Tutorial Letter 101/3/2014

Numerical Methods 1

COS2633

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module.

BAR CODE

CONTENTS

Page

1 INTRODUCTION

Dear Student

Welcome to *Numerical Methods 1* (COS2633) which is available as part of a major in Computer Science *and* Applied Mathematics. We hope that you will find it interesting and exciting. We certainly do!
This letter (COS2633/101/3/2014) contains important information that you are going to need during the year. Please read it carefully. Another publication that can be of real help is the brochure entitled *My Studies @ Unisa*, which you received with your tutorial matter. It contains information about computer laboratories, the library, *my*Unisa, assistance with study skills, etc.

1.1 Tutorial matter

You will receive a number of tutorial letters during the semester. A tutorial letter is our way of communicating with you about teaching, learning and assessment. Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible, but is also available on *my*Unisa.

Tutorial Letter 101 contains important information about the scheme of work, resources and assignments for this module. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In Tutorial Letter 101, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed textbook and how to obtain this textbook. Please study this information carefully and make sure that you obtain the prescribed textbook as soon as possible.

Please make sure that you read *all* the tutorial matter and follow the correct procedures for submitting assignments. Also note that all tutorial matter will be downloadable from the Internet. Keep an eye on *my*Unisa and if, for whatever reason, you do not receive a printed copy of a tutorial letter in time, download it from either of the two sites, *my*Unisa and osprey.

If you have internet access, please visit our departmental website for information on the Department of Mathematical Sciences. To reach our website, follow the links on the main Unisa website, http://www.unisa.ac.za

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2 PURPOSE AND OUTCOMES FOR THE MODULE

2.1 Purpose

This module is available as part of a major in Computer Science *and* Applied Mathematics.The *abbreviated syllabus* comprises the numerical solution of nonlinear equations and systems of equations, the construction and use of interpolating polynomials, least square approximation, numerical integration and differentiation.

In this module you will learn how to develop and use numerical methods to solve mathematical problems by means of a computer. While the emphasis is on the more practical aspects, a good mathematical background is essential. We therefore advise you to include second year mathematics, in particular MAT2611 and MAT2613, in your curriculum.

The module that follows Numerical Methods 1 is, of course, Numerical Methods 2 (APM3711) which is also available as a subject in Computer Science *and* Applied Mathematics. Although numerical methods are not dependent on any specific programming language, many software packages are available as an aid to the study of numerical methods. *You are therefore expected to learn one or two programming languages (like Matlab, python, C++ or Maple) on your own and to be able to write relatively simple programs in the language.* .

2.2 Outcomes

2.2.1 Be able to draw a rough graph of any given function.

2.2.2 Solve different nonlinear equations using different numerical methods and interpret the results.

2.2.3 Solve sets of equations using different numerical methods.

2.2.4 Be able to construct interpolating polynomials and fit curves to a given data

2.2.5 Be able to perform numerical Differentiation and Integration

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3 LECTURER(S) AND CONTACT DETAILS

3.1 Lecturers

The lecturer responsible for this module is as follows:
Mr.D. C. Ikpe
Tel: (012)429 6518
Room no: 5-110
Theo van Wijk Building
e-mail: ikpecd@unisa.ac.za

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer(s). Please have your study material with you when you contact your lecturer.
E-mail and telephone numbers are included above but you might also want to write to us. Letters should be sent to:

The Module leader COS2633
Department of Mathematical Sciences
PO Box 392
UNISA
0003

PLEASE NOTE: Letters to lecturers may not be enclosed with or inserted into assignments.

3.2 Department

Fax number: 012 429 6064 (RSA) +27 12 429 6064 (International)
Departmental Secretary: 012 429 6202 (RSA) +27 12 429 6202 (International)

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *my Studies @ Unisa* that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).
Always have your student number at hand when you contact the University.
Please note that all administrative enquiries should be directed to the Contact details from *my Studies @ Unisa*. Enquiries will then be channeled to the correct department.

4 MODULE RELATED RESOURCES

The Department of Dispatch should supply you with the Study Guide and Tutorial Letter 101 at registration and others later. Full particulars regarding the suggested use of the study guide in conjunction with the textbook can be found in the study guide.
If you have access to the Internet, you can view the study guide and tutorial letters for the modules for which you are registered on the University's online campus, *my*Unisa, at http://my.unisa.ac.za

4.1 Prescribed books

Your prescribed textbook for this module for this semester is:

R.L Burden & J.D. Faires
Numerical Analysis.
Brooks/Cole Cengage Learning, 9th edition, 2010.

Please buy this book without delay. Prescribed books can be obtained from the University's official booksellers. Consult the list of official booksellers and their addresses listed in the brochure *my Studies @ Unisa*. If you have difficulty in locating your book at these booksellers, please contact the Prescribed Book Section at Tel: 012 429-4152 or email: vospresc@unisa.ac.za

4.2 Recommended books

There are no recommended books for this module:

4.3 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication *my Studies @ Unisa* that you received with your study material.

myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa all through the computer and the internet.
To go to the *myUnisa* website, start at the main Unisa website, www.unisa.ac.za, and then click on the "*myUnisa*" link below the orange tab labelled "Current students". This should take you to the myUnisa website. You can also go there directly by typing my.unisa.ac.za in the address bar of your browser.
Please consult the publication *my Studies @ Unisa* which you received with your study material for more information on *myUnisa*.

6 MODULE SPECIFIC STUDY PLAN

The sections of the ninth edition that are prescribed for examination purposes are

- Chapter 2 sections 2.1 - 2.6;
- Chapter 3 sections 3.1 - 3.7;
- Chapter 4 sections 4.1 - 4.9;
- Chapter 6 sections 6.1 - 6.5;
- Chapter 7 sections 7.1, 7.3 - 7.5;
- Chapter 8 section 8.1;
- Chapter 10 section 10.2;

Semester 1		
Period	Assignment (due date)	Textbook (9th ed.)
20/01 - 21/02	1 (26/02)	**Study** chapters 1 and 2
21/02 - 31/03	2 (18/04)	**Study** chapters 3, 4, 6,7,8.1 and 10.2
20/01 - 24/04	3 (25/04)	**Study** all chapters
26/04 - examination date : **Revision**		

Table 1: Suggested study programme for Semester 1

Semester 2		
Period	Assignment (due date)	Textbook (9th ed.)
15/07 - 05/08	1 (06/08)	**Study** chapters 1 and 2
07/08 - 16/09	2 (29/09)	**Study** chapters 3,4, 6,7,8.1 and 10.2
15/07 - 05/10	3 (06/10)	**Study** all chapters
06/10 - examination date : **Revision**		

Table 2: Suggested study programme for Semester 2

In addition to the textbook you should also study the following:

- *Tutorial letter COS2633/102/3/2014*, the use of which we discuss in its preface.
- *Tutorial letters*, which include detailed discussions and model solutions of the assignments. The assignments and the corresponding tutorial letters are important since they give you an idea of what we expect of you with respect to the *types of problems* to be solved, and their *solutions*. Please note, however, that you should not rely solely on the tutorial letters for your exam preparation. The examination covers the whole syllabus, theory as well as practice, and you should prepare accordingly. We also give *additional explanations* in these letters.

  The tutorial letters are dispatched to you in the course of the year as they become available and will also be downloadable from the Internet.
- *Inventory for the current academic year* that you received on registration and which lists the items available from the Department of Dispatch in Pretoria or the regional offices at the time of registration. Please check the tutorial matter you have received against this inventory and, if necessary, take appropriate action contacting the department of dispatch.

You should read the entire tutorial letter COS2633/102/3/2014, before working through the textbook. You should work through the sections of the prescribed textbook in the order indicated in table 1 or 2 and submit assignments 1, 2 and 3 before the respective due dates.

See the brochure *my Studies @ Unisa* for general time management and planning skills.

7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment plan

Assignments are seen as part of the learning material for this module. As you do the assignments, discuss the work with fellow students or tutors, you are actively engaged in learning. It is therefore important that you complete all the assignments.

- Assignment 1 is compulsory and will determine your exam admission. It contributes 20% towards your year mark. You should note that this assignment is a **multiple choice** assignment. The reason for this is that in the semesterised tuition model that Unisa has embraced the turnaround time on submission and marking of assignments by hand is too long to allow for more than one hand marked assignment (in this module assignment 2) in the tuition period of one semester.

  We urge you to use the following approach: Solve the problems at hand on paper on your own and obtain answers to all the problems. Keep your written paper solutions and your answers in a safe place for later use. Then check which of the multiple choice answers correspond with yours and select the allocated numbers as your answers. Submit the marking sheet in adherence to the official due date. Once you have received our discussion of the solutions you should compare your (carefully kept) paper solutions with ours to see where you made mistakes, if any, or to see what our view on the solutions are. This is an important part of your learning process. In the exam there will be no multiple choice questions.

- Assignment 2 contributes 60% towards your year mark. It should be submitted and will be marked by us.

- Assignments 3 comprises of your participation in the online discussions on the module topics throughout each semester. It will also be submitted on or before the due date and *marked* by us. This assignment contributes 20% to the year mark, provided that it is clear that you made contributions in *all* the topics that are discussed. The idea is that you share with your fellow students your understanding of the module topics and how you arrive at your solutions to the assignments. This will also give you the opportunity to evaluate your own work against the work of your fellow students and against the model solutions, which will be provided by us during the discussions. You will be informed of what exactly should be submitted **two weeks** prior to the due date.

Assignments will be assessed not only on the mathematical correctness of your work, but also on whether you use mathematical notation and language to communicate your ideas clearly. Markers will comment on the work that you submit in your assignments. The assignments and the comments constitute an important part of your learning and should help you to be better prepared for the examination.

- **Meeting due dates**: Only those assignments that reach us *before* or *on* the appropriate due date will be marked. If this date falls on a Sunday or a public holiday, then the next working day will be considered as the due date.
- **Extension date**: Because of the tight two semester schedule, we will not be able to allow an extension of the due dates
  PLEASE DO NOT PHONE TO REQUEST ANY EXTENSION OF THE ASSIGNMENT DUE DATES.
- **Marking of questions**:
  - It is the prerogative of the lecturer to decide which question(s) of any particular assignment will be marked.
  - Students will not be notified of this in advance.
  - The same question or questions will be marked for all students.
  - Only the question(s) that is(are) marked will contribute towards the year mark (%) obtained for an assignment.

  It is therefore in your own interest to attempt all questions with care!
- **Examination dates**: Make a note of the dates of your examination and make arrangements for leave in advance.

Examination admission

Due to regulations imposed by the Department of Education the following applies for COS2633: In order to be considered for examination admission in COS2633, a student **must** submit assignment 1 BEFORE the due date (26 February for Semester 1 or 06 August for Semester 2). This means that assignment 1 is **compulsory**. Students who do not comply with this condition will not be considered as "active" students, will not qualify for government subsidy to the University and will therefore not be allowed to write the examination.

You will be admitted to the examination if and only if Assignment 1 reaches the Assignment Section by 26 February 2014 if you are registered for Semester 1, or by 06 August 2014 if you are registered for Semester 2.

Please note that lecturers are not responsible for examination admission, and ALL enquiries about examination admission should be directed to the Unisa Contact Centre.

Year mark

It is a University decision that the final mark of a module should consist of an examination mark and a year mark. For COS2633 the final mark will be calculated as follows:

- The June/November examination mark will constitute 80% of the final mark and the assignments will contribute 20% to the final mark.

Assignment	Evaluated by	% of year mark
1	Lecturer	20
2	Lecturer	60
3	Lecturer	20

Table 3: Summary of assignment contributions to year mark

- The year mark will not be taken into account in any special examination results.

This will have implications for you as the student, so please make sure that you *read this section very carefully*.

Composition

In this module, assignments 1, 2 and 3 will contribute towards the year mark according to the weights 20, 60 and 20. So, if you obtain P%, Q% and R% for assignments 1, 2 and 3 respectively, then the contribution of your year mark towards your final mark will be

$$M_y = \frac{1}{5}\left[\frac{20P + 60Q + 20R}{100}\right]\%.$$

Example: If you obtain 55%, 100% and 65% for assignments 1 to 3 respectively, then your year mark will contribute

$$M_y = \frac{1}{5}\left[\frac{20(55) + 60(100) + 20(65)}{100}\right] = 16.8\%$$

towards your final mark.

The June/November exam mark will contribute 80% towards the final mark. If you obtain Y% in the June/November exam, then its contribution to your final mark will be

$$M_e = \frac{80Y}{100}\%.$$

Example: If you obtain 58% in the examination, the contribution towards your final mark will be

$$M_e = \frac{80(58)}{100} = 46.4\%.$$

The final mark: $M_{final} = (M_y + M_e)\%$.

In terms of the above examples, your final mark will be 16.8 + 46.4 = 63.2%.

Table 3 contains a summary of the contributions of the various assignments to the year mark.

8.2 General assignment numbers

The assignments are numbered as 1, 2 and 3 for each semester.

Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

Due dates of assignments

The dates for the submission of the assignments are:

	ASSIGNMENT 1	ASSIGNMENT 2	ASSIGNMENT 3
SEMESTER 1	26 February 2014	18 April 2014	25 April 2014
SEMESTER 2	06 August 2014	29 September 2014	06 October 2014

8.3 Submission of assignments

You may submit assignments either by post or electronically via myUnisa. **Assignments may not be submitted by fax or e-mail**. For detailed information and requirements as far as assignments are concerned, see the brochure *my Studies @ Unisa* that you received with your study material.
To submit an assignment via myUnisa:

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left-hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.

Also, you should NEVER submit your assignment answers directly to a lecturer, no matter what the circumstances are! (For example, even if the myUnisa website is down, do NOT email your assignment answers to a lecturer!)

8.4 Assignments

- The assignments consist mainly of *numerical problems*, which may be solved with the aid of a pocket calculator or a computer. You must not only supply the answers to problems, but also explain the methods used.
- When you solve these problems, make sure that you *understand* the underlying *theory* well, since questions on the theory will be included in the examination paper.
- The *submission* of the *answers* to the assignments must be done on the prescribed paper, stapled to an assignment cover with the front page completed. The student number, subject, module code and assignment number must also be given on each page. Assignments must be sent to the address given in *my Studies @ Unisa*. They may also be submitted electronically through *my*Unisa.

  Remember that your assignments must have exactly the *same* number as the one specified in this tutorial letter.
- If you submit your assignments *electronically*, you MUST use *standard notation*, that is, the notation of the textbook. We live in an era in which word processing tools allow for professional mathematical texts to be produced. Be warned: If you devise your own primitive, crude ”all on one line” notation, your assignment will not be marked. Remember to be RATHER SAFE THAN SORRY!

- A *discussion* of the *solutions* to the assignments is sent out to all students after the due date. They will also be available electronically.

ONLY FOR SEMESTER 1 STUDENTS
ASSIGNMENT 1
Bisection Method, Fixed Point Iteration, Newton's Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomial and Müller's Method
FIXED CLOSING DATE: 26 February 2014
UNIQUE ASSIGNMENT NUMBER: 888316

Question A

Suppose we wish to develop an iterative method to compute the square root of a given positive number c, i.e., to solve the nonlinear equation

$$f(x) = x^2 - c = 0$$

given the value of c. Each of the functions g_1 and g_2 listed next gives a fixed-point problem that is equivalent to the equation $f(x) = 0$. For each of these functions, we want to determine whether the corresponding fixed-point iteration scheme

$$x_{k+1} = g_i(x_k)$$

is (locally) convergent to $\sqrt{c}$ if $c = 5$.
Note: *do exactly 3 iterations for the $x = g(x)$ method wherever needed.*.

(1) consider $g_1(x) = c + x - x^2$.

1- $g_1(x)$ is locally convergent.
2- $g_1(x)$ would have been locally convergent if $g_1(x)$ were continuous and differentiable in an interval that include $\sqrt{5}$.
3- The interval of convergence where $| g_1'(x) |< 1$ contains $\sqrt{5}$.
4- $g_1'(x)$ is not continuous.
5- None of the above is true.

(2) consider $g_1(x) = c + x - x^2$.

1- $g_1(x)$ is not locally convergent because $g_1'(x)$ is not continuous.
2- the convergence of $g_1(x)$is not guaranteed because $g_1(x)$ and $g_1'(x)$ are contiuous but the intersection of the graphs of $y = g_1(x)$ and $y = x$ is empty.
3- the convergence of $g_1(x)$is not guaranteed because the interval of convergence where $| g_1'(x) |< 1$does not contain $\sqrt{5}$.
4- $g_1'(x)$ is not continuous.
5- None of the above is true.

(3) consider $g_2(x) = 1 + x - \frac{x^2}{c}$.

1- the convergence of $g_2(x)$is guaranteed because the interval of convergence where $\mid g_2'(x) \mid < 1$contains $\sqrt{5}$.

2- the convergence of $g_2(x)$is not guaranteed because $g_2(x)$ and $g_2'(x)$ are contiuous but the intersection of the graphs of $y = g_2(x)$ and $y = x$ is empty.

3- the convergence of $g_2(x)$is not guaranteed because $g_2'(x)$ is not continuous.

4- $g_2'(x)$ is not continuous.

5- None of the above is true.

(4) consider $g_2(x) = 1 + x - \frac{x^2}{c}$

1- there is no guarantee on the convergence of $g_2(x)$.

2- the convergence of $g_2(x)$would have been guaranteed if $g_2(x)$ were continuous and differentiable in an interval that include $\sqrt{5}$.

3- the convergence of $g_2(x)$ is not guaranteed because the interval of convergence where $\mid g_2'(x) \mid < 1$ does not contains $\sqrt{5}$.

4- $g_2'(x)$ is not continuous.

5- None of the above is true.

(5) the fixed-point function, $g_3(x)$, given by Newton's method for $f(x) = x^2 - 5$ is:

1- $g_3(x) = g_1(x)$.

2- $g_3(x) = x - \frac{x^2-5}{2x}$.

3- $g_3(x) = g_2(x)$.

4- $g_3(x) = x - \frac{g_1(x)}{g_2(x)}$.

5- None of the above.

Question B

The function $f(x) = 4x^3 - 1 - \exp^{(x^2/2)}$ has values of zero near $x = 1.0$ and $x = 3.0$.

(6) What is the derivative of f?

1- $f'(x) = 12x^2 - 2xe^{(\frac{x^2}{2})}$.

2- $f'(x) = 3x^2 - xe^{(\frac{x^2}{2})}$.

3- $f'(x) = 12x^2 - xe^{(\frac{x^2}{2})}$.

4- $f'(x) = 4x^3 - xe^{(\frac{x^2}{2})}$.

5- None of the above.

(7) What happens if you begin Newton's method at $x = 1.5$?

1- nothing happens.

2- both roots near, 1.0 and 3.0, are reached.

3- the root near 1.0 is reached.

4- the root near 3.0 is reached

5- None of the above.

(8) select the appropriate answer.

1- no root is reached with the starting point in (6) and the root 1.0 can be reached if the starting point is greater than 2.5.

2- any root is reached if the starting point is not null.

3- the root 3.0 can never be reached.

4- one root is reached in the case of (6) above, and the other can be reached if the starting point is greater than 2.75.

5- the root 1.0 can never be reached.

(9) with an appropriate starting point,

1- the zero near 1.0 is reached and three iterations are needed to achieve an error of 10^{-6}

2- the zero at 3.0 is reached and at least nine iterations are needed to achieve an error of 10^{-5}

3- both roots are reached and at least ten iterations are needed to achieve an error of 10^{-5}

4- the zero near 1.0 is reached and at least five iterations are needed to achieve an error of 10^{-5}

5- at least one both roots is reached but, the number of iterations is too high to be computable.

Consider the equation

$$x(x-1) - e^x = 0 \tag{1}$$

(10) A sketch curve of the function(s) in equation (1) roughly shows that:

1- the equation has no root.

2- the equation has two roots at approximately -2 and 0.5.

3- the equation has one root at approximately -2.

4- the equation has one root at approximately -1.5.

5- none of the above is true.

ONLY FOR SEMESTER 1 STUDENTS

ASSIGNMENT 2

Linear Systems of Equations, Linear Algebra,Pivoting Strategies and Matrix Factorization, Jacobi and Gauss-Seidel Iterative Techniques and Error Bounds, Interpolation and Lagrange Polynomial,Data Approximation, Hermite Interpolation,Divided Difference, Cubic Splines, Parametric Curves and Discrete Least Squares Approximation, Numerical Differentiation and Integration.

FIXED CLOSING DATE: 18 April 2014

UNIQUE ASSIGNMENT NUMBER: 859458

Question 1

Consider the linear system

$$\begin{array}{rcrcrcrcl} 0.06x_1 & + & 0.08x_2 & + & 0.07x_3 & + & 0.08x_4 & = & 0.29 \\ 0.08x_1 & + & 0.20x_2 & + & 0.09x_3 & + & 0.07x_4 & = & 0.44 \\ 0.07x_1 & + & 0.09x_2 & + & 0.20x_3 & + & 0.10x_4 & = & 0.46 \\ 0.06x_1 & + & 0.08x_2 & + & 0.10x_3 & + & 0.20x_4 & = & 0.44 \end{array}$$

(a) Write the system in matrix notation. (2)

(b) What is the true solution of the system? (2)
Solve the system using

(c) Gaussian elimination without pivoting. (4)

(d) Gaussian elimination with pivoting. (4)

(e) LU decomposition. (4)

(f) The Jacobi method. Do three iterations, starting at $x_0 = (0, 0, 0, 0)^T$. (4)

(g) The Gauss-Seidel method. Do three iterations, starting at $x_0 = (0, 0, 0, 0)^T$. (4)

(h) Solve the system using Successive Over-Relaxation technique.
Let $w = 0.5$ and $x_0 = (0, 0, 0, 0)^T$. (4)

(i) Which method do you prefer and why? (2)

Use four-decimal arithmetic with truncation (*not* rounding) **[30]**

Question 2

Find the inverse of

$$\begin{bmatrix} 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \\ 2 & -1 & 3 & 1 \\ 3 & -1 & 4 & 3 \end{bmatrix}.$$

Use Gaussian elimination and work *exactly*. **[6]**

Question 3

Write a computer program which uses Newton's method to obtain the solutions, to within 10^{-5}, of the pair of simultaneous equations

$$\begin{aligned} x^2 + y^2 &= 4 \\ xy &= 1. \end{aligned}$$

Draw a rough graph to obtain good starting values. Compute all the solutions.

[8]

Question 4

Consider the following data:

x	$f(x)$
1	1
2	8
3	27
4	64
5	125
6	216

(a) Set up a difference table through fourth differences. (3)

(b) What is the minimum degree that an interpolating polynomial, that fits all six data points exactly, can have? Explain. (3)

(c) Give the (forward) Newton-Gregory polynomial that fits the data points with x values 2, 3, and 4. Then compute $f(3.5)$. (3)

(d) Compute an approximate bound for the error in the approximation to $f(3.5)$ in (c) using Newton's forward interpolating polynomial. (3)

(e) Compute $f(3.5)$ using the Lagrange interpolating polynomial through the data points with x values 2, 3, and 4. (3)

[15]

Question 5

If the following four points are connected in order by straight lines, a zigzag line is created:

$$(0, 0), (1, 0.3), (2, 1.7), (3, 1.5).$$

(a) Using the two interior points as controls, find the cubic Bezier curve. Plot this together with the zigzag line. (3)

(b) Use this cubic equation to find interpolates at $x = 0.5$, $x = 0.75$, and $x = 2.5$. How close are these to the zigzag line? (5)

(c) If the second and third points (the control points) are moved, the Bezier curve will change. If these are moved vertically, where should they be located so that the Bezier curve passes through all the original four points? (5)

[13]

Question 6

Compute

$$\int_0^1 \frac{\sin x}{x} dx$$

by means of

(a) the trapezoidal rule, with $h = \frac{1}{4}$; (3)

(b) a three-term Gaussian quadrature formula; (3)

(c) Simpson's $\frac{1}{3}$ rule. (3)

(d) Estimate the respective truncation errors in (a) and (c). (3)

(e) Determine the integral analytically and then compute the actual errors in (a), (b) and (c), respectively. [**Hint**: Use the Taylor series expansion of $\sin x$.] (6)

[18]

Question 7

Consider the function $f(x) = x \cos x$. Apply Richardson extrapolation to obtain $f''(0.32)$ accurate to five significant figures. Start with $h = 0.1$ and use central differences. **[12]**

Question 8

Consider the following table:

x	y
4.0	102.56
4.2	113.18
4.5	130.11
4.7	142.05
5.1	167.53
5.5	195.14
5.9	224.87
6.3	256.73
6.8	299.50
7.1	326.72

.

(a) Construct the least squares approximation polynomial of degree three and compute the error. (5)

(b) Construct the least squares approximation of the form ke^{mx} and compute the error. (5)

(c) Construct the least squares approximation of the form kx^{m} and compute the error. (5)

(d) Draw a graph of the data points and the approximations in (a), (b) and (c). (5)

[20]

TOTAL: [120]

ONLY FOR SEMESTER 1 STUDENTS

ASSIGNMENT 3

All Topics:

FIXED CLOSING DATE: 25 April 2014

UNIQUE ASSIGNMENT NUMBER: 827986

Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.

TOTAL: [100]

ONLY FOR SEMESTER 2 STUDENTS
ASSIGNMENT 1
Bisection, Secant and Regula Falsi Methods, Fixed-Point Iteration, Newton's Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomial and Müller's method
FIXED CLOSING DATE: 06 August 2014
UNIQUE ASSIGNMENT NUMBER: 823114

Question A

We consider two curves $y_1 = (\frac{20}{x^2})\sin(\frac{10}{x})$ and $y_2 = 5\cos x$ in the interval $(1, 3)$.

(1) to draw the (rough) graph of a given function, the following is sufficient:

1- consider the existence, discontinuities and singularities of the function.
2- consider the symmetries and boundedness of the function.
3- consider the behavior at very small and very large values of x and behaviour atzero.
4- consider the roots, zeros and turning points;
5- none of the above is sufficient unless we consider them all together and when necessary consider other points.

(2) the rough graph of y_1 and y_2 reveals the following:

1- the two curves have no point of intersection.
2- the two curves intersect in only one point.
3- the two curves intersect in two points.
4- the two curves intersect in three points.
5- the two curves have an infinite points of intersections but, only three in the interval $(1, 3)$.

(3) the rough graph of y_1 and y_2 in the interval $(1, 3)$ reveals that,

1- we cannot locate any intersection point.
2- we can locate more than three intersection points including 1.1, 1.4, and 1.6.
3- the two curves intersect in exactly two points (roughly) located at 1.2 and 1.3.
4- the two curves intersect in exactly three points (rougly) located at 1.1, 1.6, and 2.3.
5- the two curves intersect in exactly three points (roughly) located at 2.3, 2.5, and 2.7.

We now consider solving the equation

$$(\frac{20}{x^2})\sin(\frac{10}{x}) = 5\cos x \qquad (2)$$

by means of $x = g(x)$ method.

(4) to solve equation 2, we need to be performed following:

1- determine any function $g_i(x)$ that gives a fixed-point problem that is equivalent to equation 2.

2- determine all possible $g_i(x)$ that give a each, a fixed-point problem that is equivalent to equation 2.

3- solve directly the equation.

4- consider 2- above, and carefully analyse the convergence of each $g_i(x)$ in the given interval.

5- consider 4- above, but by applying it to appropriate starting points.

(5) the following $g_i(x)$ give(s),each, a possible fixed-point problem that is equivalent to equation 2:

1- $g_1(x) = \cos^{-1}(4x^2)\sin(\frac{10}{x})$.

2- $g_1(x) = \cos^{-1}(4x^2)\sin(\frac{10}{x})$ and $g_2(x) = \sqrt{\frac{4\sin(\frac{10}{x}}{\cos(x)}}$.

3- $g_1(x) = \cos^{-1}(\frac{4}{x^2})\sin(\frac{10}{x})$, $g_2(x) = \frac{10}{\sin^{-1}(\frac{x^2}{4})\cos(x)}$ and $g_3(x) = \sqrt{\frac{4\sin(\frac{10}{x}}{\cos(x)}}$.

4- 3- above but, without $g_1(x)$.

5- none of the above.

(6) a careful analysis of each possible fixed-point function of equation 2 with $x_0 = 0.5$,$x_0 = 1.5$ and $x_0 = 2.5$ yields to the conclusion that:

1- equation 2 has no solution.

2- the $x = g(x)$ method is not appropriate to solve equation 2.

3- the $x = g(x)$ method is useful to solve equation 2 but, the starting values are not appropriate.

4- equation 2 is not a valid equation.

5- none of the above is true.

(7) in practice Aitken' acceleration is a useful technique.

1- applying it to equation 2 will improve the result obtained in (f).

2- applying it to equation 2 is pointless since the $x = g(x)$ method does not help in the case of equation 2.

3- applying it to equation 2 is pointless since the result from the $x = g(x)$ method is error free.

4- applying it to equation 2 can help to produce new results where the $x = g(x)$ method failed.

5- none of the above is true.

Question B

Consider the function $f(x) = (\frac{20}{x^2})\sin(\frac{10}{x}) - 5\cos x$ in the interval $(1, 3)$.

(8) which one of the following statement is true?

1- the Newton's method is the general form of the $x = g(x)$ method.

2- the Newton's method is a similar but different technique from the $x = g(x)$ method.

3- the Newton's method is a special case of the $x = g(x)$ method, with $g(x) = x - \frac{f(x)}{f'(x)}$.

4- the statement in 3 above is true only in some special cases.

5- none of the above is true.

consider the equation

$$f(x) = 0 \tag{3}$$

in the interval $(1, 3)$

(9) applying the Newton's method to equation 3,

1- the convergence is not guaranteed since the $x = g(x)$ failed.

2- the convergence is guaranteed for one starting value.

3- the convergence is guaranteed for two starting values.

4- the convergence is guaranteed for three starting values.

5- the convergence is guaranteed for four starting values.

(10) applying the Newton's method to equation 3, with the starting value $x_0 = 0.5$ yields:

1- no root since the convergence is not guaranteed.

2- a root closest to 0.5, namely $x = 0.528821$, after three iterations.

3- a root closest to 0.5, namely $x = 0.498212$, after three iterations.

4- no root since the interval of convergence does not contain 0.5.

5- none of the above is true.

ONLY FOR SEMESTER 2 STUDENTS

ASSIGNMENT 2

Linear systems of Equations, Linear Algebra,Pivoting Strategies and Matrix factorization, Jacobi and Gauss-Seidel Iterative Techniques and Error Bounds, Interpolation and Lagrange Polynomial,Data Approximation, Hermite Interpolation,Divided Difference, Cubic Splines, Parametric Curves and Discrete Least square Approximation, Numerical Differentiation and Integration..

FIXED CLOSING DATE: 29 September 2014

UNIQUE ASSIGNMENT NUMBER: 888579

Question 1

Consider the linear system

$$\begin{array}{ccccccccc} 0.06x_1 & + & 0.08x_2 & + & 0.07x_3 & + & 0.08x_4 & = & 0.29 \\ 0.08x_1 & + & 0.20x_2 & + & 0.09x_3 & + & 0.07x_4 & = & 0.44 \\ 0.07x_1 & + & 0.09x_2 & + & 0.20x_3 & + & 0.10x_4 & = & 0.46 \\ 0.06x_1 & + & 0.08x_2 & + & 0.10x_3 & + & 0.20x_4 & = & 0.44 \end{array}$$

(a) Write the system in matrix notation. (2)

(b) What is the true solution of the system? (2)
Solve the system using

(c) Gaussian elimination without pivoting. (4)

(d) Gaussian elimination with pivoting. (4)

(e) LU decomposition. (4)

(f) The Jacobi method. Do three iterations, starting at $x_0 = (0, 0, 0, 0)^T$. (4)

(g) The Gauss-Seidel method. Do three iterations, starting at $x_0 = (0, 0, 0, 0)^T$. (4)

(h) Solve the system using Successive Over-Relaxation technique.
Let $w = 0.5$ and $x_0 = (0, 0, 0, 0)^T$. (4)

(i) Which method do you prefer and why? (2)

Use four-decimal arithmetic with truncation (*not* rounding) **[30]**

Question 2

Augment the coefficient matrix with all three of the right-hand sides and get all three solutions to $Ax = b_i,\ i = 1, 2, 3$, simultaneously, given

$$A = \begin{bmatrix} 4 & 2 & 1 & -3 \\ 1 & 2 & -1 & 0 \\ 3 & -1 & 2 & 4 \\ 0 & 2 & 4 & 3 \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 4 \\ 2 \\ 8 \\ 9 \end{bmatrix},\ b_2 = \begin{bmatrix} 9 \\ 1 \\ 8 \\ 4 \end{bmatrix},\ b_3 = \begin{bmatrix} 4 \\ 2 \\ -7 \\ 5 \end{bmatrix}.$$

[6]

Question 3

Write a computer program which uses Newton's method to obtain a solution, accurate to four decimal digits, of the pair of simultaneous equations

$$\begin{aligned} f(x,y,z) &= x^2 + y^2 + z^2 - 1 &= 0 \\ g(x,y,z) &= x^2 + z^3 - 0.25 &= 0 \\ h(x,y,z) &= x^2 + y^2 - 4z &= 0 \end{aligned}$$

taking $x = 1$, $y = 1$ and $z = 1$ as the initial values in the iterative process, compute three iterations of the solution. **[8]**

Question 4

Use the table below to find a value for $y(0.54)$ using a cubic polynomial that fits at $x = 0.3$, 0.5, 0.7, and 0.9.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0.1	0.003			
		0.064		
0.3	0.067		0.017	
		0.081		0.002
0.5	0.148		0.019	
		0.100		0.003
0.7	0.248		0.022	
		0.122		0.004
0.9	0.370		0.026	
		0.148		0.005
1.1	0.518		0.031	
		0.179		
1.3	0.697			

[15]

Question 5

The equation $f(x) = e^x - 3x^2 = 0$ has three real roots. An obvious rearrangement is

$$x = \pm\sqrt{\frac{e^x}{3}}.$$

Show that convergence is to the root near -0.5 if we begin with $x_0 = 0$ and use the negative value. Show also that convergence to a second root near 1.0 is obtained if $x_0 = 0$ and the positive value is used. Show, however, that this form does not converge to the third root near 4.0 even though a starting value very close to the root is used. Find a different rearrangement that will converge to the root near 4.0. **[18]**

Question 6

Evaluate the integral

$$\int_{-0.2}^{1.4}\int_{0.4}^{2.6} e^x \sin(2y)dydx$$

and in each case ((a), (b) and (c)) compare your answers to the analytical solution. Use $h = 0.1$ in both directions in parts (a) and (b):

(a) Use the trapezoidal rule in both directions,

(b) Use Simpson's $\frac{1}{3}$ rule in both directions,

(c) Use Gaussian quadrature, three-term formulas, in both directions.

[13]

Question 7

If these points are connected in order by straight lines, a zigzag line is created: (0, 0), (1, 0.3), (2, 1.7), (3, 1.5

(a) Using the two interior points as guide points, find the cubic Bézier curve plot this with the zigzag line. (6)

(b) Using the cubic equation in [(*a*)] to find the interpolates at $x = 0.5$, $x = 0.75$ and $x = 2.5$. How close are these to the zigzag line? (6)

(c) If the guide points are moved, the Bézier curve will change. if these are moved vertically, where should they be located so that the Bézier curve passes through all of the original four points? (6)

[18]

Question 8

Find the least squares straight line that fits the following data, assuming that the y-values are free from error.

x	y
1	5.04
2	8.12
3	10.64
4	13.18
5	16.20
6	20.04

.

Modify the normal equations to get the least squares line $x = ay + b$. **[12]**

TOTAL: [120]

ONLY FOR SEMESTER 1 STUDENTS
ASSIGNMENT 3
All Topics:
FIXED CLOSING DATE: 06 October 2014
UNIQUE ASSIGNMENT NUMBER: 864605

Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.

TOTAL: [100]

9 OTHER ASSESSMENT METHODS

There are no other assessment methods for this module.

10 EXAMINATIONS

This module is offered in a semester period of fifteen weeks. This means that if you are registered for the first semester, you will write the examination in May/ June 2014 and the supplementary examination will be written in October/ November 2014. If you are registered for the second semester you will write the examination in October/ November 2014 and the supplementary examination will be written in May/ June 2015.

During the relevant semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.

For general information and requirements as far as examinations are concerned, see the brochure *my Studies @ Unisa* which you received with your study material.

Examination paper

The exam is a two hour exam. You are allowed to use a **non-programmable calculator** in the exam. The examination questions will be similar to the questions asked in the study guide and in the assignments. You need not know any proofs of theorems in this module, however, you have to be able to apply the formulas stated in the theorems and definitions.
Tutorial letter with information on the examination.

To help you in your preparation for the examination, you will receive a tutorial letter that will explain the format of the examination paper, and set out clearly what material you have to study for examination purposes. This tutorial letter will also contain a previous exam and the solutions for that exam.

11 FREQUENTLY ASKED QUESTIONS

The *my Studies @ Unisa* brochure contains an A–Z guide of the most relevant study information.

12 SOURCES CONSULTED

All the sources consulted in the compilation of this tutorial letter are acknowledged in the body of this letter.

13 CONCLUSION

In conclusion we hope that you found this module interesting and enjoyable. We wish you the best of luck with the examination and look forward to welcoming you to APM3711, which covers most of the remaining topics in numerical methods for your undergraduate studies next year!