Formal logic 3

Tutorial letter 201 (first semester)
for
COS3761

Solutions to Assignment 1

School of Computing

.

TUTORIAL MATTER

SOLUTION TO ASSIGNMENT 1

Question	Semester 1
1	3
2	4
3	3
4	2
5	2
6	3
7	4
8	1
9	2
10	2
11	4
12	3
13	1
14	4
15	3
16	4
17	2
18	3
19	5
20	4

- The prescribed book for Formal Logic 2 (module COS2661or COS261C) is Barwise, J. & J. Etchemendy. 1999, 2000, 2002, 2003. *Language, proof and logic.* Stanford: Center for the Study of Language and Information. ISBN: 1-57586-374-X.
- The prescribed book for Formal Logic 3 (module COS3761or COS361F) is Logic in Computer Science: Modelling and Reasoning about Systems by Michael Huth and Mark Ryan. 2004. ISBN: 0 521 54310 X paperback. Second edition.

QUESTION 1 option 3

You should make sure that you know how "only if" is translated into FOL.

QUESTION 2 option 4

Consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of "unless".

QUESTION 3 option 3

"If and only if" is translated with $\leftrightarrow$.

QUESTION 4 option 2

Hopefully this is quite clear. If not, consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of "only if".

QUESTION 5 option 2

Consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of "unless".

QUESTION 6 option 3

Hopefully this is quite clear.

QUESTION 7 option 4

$p \rightarrow q \wedge r \vdash (p \rightarrow q) \wedge (p \rightarrow r)$

1	$p \rightarrow q \wedge r$	premise
2	p	assumption
3	$q \wedge r$	$\rightarrow$ e 1, 2
4	q	$\wedge$ e 3
5	$p \rightarrow q$	$\rightarrow$ i 2-4
6	p	assumption
7	$q \wedge r$	$\rightarrow$ e 1, 6
8	r	$\wedge$ e 7
9	$p \rightarrow r$	$\rightarrow$ i 6-8
10	$(p \rightarrow q) \wedge (p \rightarrow r)$	$\wedge$ i 5, 9

QUESTION 8 option 1

The second line of option 2 is already incorrect because an assumption has to be the first step inside a subproof.
The rule in line 5 of option 3 is incorrect.
The rule in line 5 of option 4 is incorrect and line 6 should be outside the subproof.

QUESTION 9 option 2

Option 1 is incorrect because there is no premise.
Option 3 is incorrect because $\neg p$ should be assumed in line 1, not p.
Option 4 is incorrect because $p \rightarrow (p \rightarrow q)$ should be derived in line 7, not $\neg p \rightarrow (p \rightarrow (p \rightarrow q))$.

QUESTION 10 option 2

Unfortunately three options were numbered "Option 2", but hopefully you realised what they should have been – you are after all third level students. It was also announced on myUnisa.

If the answer is not clear, study the basic natural deduction rules again. They are summarised on page 27 of your textbook. Note that the "$\neg$ e rule" was called the "$\perp$ Intro" rule in the textbook of Formal Logic 2.

QUESTION 11 option 4

If this is not clear, study the basic natural deduction rules again. They are summarised on page 27 of your textbook (and from page 557 of the textbook of Formal Logic 2). Note again that the "$\neg$ e rule" was called the "$\perp$ Intro" rule in the textbook of Formal Logic 2.

QUESTION 12 option 3

Remember that $\wedge$ and $\vee$ binds more tightly than $\rightarrow$.

The main connective of the formula is $\rightarrow$: $(((s \rightarrow (r \vee t)) \vee ((\neg q) \wedge r)) \rightarrow ((\neg(p \rightarrow s)) \rightarrow r))$
The main connective of the right hand side $((\neg(p \rightarrow s)) \rightarrow r)$ is again $\rightarrow$.
The main connective of the left hand side $((s \rightarrow (r \vee t)) \vee ((\neg q) \wedge r))$ is $\vee$.

QUESTION 13 option 1

Make sure that you agree with this.

QUESTION 14 option 4

QUESTION 15 Option 3

QUESTION 16 option 4

The sequent would be valid only if the right hand side formula is true for *all* valuations that make all the left hand side formulas true. In this case the valuation indicated by option 4 makes both left hand side formulas true but the right hand side formula false, thus we have a counter-example. Option 3 is incorrect – the valuation given there is not a counter-example.

QUESTION 17 option 2

Note that option 4 does not involve the entailment relation $\models$.

See page 46 of your textbook for an explanation of semantic entailment for propositional logic.

p	**q**	**r**	$\neg$ **r**	$\neg$ **r** $\rightarrow$ **p**
T	T	T	F	T
T	T	F	T	T
T	F	T	F	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

p	**q**	**r**	$\neg$ **r**	$\neg$ **r** $\wedge$ **q**
T	T	T	F	F
T	T	F	T	T
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T
F	F	T	F	F
F	F	F	T	F

p	**q**	**r**	**p ∨ q**	**p ∨ q → r**
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

QUESTION 18 option 3

See page 46 of your textbook for an explanation of the semantic entailment relation $\models$.

QUESTION 19 option 5

After the first step we have

$(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s)$

Now we mark all occurrences of

r (because of $\underline{T} \rightarrow r$)

and q (because of $\underline{T} \rightarrow q$

Thus

$(p \wedge \underline{q} \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (\underline{r} \rightarrow p) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{r} \wedge u \rightarrow w) \wedge (u \rightarrow s)$

None of options 1 to 4 is correct.

QUESTION 20 option 4

Option 1: Incorrect: any valuation (in propositional logic) is a model.
Option 2: Incorrect: a sequent is valid if all valuations that make the premises true, also make the conclusion true.
Option 3: Incorrect: a formula is semantically entailed by other formulas if the formula is true whenever the other formulas are all true.
Option 4: Correct: propositional logic is complete.