Memo 201/0/2018

Control Systems III (Theory)

CSY3601

Year Module

Department of Electrical and Mining Engineering

IMPORTANT INFORMATION:

Only prescribed book will be allowed in the examination venue.

BAR CODE

ASSIGNMENT 1 (monomial-choice)

Question 1

Answer: 1)

Question 2

Answer: 2)

Question 3

Answer: 2)

Question 4

Answer: 4)

Question 5

Answer: 3)

Question 6

Answer: 1)

Question 7

Answer: 4)

Question 8

Answer: 2)

Question 9

Answer: 1)

Question 10

Answer: 2)

ASSIGNMENT 2

Question 1

Use block-diagram-reduction techniques to find the transfer function $T(s) = \frac{C(s)}{R(s)}$ of the system represented by the block diagram shown in Figure 2.1.

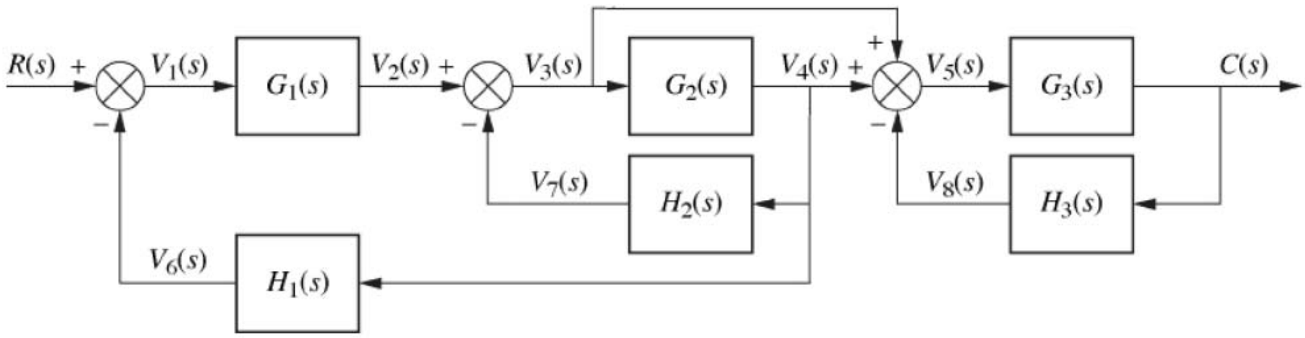


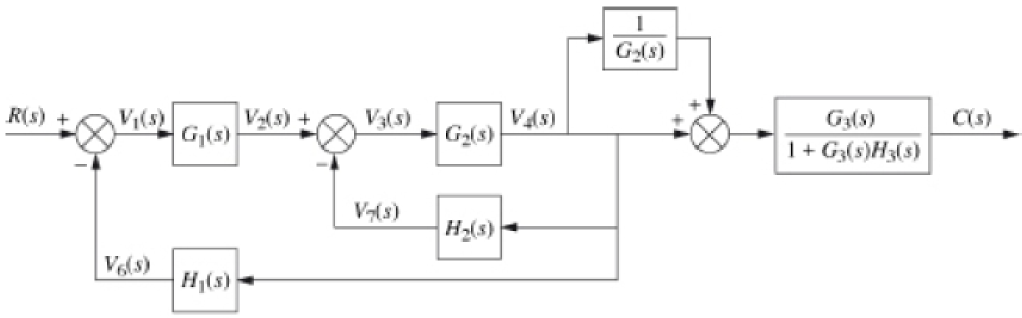
Figure 2.1 Block diagram for Question 1

[20]

Answer:

Rightmost feedback loop can be reduced

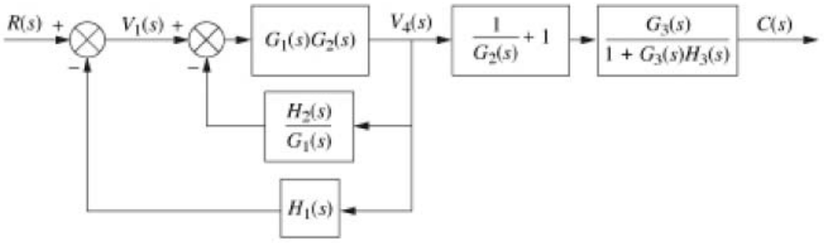
Create parallel form by moving G_2 left



✓✓✓✓✓

Reduce parallel form involving $1/G_2$ and unity

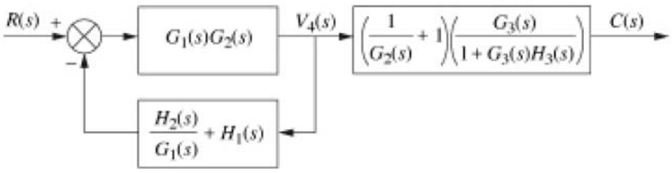
Push G_1 to the right past the summing junction to create a parallel form in the feedback path



✓✓✓✓✓

Reduce parallel form on left

Recognize cascade form on right



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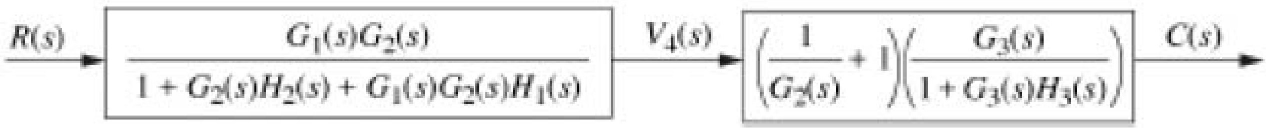
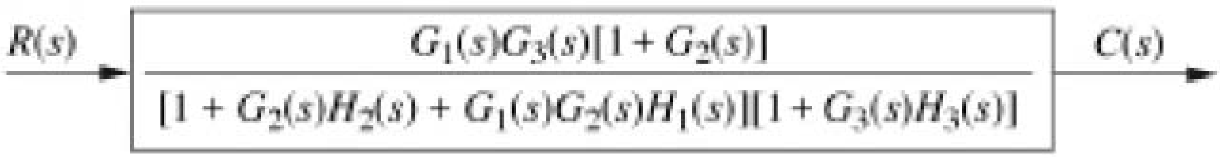




✓✓✓✓✓

Question 2

Convert the block diagram in Figure 2.2 to a signal-flow graph and use Mason's Rule to find the transfer function $T(s) = \dfrac{C(s)}{R(s)}$

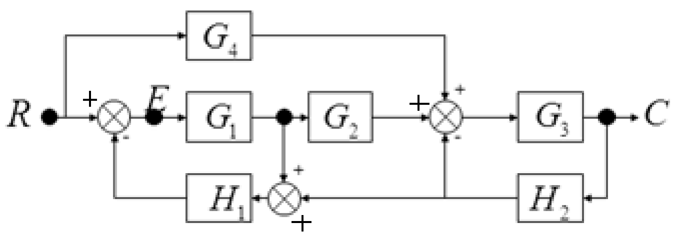


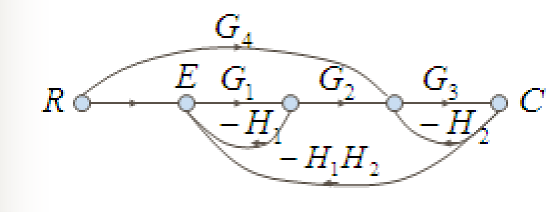
Figure 2.2 Block diagram for Question 2

Hint: use the website materials to find how you can use Mason's gain formula.

[30]

Answer:

Connect the node and label the subsystems



✓✓✓✓✓✓✓✓

Forward paths:

$T_1 = G_1G_2G_3$ ✓✓, $T_2 = G_3G_4$ ✓✓

Loops:

$L_1 = -G_1H_1$, ✓✓

$L_2 = -G_3H_2$ ✓✓

$L_3 = -G_1G_2G_3H_1H_2$ ✓✓

There are two non-touching loops

$\Delta = 1 - \sum L_a + \sum L_b L_c$ ✓✓

$\Delta_1 = 1$, $\Delta_2 = 1 + G_1H_1$ ✓✓✓✓

$$T = \frac{1}{\Delta}\sum_{k=1}^{2} T_k \Delta_k$$
$$= \frac{G_1G_2G_3 + G_3G_4 + G_1G_3G_4H_1}{1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_1H_2 + G_1G_3H_1H_2}$$ ✓✓✓✓✓✓

Question 3

Solve the following differential equation using Laplace transform

$$\frac{d^2x}{dt^2} + 4x = t^2$$
$$x(0) = 2, x'(0) = 3$$

[20]

Answer:

$$s^2X(s) - sx(0) - x^{'}(0) + 4X(s) = \frac{2}{s^3}$$

$$s^2X(s) - 2s - 3 + 4X(s) = \frac{2}{s^3}$$

$$X(s) = \frac{2}{s^3(s^2+4)} + \frac{2s}{(s^2+4)} + \frac{3}{(s^2+4)}$$

(10)

Since

$$\ell[\int_0^t f(t)dt] = F(s)/s$$

and $\ell[\omega t - \sin \omega t] = \dfrac{\omega^3}{s^2(s^2+\omega^2)}$,

$$\ell^{-1}[\frac{2}{s^3(s^2+4)}] = \ell^{-1}[\frac{1}{4s}\frac{2^3}{s^2(s^2+4)}] = \frac{1}{4}[\int_0^t (2t - \sin 2t)dt = \frac{1}{4}t^2 + \frac{1}{8}\cos 2t - \frac{1}{8}.$$

$$\ell^{-1}[\frac{2s}{(s^2+4)}] = 2\cos 2t$$

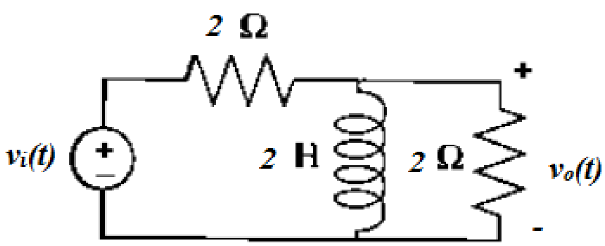
$$\ell^{-1}[\frac{3}{(s^2+4)}] = \frac{3}{2}\sin 2t$$

Hence $x(t) = \dfrac{1}{4}t^2 + \dfrac{17}{8}\cos 2t + \dfrac{3}{2}\sin 2t - \dfrac{1}{8}$

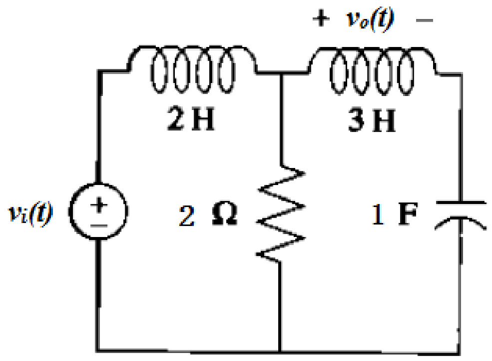
(10)

Question 4

Find the transfer functions, $V_O(s)/V_i(s)$,for the following circuits.



(a) Circuit 1 for Question 4



(b) Circuit 2 for Question 4

[30]

Answer:

(a)

$$Z_L(s) = Ls = 2s$$ ✓✓

$$Z_L // R_{2\Omega} = \frac{2s}{1+s}$$

$$V_o = \frac{Z_L // R_{2\Omega}}{2 + Z_L // R_{2\Omega}} V_i = \frac{s}{2s+1} V_i$$

$$\frac{V_o}{V_i} = \frac{s}{2s+1}$$

✓✓✓✓✓✓✓✓✓✓✓✓✓

(b)

$$Z_{L1} = 2s, Z_{L2} = 3s, Z_C = \frac{1}{s}$$ ✓✓✓

$$Z_2 = Z_{2\Omega} // Z_{L2C} = \frac{1}{\frac{1}{2} + \frac{1}{3s + \frac{1}{s}}} = \frac{6s^2 + 2}{3s^2 + 2s + 1}$$

✓✓✓✓✓

$$V_o = \frac{Z_2}{2s + Z_2} \frac{3s}{3s + \frac{1}{s}} V_i = \frac{3s^2}{3s^3 + 5s^2 + s + 1} V_i$$

$$\frac{V_o}{V_i} = \frac{3s^2}{3s^3 + 5s^2 + s + 1}$$

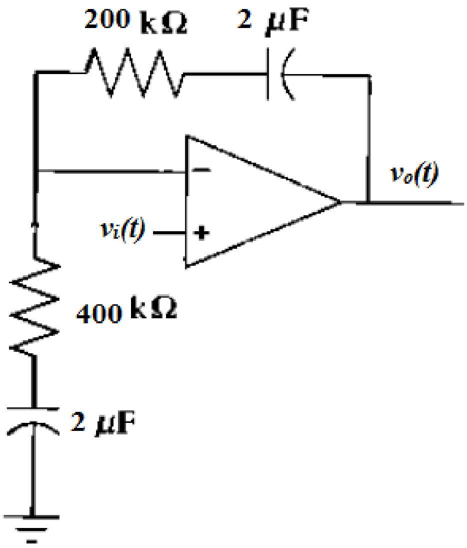
✓✓✓✓✓✓✓

Total: 100

ASSIGNMENT 3

Question 1

Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for the operational amplifier circuits as shown in the following figure.



[12]

Answer:

$$Z_{C1} = Z_{C2} = \frac{1}{2\times10^{-6}s}$$

$$I_{C1} = \frac{V_i}{400*10^3 + \frac{1}{2\times10^{-6}s}}$$

$$V_o = (R_1 + R_2 + Z_{C1} + Z_{C2})I_{C1} = \frac{6s+10}{4s+5}V_i$$

$$\frac{V_o}{V_i} = \frac{6s+10}{4s+5}$$

Question 2

Assume that the motor, whose transfer functions is shown in the following figures, is used as the forward path of a closed-loop, unity feedback system.

a. Calculate the percent overshoot and settling time that could be expected.

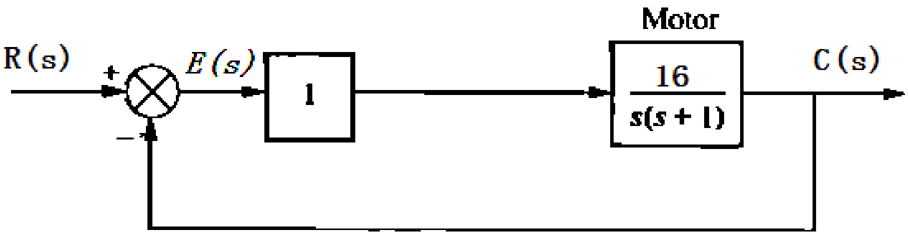
(15)

b. Find the values of K_1 and K_2 to yield a 14% overshoot and a settling time of 0.3 second.

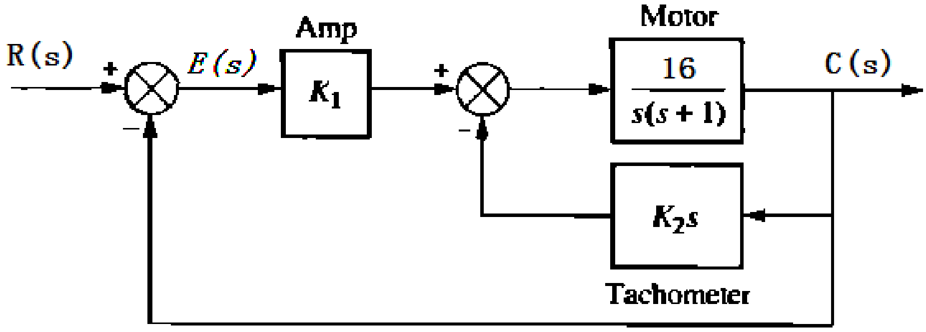
(17)

[32]

Here 5% criterion is used.



(a)



(b)

Answer:

a.

$$C/R = \frac{16}{s^2 + s + 16}$$

$$\omega_n = 4rad/s; \zeta = 0.125$$

This system is underdamped. ✓✓✓✓✓

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} * 100\% = 67.3\%$$

$$Ts = \frac{3}{\omega_n \zeta} = 6s$$

✓✓✓✓✓✓✓✓✓✓

b.

$$C/R = \frac{16K_1}{s^2 + (1 + 16K_2)s + 16K_1} = \frac{\omega_n^2}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$ ✓✓✓✓✓

Since

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} * 100\% = 14\%$$

$$Ts = \frac{3}{\omega_n \zeta} = 0.3s$$

we can get $\omega_n = 18.87 rad/s; \zeta = 0.53$ ✓✓✓✓✓

$$K_1 = \frac{\omega_n^{\ 2}}{16} = 22.25$$

$$K2 = 1.19$$

✓✓✓✓✓✓✓

Question 3

For the unity negative feedback system with closed-loop transfer function

$$G(s) = \frac{2}{s^4 + 5s^3 + s^2 + 10s + 1}$$

Determine the closed-loop system is stable or not based on Routh's stability criterion, and how many poles are in the right half-plane if this system is unstable.

[16]

Answer:

s^4	1	1	1	
s^3	5	10		✓✓✓✓
s^2	$b_1 = -1$	$b_2 = 1$		✓✓✓
s^1	$c_1 = 15$	$c_2 = 0$		✓✓✓
s^0	$d_1 = 1$			✓✓

$$b_1 = \frac{5\times1 - 1\times10}{5} = -1$$

$$b_2 = \frac{5\times1 - 1\times0}{5} = 1$$

$$\frac{-1\times10 - 5\times1}{-1} = 15$$

$$\frac{-1\times0 - 5\times0}{-1} = 0$$

$$d_1 = \frac{15\times1 - (-1)\times0}{15} = 1$$

Sign: changed twice => two poles in r.h.p.

And the system is unstable. ✓✓✓✓

Question 4

For the unity negative feedback system with a closed-loop transfer function

$$G(s) = \frac{1}{s^3 + 3s^2 + 3s + 1 + k}$$

Find the range of K for closed-loop stability based on Routh's stability criterion.

Here, k can be negative.

[15]

Answer:

s^3	1	3
s^2	3	$1+k$
s^1	$b_1=(8-k)/3$	✓✓✓✓
s^0	$c_1=1+k$	✓✓✓
		✓✓

$$b_1=\frac{3\times 3-1\times(1+k)}{3}=\frac{8-k}{3}$$

$$c_1=\frac{(8-k)/3\times(1+k)-3\times 0}{(8-K)/3}=1+k$$

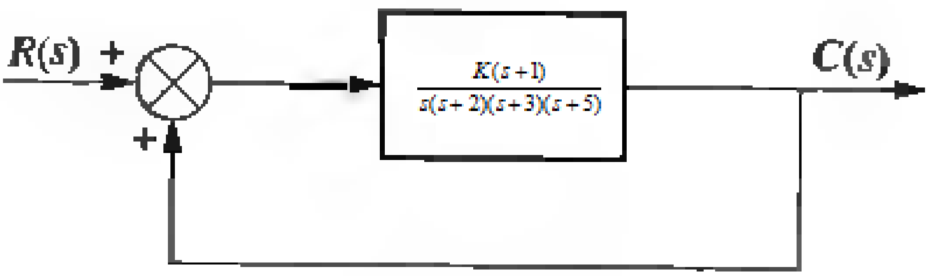
Stable system

$$\left.\begin{array}{l}\dfrac{8-k}{3}>0\Rightarrow 8>k\\ 1+k>0\Rightarrow k>-1\end{array}\right\}\Rightarrow 8>k>-1 \qquad \text{to ensure system stable!}$$

✓✓✓✓✓✓

Question 5

Sketch the root locus for the **positive-feedback** control system shown in the following figure.



[10]

Answer:

Open loop poles: 0, -2, -3, -5

Open loop zeros: -1

Real-axis segments: [0, +inf),[-2, -1], [-5,-3]

✓✓

$$\sigma_a=-3$$

$$\theta_a=\frac{k2\pi}{3}=\pm 120^2$$

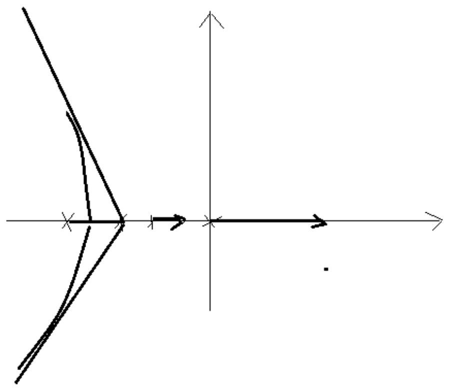
✓✓

Break out point:

$$K = \frac{s(s+2)(s+3)(s+5)}{(s+1)}$$

$$\frac{dK}{ds} = 0 \;\; => s_1 = -2.42(invalid), s_2 = -4.2$$

✓✓

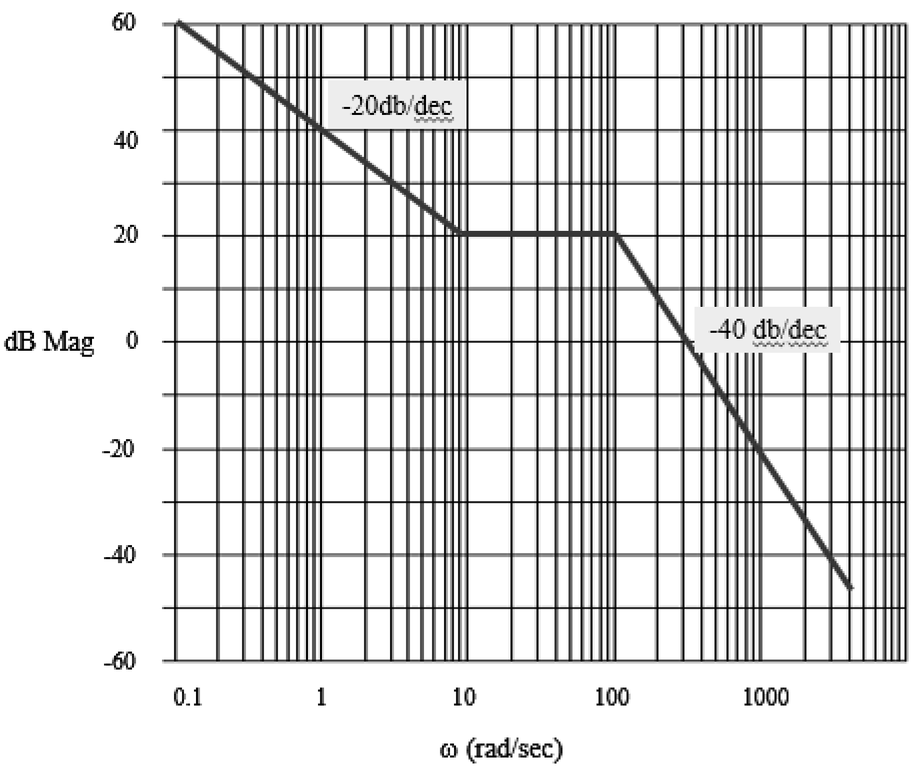


✓✓✓✓

Question 6

The Bode diagram of a minimum phase system is given in the following figure.

Find the open loop transfer function of the system;



[15]

ANSWER:

I type system.
One open loop zero: -10
Two Open loop poles: -100
✓✓✓✓

The system format: $G(s) = \dfrac{K(1+s/10)}{s(1+s/100)^2}$ ✓✓✓✓✓✓

$20\log\left|G(j\omega)\right|_{\omega=100} = 20$

Hence $K \approx 100$.
And

$G(s) = \dfrac{100(1+s/10)}{s(1+s/100)^2}$ ✓✓✓✓✓

Total:100

1 OTHER ASSESSMENT METHODS

None

2 EXAMINATION

Use your *my Studies @ Unisa* brochure for general examination guidelines and examination preparation guidelines.

Examination type: Partial open book

Examination duration: 3 hours

Examination language: English

Calculators allowed: Yes

3 FREQUENTLY ASKED QUESTIONS

The my Studies @ Unisa brochure contains an A-Z guide of the most relevant study information.

4 SOURCES CONSULTED

None

5 CONCLUSION

Please ensure that you have all the tutorial letters and prescribed book available before starting with your studies.

6 ADDENDUM

None