Tutorial letter 101/3/2018

Quantitative Modelling 1
DSC1520

Semesters 1 and 2

Department of Decision Sciences

This is a fully online module.
Information available on myUnisa at DSC1520-2018-S1 or DSC1520-2018-S2.
Communication through myLife e-mail account.
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1 Introduction and welcome

It is a pleasure to welcome you to this module, Quantitative Modelling 1.

We hope that you will find this module interesting and enjoyable and that you will complete it successfully. This is a fully online module. You therefore need to go online to see your study material and read about the module. The myUnisa website is at http://my.unisa.ac.za/ where you need to login with your student number and password. If you do not have a password yet, go to Claim UNISA login (at the left) to register. Once logged in, you will see the module site DSC1520-18-S1 or DSC1520-18-S2 at the top, depending on the semester you are registered for. If you don’t see it, check under More Sites. This tutorial letter (Tutorial letter 101) and the online study guide are supplied in printed format so you can study from it even when you cannot go online.

2 Lecturer and contact details

You are welcome to contact your lecturer(s), the Department of Decision Sciences or the university when you encounter study related problems during the semester.

Provide your student number and module code whenever you contact us.

2.1 Lecturer

You will find the name of the lecturer responsible for this module on the DSC1520 site on myUnisa. We suggest that you write the name and contact details of the lecturer in the space below so it will be available whenever you need to contact him/her.

Your lecturer will assist you with problems you may have with the study material (not administrative matters). You may contact him/her by means of e-mail, telephone or fax. You may also visit him/her at the office, but only by appointment.

2.2 Department

You can reach the Department of Decision Sciences in one of the following ways:

E-mail: qm@unisa.ac.za
Tel: 012 433-4684
Fax: 012 429-4898
2.3 University

We assume that you are acquainted with the contents of the brochure Study @ Unisa. This helpful document is available on the myUnisa homepage — go to Undergraduate & honours. You may consult the online version here or open the printable PDF version at the bottom of the list on the left.
Consult this brochure if you need to contact the university about matters not related to the content of this module.

3 Student support system

Consult Study @ Unisa for information on the various student support systems and services available at Unisa. For example, student counselling, tutorial classes and language support.

Unisa also provides student support through e-tutors and face-to-face tutors. These tutors are qualified experts in the subjects for which they have been appointed.

3.1 E-tutors

E-tutoring entails the delivery of teaching and learning online via the internet. The e-tutors for DSC1520 undertake to support students in their groups to be successful in this module.

Students are grouped to e-tutors after the registration process has ended. You will receive a myLife e-mail notification when you have been allocated to an e-tutor. You can contact and communicate with your e-tutor through the e-tutor site on myUnisa named DSC1520-18-S1-E1 or DSC1520-18-S1-E2 (semester 1) or DSC1520-18-S2-E1 or DSC1520-18-S2-E2 (semester 2).

3.2 Face-to-face tutors

For face-to-face contact sessions with a tutor, you need to go to the Unisa Regional Learning Centre (RLC) nearest to you to enrol. In such face-to-face sessions students meet with their tutors in a classroom setup at the RLCs.

The RLC will supply you with the dates and times of the sessions for the modules you have enrolled for. Note that tutor classes are not presented at all RLCs, since there must be a certain minimum number of students enrolled at an RLC for a face-to-face tutor to be appointed there.

4 Online services

This module is presented fully online and you need to have access to a computer that is linked to the internet. If you do not have your own computer with internet access, consult Study @ Unisa for guidelines on how and where to get access. (Under Prepare for your study success, go to Connect online.)

As a registered Unisa student you have free access to myUnisa (Unisa’s online campus) and you receive a free myLife email address.
4.1 myUnisa

myUnisa is a platform where you can communicate electronically with your lecturers, other students and the administrative departments of Unisa.

This is also where you will submit assignments, view results, join online discussion forums, etc.

4.2 myLife

Announcements, marked assignments and other forms of communication from Unisa will be sent to your myLife e-mail address.

If you regularly use another e-mail account, we suggest that you configure your myLife account to forward e-mails to this account.

5 Study material

The study material for this module consists of the following:


3. A module online study guide, intended to guide you through the textbook.

4. Additional tutorial letters that will be posted on myUnisa (under Additional Resources) during the semester.

   **Important:**
   
   The textbook is the basis of the study material for this module.
   
   Purchase the textbook as soon as possible.
   
   *It is not possible to pass this module without the textbook.*

5.1 Printed study material

Printed copies of Tutorial letter 101 and the Study guide will be sent to all students.

These are exactly the same as the electronic versions available on myUnisa. If you haven’t received the printed version, please do not waste valuable time by waiting for it to arrive before starting with your studies. Download it from myUnisa and print it for yourself.

5.2 Where to buy the textbook

To find out where you can buy the textbook, consult the list of booksellers and their addresses that is available on the myUnisa homepage under Books.

If you cannot obtain the textbook from these booksellers, please contact the Processing Section at the library as soon as possible by sending an e-mail to vospresc@unisa.ac.za.
6 Overview of module

6.1 Purpose and outcomes

The purpose, specific outcomes and assessment criteria of this module are available on myUnisa, under Additional resources.

6.2 Syllabus

The study material is subdivided into five study units, namely

<table>
<thead>
<tr>
<th>Study unit</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear functions</td>
</tr>
<tr>
<td>2</td>
<td>Simultaneous linear functions</td>
</tr>
<tr>
<td>3</td>
<td>Nonlinear functions</td>
</tr>
<tr>
<td>4</td>
<td>Differentiation</td>
</tr>
<tr>
<td>5</td>
<td>Integration</td>
</tr>
</tbody>
</table>

Details regarding the contents of each study unit can be found in the table of contents of the Study guide.

7 Calculator

You may use any (programmable) scientific or financial pocket calculator for this module. Although we say “programmable”, you will not need the programmable features of such a calculator in this module.

8 Study approach

8.1 Study time

A semester is quite short and it is essential that you plan your study program carefully. Use the assignment due dates as a guideline to manage your time in such a way that you master the study material, submit the assignments on time and have enough time to prepare for the examination.

You will have to work consistently throughout the semester if you wish to be successful in this module.

8.2 Study method

We suggest that you approach the study material as follows:

▷ Work through each study unit in the study guide together with the textbook.
▷ Follow the instructions carefully and do the activities on your own (with pen on paper).
▷ Contact your lecturer (or e-tutor) if you need help with the study material, preferably before you carry on with a new study unit.
9 Assessment

This module is assessed by means of three compulsory assignments (formative assessment) and a written examination (summative assessment).

9.1 Compulsory assignments

To benefit fully from our formative tuition and assessment, you should complete and submit the three compulsory assignments before or on the due dates. Assignments 01 and 03 consist of multiple-choice questions (MCQ) and Assignment 02 is a written assignment.

9.2 Examination

To gain admission to the examination, you must submit the compulsory assignments before or on the due dates.

The examination at the end of the semester will consist of a two hour MCQ paper.

9.3 Semester and examination marks

The assignments all count towards your semester mark, which contributes 20% to your final mark for the module. Assignments 01 and 03 (MCQ) each contributes 30% to the semester mark, while Assignment 02 (written) contributes 40%.

The written examination contributes 80% to the final mark. A subminimum of 40% applies to the examination. This means that your year mark will not be taken into account if your examination mark is less than 40%. In this case your examination mark will also be your final mark.

9.4 Assignment due dates and unique numbers

The assignment due dates and unique numbers for the first and second semesters are as follows:

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Type</th>
<th>Dates and unique numbers</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First semester</td>
<td>Second semester</td>
</tr>
<tr>
<td>01</td>
<td>MCQ</td>
<td>6 March 2018</td>
<td>17 August 2018</td>
</tr>
<tr>
<td>02</td>
<td>Written</td>
<td>29 March 2018</td>
<td>7 September 2018</td>
</tr>
<tr>
<td>03</td>
<td>MCQ</td>
<td>23 April 2018</td>
<td>4 October 2018</td>
</tr>
</tbody>
</table>

9.5 Submission of assignments

You must submit all assignments electronically through myUnisa. See Study @ Unisa for instructions to submit MCQ assignments.

To submit a written assignment electronically, you can either write it by hand and scan it in PDF format, or you can type it in MSWord (or a similar word processing package) and then save it as a PDF before submitting it.
10 Semester 1 – Compulsory assignments

10.1 Assignment 01 (S1)

This MCQ assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 March 2018</td>
<td>811139</td>
</tr>
</tbody>
</table>

Instructions

▷ Work through Study units 1 and 2 in the Study guide before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

Question 1

Find the slope of the line represented by the equation

\[
0 = 6 + 3x - 2y.
\]

[1] \( \frac{2}{3} \)  
[2] \( \frac{3}{2} \)  
[3] 3  
[4] 2  
[5] None of the above.

Question 2

Consider the demand function

\[
P = 100 - 5Q,
\]

with \( P \) being price and \( Q \) quantity. Find the price elasticity of demand when \( Q = 8 \). Is demand elastic or inelastic? Justify your answer.

[1] \( \varepsilon_d = -1.5; \text{ demand is elastic; } |\varepsilon_d| < 1 \)
[2] \( \varepsilon_d = -0.03; \text{ demand is inelastic; } |\varepsilon_d| < 1 \)
[3] \( \varepsilon_d = -1.5; \text{ demand is elastic; } |\varepsilon_d| > 0 \)
[4] \( \varepsilon_d = 1.5; \text{ demand is inelastic; } \varepsilon_d > 1 \)
[5] None of the above.
Question 3
Given the demand function $P = 60 - 0.2Q$. What is the arc price elasticity of demand when price decreases from R50 to R40?

[1] $-\frac{1}{3}$
[2] $\frac{1}{3}$
[3] $-3$
[4] $3$
[5] None of the above.

Question 4
In the following market:

Demand function: $Q = 50 - 0.1P$
Supply function: $Q = -10 + 0.1P$

where $P$ and $Q$ are the price and quantity respectively. Calculate the equilibrium price and quantity.

[1] $P = 300; Q = 20$
[2] $P = 200; Q = 30$
[3] $P = 20; Q = 300$
[4] $P = 30; Q = 200$
[5] None of the above.

Question 5
Determine the value of $x$ that solves the inequality:

$$-3(x + 1) + 6 \left(x + \frac{1}{3}\right) \leq 4 \left(x - \frac{1}{2}\right).$$

[1] $x \leq -\frac{3}{7}$
[2] $x \leq -1$
[3] $x \geq 1$
[4] $x \leq 1$
[5] None of the above.
Question 6

The linear function

\[ 2P = 20 - Q \]

can be graphically represented as:

[5] None of the above.
**Question 7**
Workers at a building site have the option to eat lunch provided by an outside company. When the price per lunch is R40, there is a demand for 80 lunches. It is known that for each R5 increase in price, demand decreases by three lunches. Determine the demand function for lunches with quantity demanded \( Q \) as a function of price \( P \).

\[ Q = 104 - 0.6P \]
\[ Q = 136.67 - 1.67P \]
\[ Q = 56 + 0.6P \]
\[ Q = 40 - \frac{3}{5}P \]
[5] None of the above.

**Questions 8 and 9 are based on the following information:**
The demand and supply functions for accommodation at a B & B are given by

\[ P_d = 952 - 8Q \text{ and } P_s = 400 + 4Q, \]

where \( P \) is the price per room and \( Q \) is the number of rooms occupied. The following graph represents these functions:

**Question 8**
The coordinates of the intercepts at \( A, B \) and \( C \) are

\[ A = (0; 95.2), B = (0; 40), C = (11.9; 0). \]
\[ A = (0; 100), B = (0; 119), C = (952; 0). \]
\[ A = (0; 119), B = (0; 100), C = (400; 0). \]
\[ A = (0; 952), B = (0; 400), C = (119; 0). \]
[5] None of the above.
Question 9
What does point D represent? Provide the coordinates at point D.

[1] Break-even; $D = (46, 67; 586, 68)$
[2] Equilibrium; $D = (46; 584)$
[3] Equilibrium; $D = (587; 47)$
[4] Break-even; $D = (47; 587)$
[5] None of the above.

Question 10
Solve the following system of linear equations:

\[
\begin{align*}
  x + y + z &= 8 \\
  x - 3y &= 0 \\
  5y - z &= 10.
\end{align*}
\]

The sum of the values of $x, y$ and $z$ of the solution is


Question 11
The demand and supply functions for a product are given by

\[
P_d = 136 - 4Q \text{ and } P_s = 14 + 5Q,
\]

where $P$ is the price and $Q$ is quantity. By working throughout with values rounded to two decimals, the producer surplus at equilibrium is

Question 12
An animal feed is to be made from corn, soybean and cottonseed. The feed should supply at least 1,800 units of fiber, at most 2,800 units of fat and at least 2,200 units of protein. The number of units of fiber, fat and protein that one unit of each ingredient provides, is shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Corn</th>
<th>Soybean</th>
<th>Cottonseed</th>
<th>Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>1,800</td>
</tr>
<tr>
<td>Fat</td>
<td>30</td>
<td>20</td>
<td>40</td>
<td>2,800</td>
</tr>
<tr>
<td>Protein</td>
<td>20</td>
<td>40</td>
<td>25</td>
<td>2,200</td>
</tr>
</tbody>
</table>

If X, Y and Z are the number of units of corn, soybean and cottonseed required, respectively, then the system of equations representing the constraints on making the feed with these requirements is

\[ \begin{align*}
X + 2Y + 3Z & \leq 18; \\
3X + 4Y + 2.5Z & \leq 28; \\
2X + 4Y + 2.5Z & \leq 22. \\
\end{align*} \]

\[ \begin{align*}
X + 3Y + 2Z & \leq 180; \\
2X + 2Y + 4Z & \leq 280; \\
3X + 4Y + 2.5Z & \leq 220. \\
\end{align*} \]

\[ \begin{align*}
10X + 20Y + 30Z & \geq 1,800; \\
30X + 20Y + 40Z & \leq 2,800; \\
20X + 40Y + 25Z & \geq 2,200. \\
\end{align*} \]

\[ \begin{align*}
10X + 30Y + 20Z & \geq 1,800; \\
20X + 20Y + 40Z & \leq 2,800; \\
30X + 40Y + 25Z & \geq 2,200. \\
\end{align*} \]

Questions 13 and 14 are based on the following LP model:

\[ \text{Minimise } Z = 1.8x + 1.2y \]

subject to

\[ \begin{align*}
2x + 6y & \geq 30 \\
4x + 2y & \geq 20 \\
y & \geq 2; \quad x \geq 0. \\
\end{align*} \]
Question 13

The feasible region for this model is given by

[5] None of the above.
Question 14

The optimal solution is

[1] \( x = 5; \ y = 0; \ Z = 9,00. \)
[2] \( x = 3; \ y = 4; \ Z = 10,20. \)
[3] \( x = 0; \ y = 10; \ Z = 12,00. \)
[4] \( x = 9; \ y = 2; \ Z = 18,60. \)

Question 15

The consumer surplus for the demand function

\[ P = 90 - 5Q \]

when the market price is \( P = 20 \) is

[1] 140.
10.2 Assignment 02 (S1)

This written assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 March 2018</td>
<td>737135</td>
</tr>
</tbody>
</table>

Instructions

- Work through *Study unit 3* in the Study guide before attempting this assignment.
- Answer all the questions.
- Show all steps you followed to find the answers (including graphs). Marks will be given for these and not only for the answer.
- Submit your answers in PDF format on myUnisa.
- You may use Excel or Maxima for assistance when you need to draw graphs and/or solve equations.

Only a selection of questions will be marked, but you will receive solutions to all the questions.

**Question 1**

Simplify:

1. \(\frac{(2a^2b^3)^2 \times (ab^4)^3}{(2a^3b^2)^4}\).

2. \(\sqrt{4x/y^4}\).

**Question 2**

1. Determine the roots of \(f(x) = -x^2 + 8x - 16\).

2. Approximately how many units must be manufactured to maximise a profit defined by the function \(f(x) = -x^2 + 8x - 16\)?

**Question 3**

A company manufactures and sells \(x\) radios per week. The weekly cost and price equations are given by

\[ c(x) = 5000 + 2x \quad \text{and} \quad p(x) = 10 - \frac{x}{1000}, \text{ for } 0 \leq x \leq 8000, \]

respectively.

1. What is the revenue function?
2. Derive for each week
   (a) the price at which revenue is maximum,
   (b) the maximum revenue,
   (c) the number of radios to produce to realise a maximum profit,
   (d) the maximum profit,
   (e) the price that the company should charge for each radio to realise a maximum profit.

**Question 4**
Consider the quadratic function

\[ f(x) = -2x^2 + 10x - 8. \]

1. Find the roots and the turning point.
2. Hence draw the graph.

**Question 5**
An investment in a bank is said to grow according to the following formula:

\[ P(t) = \frac{6000}{1 + 29e^{-0.4t}} \]

where \( t \) is time in years and \( P \) is the amount (principle plus interest).

1. What is the initial amount invested?
2. Determine the time in years when the amount will be R4 000.

**Question 6**

1. Solve for \( Q \) if \( \log Q - \log \left( \frac{Q}{Q+1} \right) = 0.8. \)
2. Simplify \( \left( \frac{4t^2}{L^2} \right)^2. \)

**Question 7**

1. Use rules of logarithms to solve the equation \( 3 \ln(2x^2) - 5 \ln x = 7. \)
2. Evaluate \( \frac{\log_{12.34} 12.34}{\ln 12.34}. \)
Question 8
ABC intends manufacturing and marketing a new product. It has been determined that the cost of producing the product, as a function of price, is given by

\[ C(P) = 432000 - 1800P. \]

The revenue generated when units are sold at price \( P \) rand each, is given by

\[ R(P) = 6000P - 30P^2. \]

Plot the income and cost functions on the same graph. Indicate clearly on the graph, the break-even point(s) and profit area.

Question 9
Consider the cubic function

\[ f(x) = -4Q^3 + 2Q^2. \]

1. Draw the graph of \( f \).
2. From the graph, estimate the roots of \( f \) and the coordinates of the turning point of \( f \).

Question 10
The demand and supply functions of a certain product are given by

\[ P_d = Q^2 + 2Q + 5 \quad \text{and} \quad P_s = 29 - 3Q, \]

where \( P \) is the price per unit and \( Q \) the number of units produced. Determine the equilibrium price and quantity for the product.
10.3 Assignment 03 (S1)

This MCQ assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 April 2018</td>
<td>731948</td>
</tr>
</tbody>
</table>

Instructions

▷ Work through Study units 4 and 5 in the Study guide before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

Question 1

Find the derivative of the function: \( G(x) = x(x^2 - 4\sqrt{x} + 4) \).

- \[1\] \( x^3 - 4\sqrt{x^3} + 4x \)
- \[2\] \( 3x^3 - \frac{3}{2}\sqrt{x^4} + 4 \)
- \[3\] \( 3x^4 - 6\sqrt{x^3} + 4x^2 \)
- \[4\] \( 3x^2 - 6\sqrt{x} + 4 \)
- \[5\] None of the above.

Question 2

Differentiate the function

\[ f(x) = \frac{x^2 + 6}{2x + 5}. \]

Simplify your answer.

- \[1\] \( -\frac{2(x+6)(x-1)}{(2x+5)^2} \)
- \[2\] \( \frac{2(x+6)(x-1)}{(2x+5)^2} \)
- \[3\] \( \frac{2x-12}{(2x+5)^2} \)
- \[4\] \( \frac{2(x^2+6)}{(2x+5)^2} \)
- \[5\] None of the above.
Question 3
Find the derivative of the function
\[ P(x) = x^5 e^{3x} + \frac{x + 1}{x}. \]

[1] \( e^{3x} (3x^5 + 5x^4) - \frac{1}{x^2} \)
[2] \( e^{3x} (3x^5 + 5x^4) - \frac{2}{x} \)
[3] \( e^{3x} (x^5 + 5x^4) - \frac{1}{x^2} \)
[4] \( e^{3x} (3x^5 + 5x^4) + \frac{1}{x^2} \)
[5] None of the above.

Question 4
Find the derivative of \( \ln x + 4x^{-2} \).

[1] \( \frac{1}{x} - \frac{8}{\sqrt{x}} \)
[2] \( \frac{1}{x} + 8x^{-3} \)
[3] \( \frac{1}{2} - \frac{8}{\sqrt{x}} \)
[4] \( \ln x - 8x^{-3} \)
[5] None of the above.

Question 5
Evaluate
\[ \int x^2 \left( 1 + \frac{1}{x^2} \right) \, dx. \]

[1] \( x^3 + x + c \)
[2] \( \frac{3}{4} x^3 + x + c \)
[3] \( x^2 + 1 \)
[4] \( \frac{1}{2} x^2 + x + c \)
[5] None of the above.

Question 6
Evaluate the following definite integral:
\[ \int_{-2}^{2} (x^2 - 3) \, dx \]

[1] \( 6 \frac{2}{3} \)
[2] \( -6 \frac{2}{3} \)
[3] \( 3 \frac{1}{3} \)
[4] \( -3 \frac{1}{3} \)
[5] None of the above.
Question 7
Evaluate the following integral:
\[ \int \sqrt{9x - 5} \, dx. \]

[1] \( \frac{1}{5} \sqrt{(9x - 5)^3} + c \)
[2] \( \frac{1}{9} \sqrt{(9x - 5)^3} + c \)
[3] \( \frac{2}{27} \sqrt{(9x - 5)^3} + c \)
[4] \( \frac{2}{27} \sqrt{(9x - 5)^3} + c \)
[5] None of the above.

Question 8
Evaluate the following integral:
\[ \int \frac{x^2 + 4}{x^3} \, dx. \]

[1] \( \ln x + \frac{2}{x^2} + c \)
[2] \( \ln x - 2x^{-2} + c \)
[3] \( \frac{1}{x} - 2x^{-2} + c \)
[4] \( \frac{1}{x} + 2x^{-2} + c \)
[5] None of the above.

Question 9
What is the value of maximum revenue if total revenue is given by
\[ R(x) = -\frac{1}{5}x^2 + 30x + 81 \]
where \( x \) is the quantity?

[1] 75
[2] 1 206
[3] 152,65
[4] 81
[5] None of the above.
**Question 10**

Total revenue is given by

\[ TR = 2x^5 - \frac{1}{2}x^2 + 10x + 15, \]

where \( x \) is the number of units sold. What is the marginal revenue when five units are sold?

- [1] 55
- [2] 630
- [3] 6255
- [4] 6270
- [5] None of the above.

**Question 11**

Suppose the total cost (in rand) of manufacturing radios is given by

\[ TC = 2Q^3 - Q^2 + 80Q + 150, \]

where \( Q \) is the number of radios manufactured. What is the marginal cost if 10 radios are manufactured?

- [1] R80
- [2] R660
- [3] R700
- [4] R2 850
- [5] None of the above.

**Question 12**

The annual revenue (in millions of rand) generated by a television company can be approximated by the function

\[ f(t) = 5.78 + 8.59 \ln t, \]

where \( t \) is the number of years since the company started. The rate of change in revenue 15 years after the company started, is

- [1] R0.57 per annum.
- [5] None of the above.
Question 13
The demand for seats at a mini soccer match is given by
\[ Q = 192 - P^2, \]
where \( Q \) is the number of seats and \( P \) is the price per seat. Find the price elasticity of demand if seats cost R6 each. What does this value mean?

[1] \( \varepsilon_d = -0.46 \); inelastic since \( |\varepsilon_d| < 1 \), a 1%, price increase will result in 0.46% less seats to be sold
[2] \( \varepsilon_d = 0.46 \); elastic since \( |\varepsilon_d| > 0 \), a 1%, price increase will result in 0.46% more seats to be sold
[3] \( |\varepsilon_d| = -0.5 < 0 \); demand is inelastic
[4] \( \varepsilon_d = \frac{-P^2}{Q} \) which is always elastic
[5] None of the above.

Question 14
Calculate the consumer surplus for the demand function
\[ P = \frac{40}{Q + 3}, \]
when the market price is \( P = 10 \).

[1] 0.3
[2] 1.5
[3] 11.5
[5] None of the above.

Question 15
The marginal cost function for a good is given by
\[ MC = 2Q^2 - 1. \]
Find the total cost function if fixed costs are 300.

[1] \( \frac{2Q^3}{3} - Q + 300 \)
[2] \( \frac{2Q^3}{3} - Q + C \)
[3] \( -\frac{2Q^3}{3} - 1 + C \)
[4] \( 2Q^3 - Q + 300 \)
[5] None of the above.
11 Semester 2 - Compulsory assignments

11.1 Assignment 01 (S2)

This MCQ assignment is compulsory.

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Instructions

▷ Work through Study units 1 and 2 in the Study guide before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

Question 1

Suppose the cost of manufacturing 10 units of a product is R40 and the cost of manufacturing 20 units is R70. If the cost $C$ is linearly related to output $Q$ (units produced), the cost of producing 35 items, is

- [1] R115.00.
- [4] R65.00

Question 2

Find the equation of the line passing through the points $(3; 1)$ and $(\frac{4}{3}; 2)$.

- [1] $y = \frac{-3}{5}x + \frac{14}{5}$
- [2] $y = \frac{-3}{5}x + 1$
- [3] $y = x + \frac{14}{5}$
- [4] $y = \frac{-3}{5}x - \frac{14}{5}$
- [5] None of the above.
Question 3

If the demand function is

\[ P = 70 - 0.5Q, \]

where \( P \) and \( Q \) are the price and quantity, respectively, determine the expression for price elasticity of demand in terms of \( P \) only.

\[ \begin{align*}
1 & \frac{P - 140}{P} \\
2 & \frac{P}{P - 35} \\
3 & \frac{P}{P - 70} \\
4 & \frac{P - 140}{P} \\
5 & \text{None of the above.}
\end{align*} \]

Question 4

Solve the following system of linear equations for \( x, y \) and \( z \).

\[ \begin{align*}
x - 2y + 3z & = -11 \quad (1) \\
2x - z & = 8 \quad (2) \\
3y + z & = 10 \quad (3)
\end{align*} \]

\( x + y + z \) is

\[ \begin{align*}
1 & -4. \\
2 & -2. \\
3 & 4. \\
4 & 5. \\
5 & \text{none of the above.}
\end{align*} \]

Question 5

Determine the value of \( x \) that solves the inequality:

\[ -2x + \frac{5}{6} + \frac{x}{2} \geq -2x - 4 \left( -\frac{x}{3} - 1\frac{1}{4} \right). \]

\[ \begin{align*}
1 & x \leq -5 \\
2 & x \leq -1 \\
3 & x \geq -5 \\
4 & x \leq - \frac{1}{2} \\
5 & x \leq 5.
\end{align*} \]
Question 6

The graphical representation that correctly represents of the following set of inequalities is

\[ y \geq -\frac{1}{2}x + 2 \quad (1) \]
\[ y \leq x + 3 \quad (2) \]
\[ 0 \leq y \leq 4 \quad (3) \]
\[ x \geq 0. \quad (4) \]

[Diagram 1]

[Diagram 2]

[Diagram 3]

[Diagram 4]

[5] None of the above.
Question 7
If the demand function of a commodity is
\[ P = 70 - 0.5Q, \]
where \( P \) and \( Q \) are price and quantity respectively, determine the arc price elasticity of demand when the price decreases from R7 to R5. Indicate whether the demand is elastic or not.

[1] \( \varepsilon_d = -0.4; \) elastic
[2] \( \varepsilon_d = -1.5; \) elastic
[3] \( \varepsilon_d = 0.4; \) inelastic
[4] \( \varepsilon_d = 1.5; \) elastic
[5] None of the above.

Question 8
Price elasticity of demand measures

[1] the changes in a good’s own price.

Question 9
A furniture manufacturing company manufactures dining room tables and chairs. A table requires 540 minutes for assembly and 180 minutes for finishing. A chair requires 150 minutes for assembly and 60 minutes for finishing. The number of hours available per week for assembly is at least 150 and for finishing it is at most 100. If \( x \) is the number of tables and \( y \) the number of chairs produced per week, choose the system of inequalities that best describes the situation.

[1] \[9x + 2.5y \leq 150; \] \[3x + y \geq 100; \] \[x,y \geq 0]\]
[2] \[9x + 2.5y \geq 150; \] \[3x + y \leq 100; \] \[x,y \geq 0]\]
[3] \[2.5x + 9y \geq 150; \] \[3x + y \leq 100; \] \[x,y \geq 0]\]
[4] \[540x + 150y \geq 150; \] \[180x + 60y \leq 100; \] \[x,y \leq 0]\]
[5] None of the above.
Question 10

The supply function of a certain product is

\[ P = 50 + 2Q, \]

where \( P \) is the price and \( Q \) is the number of units produced. Find the producer surplus at market equilibrium if the market price is \( P = 90 \).

[1] 260
[2] 290
[3] 400
[4] 600
[5] None of the above.

Question 11

In the graph below the set of inequalities

\[
\begin{align*}
2x + y & \leq 120 \\
x + 2y & \leq 140 \\
x + y & \leq 80 \\
x; y & \geq 0 
\end{align*}
\]

were drawn and the feasible region of the set of inequalities shaded in grey. Determine the maximum value of the function \( P = 20x + 30y \), subject to the set of inequalities above.
[1] 1 200
[2] 1 800
[3] 2 100
[4] 2 200
[5] None of the above.

**Question 12**

Consider the market defined by the following demand and supply functions,

\[ P_d = 100 - 0.5Q \] \[ P_s = 10 + 0.5Q \]

where \( P \) and \( Q \) are the price and quantity respectively. Calculate the equilibrium price and quantity.

[1] \( P = 55; \ Q = 90 \)
[2] \( P = 90; \ Q = 55 \)
[3] \( P = 45; \ Q = 90 \)
[4] \( P = 90; \ Q = 45 \)
[5] None of the above.

**Question 13**

A company manufactures radios. If \( x \) is the number of radios that retailers are likely to purchase at a price \( p(x) = 5 - \frac{x}{1000} \) rand per unit and the cost function is given by \( c(x) = 5000 + 2x \), what is the revenue function of the manufacturing company?

[1] \( R(x) = 5 - \frac{x}{1000} \)
[2] \( R(x) = 5 - \frac{x}{1000} - 5000 + 2x \)
[3] \( R(x) = 5x - 0.001x^2 \)
[4] \( R(x) = 5000 - x \)
[5] None of the above.

**Question 14**

If the variable cost of a product increases by R4 for each unit produced and the fixed cost is R64, determine what the total cost of producing 200 units will be.

[1] R736
[2] R864
[3] R12 796
[4] R13 600
[5] None of the above.

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Question 15
The consumer surplus for the demand function $P = 60 - 4Q$ when the market price $P = 16$, is

11.2 Assignment 02 (S2)

This written assignment is compulsory.

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Instructions

- Work through Study unit 3 in the Study guide before attempting this assignment.
- Answer all the questions.
- Show your steps to find the answers. Marks will be given for these and not only for the answer.
- Submit your answers in PDF format on myUnisa.
- Always provide your answer in a full sentence with the appropriate units.
- You may use Excel or Maxima for assistance when you need to draw graphs and/or solve equations.

Only a selection of questions will be marked, but you will receive solutions to all the questions.

Question 1

Simplify:

1. \( \frac{x^2 + x - 12}{x^2 - x - 20} \).
2. \( \frac{4(0.6) K^{0.4} L^{-0.4}}{4(0.4) K^{-0.4} L^{0.6}} \).

Question 2

The demand function for a commodity is

\[ Q = 6000 - 30P. \]

Fixed costs are R72 000 and the variable costs are R60 per additional unit produced.

1. Write down the equation of total revenue and total costs in terms of \( P \).
2. Determine the profit function in terms of \( P \).
3. Determine the price at which profit is a maximum, and hence calculate the maximum profit.
4. What is the maximum quantity produced?
5. What is the price and quantity at the break-even point(s)?
Question 3
Find the quadratic profit function $P(x)$ which satisfies all of the following conditions:

1. the company breaks even when $x = 100$ and $x = 300$ units are produced.
2. the maximum profit is R40 000.

Question 4
1. Determine the roots of $f(x) = -x^2 + 6x + 9$.
2. Approximately how many units must be manufactured to maximise a profit defined by the function $f(x) = -x^2 + 6x + 9$?

Question 5
1. Find the roots and the turning point of $y = x^2 - 2x - 3$.
2. Hence draw the graph.

Question 6
Solve for $x$:
1. Simplify $5^{3x+8} = 3125$.
2. $0,0625 = 2^{-x}$.

Question 7
1. Evaluate $\log_5 \left( \frac{15}{0,45} \right)$.
2. Use rules of logarithms to solve the equation $2 \ln(3x^2) - 3 \ln x = 3$.

Question 8
The daily rate of sales of a product (in units per day) is approximated by the exponential equation

$$S(t) = 1 800 + 1 500e^{-0.3t+1.5},$$

with $t$ the number of days it has been on the market.

1. Approximately how much is the rate of sale of a product after 10 days?
2. After how many days, rounded to a whole number, will the rate of sale be 2 010 units per day?
Question 9
A virus is thought to spread through a game reserve’s impala population according to the equation
\[ N = 125.5e^{0.12t} \]
where \( N \) is the number of impala infected after \( t \) days. After approximately how many days are 200 impala infected?

Question 10
The demand and supply functions of a certain product are given by
\[ P_d = \frac{500}{Q + 1} \quad \text{and} \quad P_s = 16 + 2Q, \]
where \( P \) is the price per unit and \( Q \) the number of units produced.

1. Determine the equilibrium price and quantity for the product.
2. Draw the graph of the demand, clearly indicating the equilibrium point.
11.3 Assignment 03 (S2)

This MCQ assignment is compulsory.

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Instructions

▷ Work through Study units 4 and 5 in the Study guide before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

Question 1

Differentiate

\[ \frac{d}{dx} \left[ \frac{x - x^2}{\sqrt{x}} \right] \]

[1] \( \frac{2}{3} \sqrt{x} + \frac{1}{2 \sqrt{x}} \)

[2] \( \frac{1}{\sqrt{x}} - \frac{3}{2} \sqrt{x} \)

[3] \( \frac{3}{2} \sqrt{x} - \frac{1}{2 \sqrt{x}} \)

[4] \( \frac{3}{2} \sqrt{x} + \frac{1}{2 \sqrt{x}} \)

[5] None of the above.

Question 2

Find the derivative

\[ \frac{d}{dQ} (20 + 20 \ln Q)^2. \]

[1] \( (20 + 20 \ln Q) \)

[2] \( \frac{800 \ln Q}{Q} (1 + \ln Q) \)

[3] \( \frac{40 \ln Q}{Q} (1 + 20 \ln Q) \)

[4] \( \frac{40 \ln Q}{Q} (1 + \ln Q) \)

[5] None of the above.
Question 3
Find the derivative of

\[ \frac{1}{x^4 + \sqrt{x}} \]

[1] \(4x^3 + \frac{1}{2\sqrt{2}}\)
[2] \(\frac{4x^3 + 1}{(x^4 + \sqrt{x})^2}\)
[3] \(\frac{1}{(x^4 + \sqrt{x})^2}\)
[4] \((x^4 + \sqrt{x})^2\)
[5] None of the above.

Question 4
The derivative of

\[ \ln(x^2 + 1) + e^{-2x^3} \]

is

[1] \(\frac{2x}{x^2 + 1} - 6x^2e^{-2x^3}\).
[2] \(\frac{2x}{x^2 + 1} + 6x^2e^{-2x^3}\).
[3] \(\frac{1}{x^2 + 1} - 6x^2e^{-2x^3}\).
[4] \(\frac{1}{x^2 + 1} + e^{-2x^3}\).

Question 5
Integrate the function

\[ f(x) = \frac{2}{(2x + 1)^3}. \]

[1] \(-\frac{1}{(2x + 1)^2} + c\)
[2] \(-\frac{2}{(2x + 1)^3} + c\)
[3] \(-\frac{1}{3(2x + 1)^3} + c\)
[4] \(-\frac{1}{3(2x + 1)^3} + c\)
[5] None of the above.
Question 6
Integrate the function
\[ f(t) = \frac{t^{10} - \sqrt{t}}{t^3}. \]

\[ f(t) = t^{10} - t^{1.5} + c \]
\[ 1 \]
\[ 7t^6 + 2.5t^{-3.5} \]
\[ 2 \]
\[ \frac{t^8}{8} + \frac{2}{3\sqrt{t^3}} + C \]
\[ 3 \]
\[ \frac{10t^9 - 0.5t^{-0.5}}{3t^2} \]
\[ 4 \]
None of the above.

Question 7
Evaluate the following definite integral:
\[ \int_{x=1}^{x=5} (2x + x^3)dx \]

\[ [1] \; 69,25 \]
\[ [2] \; 132 \]
\[ [3] \; 180 \]
\[ [4] \; 180,50 \]
\[ [5] \; None of the above. \]

Question 8
Integrate:
\[ \int \frac{x + \sqrt[3]{x}}{x} \; dx. \]

\[ [1] \; x + 3\sqrt[3]{x} + c \]
\[ [2] \; x + \frac{1}{3}\sqrt[3]{x} + c \]
\[ [3] \; 1 + 3\sqrt[5]{x} + c \]
\[ [4] \; 1 - \frac{3}{5}\sqrt[5]{x^5} + c \]
\[ [5] \; None of the above. \]
Question 9
Find the values of $x$ for which the function
\[ y = -x^3 + 9x^2 - 24x + 26 \]
has a maximum or a minimum value.

[1] $x = -4; x = -2$
[2] $x = 4; x = 2$
[3] $x = 4; x = -2$
[4] $x = -4; x = 2$
[5] None of the above.

Question 10
What is the marginal cost when $Q = 10$ if the total cost is given by:
\[ TC = Q^4 - 30Q^2 + 300Q + 500? \]

[1] 3,700
[2] 4,900
[3] 4,200
[4] 6,400
[5] None of the above.

Question 11
A firm has the following total revenue and cost functions:
\[ TR = 40Q - 8Q^2 \text{ and } TC = 8 + 16Q - Q^2,\]
where $Q$ is the number of units produced and sold (in thousands). How many units should be produced to maximise profit?

[1] 1,714
[2] 3,111
[3] 1,714
[4] 3,111
[5] None of the above.
Question 12
The demand function is given by
\[ Q_d = 100 - 2P^{\frac{1}{2}}, \]
with \( P \) and \( Q \) the price and quantity respectively. Find the price elasticity of demand if \( P = 400 \) and \( Q \) assumed to be 68. What does this value mean?

1. \( \varepsilon_d = -0.29; \) inelastic since \( |\varepsilon_d| < 1, \) a 1\%, price increase will result in 0.29\% less in units demanded
2. \( \varepsilon_d = 0.29; \) elastic since \( |\varepsilon_d| > 0, \) a 1\%, price increase will result in 0.29\% more in units demanded
3. \( |\varepsilon_d| = -0.5 < 0; \) demand is inelastic
4. \( \varepsilon_d = -\frac{P^2}{Q}, which is always elastic \)
5. None of the above.

Question 13
The marginal labour cost function is given by the equation
\[ MLC = 3 + 4L. \]
Calculate the cost of employing the first seven labourers.

1. -119
2. -114
3. 114
4. 119
5. None of the above.

Question 14
Calculate the consumer surplus for the demand function
\[ P = \frac{25}{Q + 2}, \]
when the market price is \( P = 5.\)

1. 0.3
2. 1.5
3. 7.9
4. 11.5
5. None of the above.
Question 15
Demand for a certain type of bicycle is modelled by the function

\[ P = 850 - 8Q^2, \]

where \( P \) is the price per bicycle and \( Q \) the number of bicycles demanded. Find the consumer surplus when six bicycles are demanded. What does this value means?

[1] \( CS = 562; \) consumers are willing to pay R562 more per bicycles than they actually pay

[2] \( CS = -4250; \) consumers pay R4250 more for six bicycles than they should

[3] \( CS = -1152; \) consumers are willing to pay R1152 less than they actually pay

[4] \( CS = 1152; \) consumers are willing to pay R1152 more for 6 bicycles than they actually pay

[5] None of the above.