}}$ , is required in this question.

## Method 1: By using the financial calculator.



342321,4765 appears on the screen.

The **future value** of this annuity is thus the amount that John will have saved in 9 years and is R342 321,48.

## Method 2: By substituting into the relevant formula.

The relevant formula to use in this case is the **future value** formula.

$$S = R \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$= 500 \left[ \frac{\left(1 + \frac{0.08}{52}\right)^{52 \times 9} - 1}{\frac{0.08}{52}} \right]$$

$$= 342321,4765 (See calculator steps below.)$$

The future value of this annuity is thus the amount that John will have saved in 9 years, which is R342 321,48.

The correct option is [4].

## Ordinary calculator steps:

END OF THE SOLUTIONS FOR ASSIGNMENT 02, SEMESTER 1