# Tutorial letter 101/3/2018

## Ordinary Differential Equations

# MAT3706

## Semesters 1 & 2

## Department of Mathematical Sciences

This tutorial contains important information for the course as well as the assignments

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**CONTENTS**

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# 1 INTRODUCTION AND WELCOME

Dear Student,

I hereby wish to extend a word of hearty welcome to you as a student in our department. I trust that you will enjoy studying this module and that you will find it both interesting and rewarding.

This tutorial letter contains important information to facilitate your studies. Please read it carefully and keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturer. In Tutorial Letter 101, you will find the assignments as well as instructions on the preparation and submission of the assignments. Please study this information carefully.

## 1.1 Tutorial matter

The MAT3706 tutorial matter is available online. As a student, you should have access to the internet, and therefore you can view the tutorial letters for the modules for which you are registered on the University online campus, **myUnisa**, by typing in http://my.unisa.ac.za. Note that you need to register on **myUnisa**.

Through **myUnisa** you can submit assignments, access library resources, download study materials as well as communicate with the university, your lecturers and other students. It is important to visit **myUnisa** on a regular basis. Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be made available on **myUnisa** in due course.

# 2 PURPOSE AND OUTCOMES OF THE MODULE

**Purpose**

## 2.1 Purpose

The purpose of this module is to enable students to master the fundamental concepts of ordinary differential equations and apply the methods described below in the outcomes for the solution of homogeneous and nonhomogenous systems of differential equations.

## 2.2 Outcomes

The broad outcomes of this course are :

1. To solve systems of ordinary differential equations using:
    - the operator method,
    - the method of triangularization,

- the eigenvalue-eigenvector, root vector methods,
- fundamental matrices, and
- the power series method.

2. To apply the *Gronwall's Inequality* to determine an estimate on the norm of a solution.
3. To determine the stability of solutions of linear systems.
4. Linearization of non-linear systems.

Specific outcomes are listed in the study guide.

# 3 LECTURERS AND CONTACT DETAILS

## 3.1 Lecturers

The contact details for the lecturers for this module is :

| | **Prof. H. Jafari** | | **Dr. Partha Pratim Ghosh** |
|---|---|---|---|
| **Office:** | C-Block, Room C6-49<br>GJ Gerwel Building<br>Florida Campus | **Office:** | C-Block, Room C6-39<br>GJ Gerwel Building<br>Florida Campus |
| **Telephone:** | +27(0)11-471-3732 | **Telephone:** | +27(0)11-670-9162 |
| **Fax:** | +27(0)11-670-9171 | **Fax:** | +27(0)11-670-9171 |
| **E-mail:** | jafarh@unisa.ac.za | **E-mail:** | ghoshpp@unisa.ac.za |

A tutorial letter will be sent to you (or you will be informed through **myUnisa**) if there are any changes and/or an additional lecturer is appointed to this module. All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer. Please have your study material with you when you contact the lecturer.

Email is the preferred form of communication to use. If you communicate by phone, please have your study material with you when you do. If you cannot get hold of your lecturer, leave a message with the Departmental Secretary. Please clearly state your name, time of call and how your lecturer can get back to you.

Another possibly convenient way to contact your lecturer could be to have *Skype/GTalk* sessions for which you need to contact the lecturer first and then proceed forward therefrom.

You are always welcome to come and discuss your work with your lecturer, but please make an appointment before coming to your lecturers office. Please come to these appointments well prepared with specific questions that indicate your own efforts to have understood the basic concepts involved.

You are also free to write to your lecturer about any of the difficulties you encounter with your work for this module. If these difficulties concern exercises which you are unable to solve, you must send your attempts so that your lecturer can see where you are going wrong, or what concepts you do not understand.

**PLEASE NOTE: Letters to lecturers may not be enclosed with or inserted into assignments.**

### 3.2 Department

| | | |
|---|---|---|
| Fax number: | 011 670 9171 (RSA) | +27 11 670 9171 (International) |
| Departmental Secretary: | 011 670 9147 (RSA) | +27 11 670 9147 (International) |
| | 011 670 9148 (RSA) | +27 11 670 9148 (International) |

### 3.3 University (contact details)

If you need to contact the University about matters not related to the content of this module, please consult the publication *My studies @ Unisa* that you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

## 4 MODULE RELATED RESOURCES

Apart from Tutorial letter 101 you will also receive other communication from your lecturer during the year in the form of emails, phone calls or letters that will be available on myUnisa. You can view tutorial letters for the modules for which you are registered on the Universitys online campus, myUnisa, at the following website http://my.unisa.ac.za

### 4.1 Prescribed book

The prescribed book for this module is:

*Differential Equations with Boundary-Value Problems*, 8th edition,
by *Dennis G. Zill* and *Warren S. Wright*,
Brooks/Cole, Cengage Learning, 2013.
ISBN-13: 978-1-133-49246-7
ISBN-10: 1-133-49246-0

(This is also the prescribed book for APM2611 which is a prerequisite for MAT3706 so most of you probably already have this book.)

### 4.2 Recommended books

There are no *Recommended Books* for this module. However, there is a *Study Guide* that is actively referred to in this module.

### 4.3 e-Reserves

There are no *e-Reserves* for this module.

Please refer to the forums on *myUnisa* where several online references and relevant materials shall be posted in the form of web links.

## 5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support systems and services available at Unisa (e.g. student counselling, language support), please consult the publication *My studies @ Unisa* that you received with your study material.

### 5.1 Study groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. The addresses of students in your area may be obtained from the following department:

Directorate: Student Administration and Registration
PO Box 392
UNISA
0003

**myUnisa**

All resources can be accessed through the ***myUnisa*** learning management system. The ***myUnisa*** learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa all through the computer and the internet.

To go to the **myUnisa** website, start at the main Unisa website, http://www.unisa.ac.za, and then click on the "***myUnisa***" link below the orange tab labelled "Current students". This should take you to the **myUnisa** website. You can also go there directly by typing my.unisa.ac.za in the address bar of your browser.

Please consult the publication *My studies @ Unisa* which you received with your study material for more information on ***myUnisa***.

## 6 MODULE SPECIFIC STUDY PLAN

In this module various approaches are used in solving systems of ordinary differential equations (ODEs). You should be able to apply these techniques to solve given systems of ODEs. Work out as many examples as you can find.

Definitions are extremely important since they form the building blocks for the whole material to follow. You **must** know them. The best way to remember a definition is to *clearly understand* the words used. Definitions generate concepts which are linked by means of theorems.

Typically a theorem states, *If* A *is true, then* B *is true*. One then engages in a process whereby the truth of A is assumed and through a sequence of logical deductions one arrives at the truth of B. Such a process is called a *proof* of a theorem. This is really the essence of mathematics and it is extremely important that you **understand the proofs of theorems**.

**All the theorems are examinable**.

Further, regarding the classification of critical points, i.e., **Chapter 8: (Chapter 10 of the prescribed book)**:

- The classification of a critical point of a linear system as summarized in Figure 10.2.12 on p. 398 of the prescribed book.
- The classification of a critical point of a nonlinear system as summarized in Figure 10.3.7 on p. 405 of the prescribed book.

You should know and be able to apply the information on these diagrams in order to analyse the critical points of linear and nonlinear systems of differential equations.

For general time management and planning skills please refer to the `myStudies@Unisa` brochure.

| Study Plan | Semester 1 | Semester 2 |
|---|---|---|
| Outcomes of Assignment 01 to be achieved by | March 15, 2018 | August 15, 2018 |
| Outcomes of Assignment 02 to be achieved by | April 04, 2018 | September 14, 2018 |
| Work through previous examination papers | | |
| Revision | | |

See the brochure `myStudies@Unisa` for general time management and planning skills

# 7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

# 8 ASSESSMENT

## 8.1 Assessment plan

Assignments are seen as part of the learning material for this module. As you do the assignments, discuss the work with fellow students or tutors, you are actively engaged in learning. It is therefore important that you complete all the assignments.

There are two assignments, numbered 01 and 02 for each semester.

In mathematics, computations are as much important as the results which ensure their validity. Hence, it is **important** that you *understand* **all** the theoretical material — the definitions and theorems in your *Study Guide* well.

**Note that the application of the theorems of the Study Guide must be known.** and **the proof of some of theorems will be provided for exam purpose.**.

For Assignments 01 and 02 you must plan to submit your answers by the dates listed below. You will receive the solutions for Assignments 01 and 02 automatically, even if you did not submit the relevant assignment. These solutions will be posted to ALL the students registered for this module about one week after the closing date of the relevant assignment, so it is important to submit your assignments so that they reach the Assignment Department at Unisa by the closing date.

Please do not wait for an assignment to be returned to you before starting to work on the next assignment.

Assignments will be assessed not only on the mathematical correctness of your work, but also on whether you use mathematical notation and language to communicate your ideas clearly and correctly.

### 8.1.1 Examination Admission

Please note that lecturers are not responsible for examination admission.

**You will be admitted to the examination if and only if:**

| **Students registered for** | **Assignment reaches Assignment Section by the date:** |
|---|---|
| **Semester 1** | **March 15, 2018** |
| **Semester 2** | **August 15, 2018** |

### 8.1.2 Semester Examination and Semester Marks

The assignments 01 and 02 shall be given percentage marks Asgn01 and Asgn02. Your Semester Mark for MAT3706 is then calculated as follows:

$$\text{Semester Mark} = 0.5 \times \text{Asgn01} + 0.5 \times \text{Asgn02}.$$

The final mark Final Mark depends on the semestral examination mark Exam Mark (in percentage) and the Semester Mark, and is calculated as follows:

$$\text{Final Mark} = 0.8 \times \text{Exam Mark} + 0.2 \times \text{Semester Mark}.$$

## 8.2 General assignment numbers

The assignments are numbered as $01, 02$.

### *8.2.1 Unique assignment numbers*

In addition to the general assignment number (e.g., $02$) there is a unique assignment number. This must also be supplied when you submit your assignment.

### *8.2.2 Due dates of assignments*

The deadlines for submitting the assignments are :

| Semester | Assignment | Unique Assignment Number | Due Date |
|---|---|---|---|
| 1 | 01 | 754254 | March 15, 2018 |
| | 02 | 810189 | April 04, 2018 |
| 2 | 01 | 788820 | August 15, 2018 |
| | 02 | 870261 | September 14, 2018 |

**Please take heed of the closing dates for the assignments. In order to obtain full credit for your work, you must see to it that your assignments reach us on or before the closing dates.**

## 8.3 Submission of assignments

You may submit your assignments either by post or electronically via ***myUnisa***. Assignments may **not** be submitted by fax or e-mail. For detailed information on assignments, please refer to the *myStudies @ Unisa* brochure, which you received with your study package. Assignments should be sent to

The Registrar
P.O. Box 392
UNISA
0003

To submit an assignment via **myUnisa**:

- Go to **myUnisa**.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

**PLEASE NOTE**: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. In other words, you must submit your own calculations in your own words. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

## 8.4 Assignments

### *8.4.1 Assignments for Semester 1*

**Only For Semester One Students**

**Assignment Number 01**

**Unique Assignment Number: 754254**

**Compulsory Assignment for Exam Admission**

**Last Date of Submission: March 15, 2018**

Answer ***ALL*** the questions.
Some of them shall be marked.

**Question 1.** *Show that the system:*

$$\begin{aligned}(D-2)[x] + 2D[y] &= 2 - 4\mathrm{e}^{2x}\\ (2D-3)[x] + (3D-1)[y] &= 0\end{aligned}$$

*is equivalent to both the following triangular systems:*

$$\begin{cases}(D^2+D-2)[x] = 2 + 20\mathrm{e}^{2x}\\ D[x] - 2y = 12\mathrm{e}^{2x} - 6\end{cases} \quad \textit{and} \quad \begin{cases}(D^2+D-2)[y] = -6 - 4\mathrm{e}^{2x}\\ x - (D+1)[y] = 8\mathrm{e}^{2x} - 4\end{cases} .$$

*Given the three systems above, separately, state (with complete justifications) your strategy in solving them completely*

**Question 2.** *Solve the system:*

$$\begin{aligned}(D^2+1)[x_1] - 2D[x_2] &= 2t,\\ (2D-1)[x_1] - (2-D)[x_2] &= 5,\end{aligned}$$

*by using the elimination method (operator method).*

**Question 3.** *Consider the system of linear differential equations:*

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}.$$

*(i) Suppose $\lambda$ is an eigen value of $\mathbf{A}$ of multiplicity $k > 1$. Define a root vector corresponding to the eigen value $\lambda$.*

*(ii) Show that each eigen value $\lambda$ of $\mathbf{A}$ of multiplicity $k$ has $k$ linearly independent root vectors.*

*(iii) Suppose that $\lambda$ is an eigen value of $\mathbf{A}$ of multiplicity $k$ and $\mathbf{U}_{\lambda,1}$, $\mathbf{U}_{\lambda,2}$, ..., $\mathbf{U}_{\lambda,k}$ are root vectors corresponding to $\lambda$ of orders $1$, $2$, ..., $k$, respectively. Further, let:*

$$\mathbf{X}_i(t) = \mathrm{e}^{\lambda t}\big[\mathbf{U}_{\lambda,i} + t\mathbf{U}_{\lambda,i-1} + \frac{t^2}{2!}\mathbf{U}_{\lambda,i-2} + \dots \frac{t^{i-1}}{(i-1)!}\mathbf{U}_{\lambda,1}\big], \text{ for } i = 1, 2, \dots k.$$

*Show that $\Big\{\mathbf{X}_1(t), \mathbf{X}_2(t), \dots, \mathbf{X}_k(t)\Big\}$ is a set of linearly independent solutions of the system of linear differential equations and every solution of the system is a linear combination of the solutions in this set.*

**Question 4.** *Solve the system:*

$$\dot{\mathbf{X}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix} \mathbf{X}, \quad \mathbf{X}(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

*Why is there a unique solution to the above system?*

**Question 5.** *Solve the system:*

$$\dot{\mathbf{X}} = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -1 & -2 \end{pmatrix} \mathbf{X}$$

*completely.*

*Find the unique solution that passes through the point $(11, 3, -7)$ when $t = 0$.*

# Only For Semester One Students

# Assignment Number 02

## Unique Assignment Number: 810189

**<u>Last Date of Submission</u>: April 04, 2018**

Answer ***ALL*** the questions.
<u>Some of them</u> shall be marked.

**Question 1.** *Solve the following initial value problem*

$$\dot{\mathbf{X}} = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix}, \quad \mathbf{X}(0) = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

**Question 2.** *If $\phi$ is a normalized fundamental matrix $\dot{\mathbf{X}} = \mathbf{AX}$ at $t_0 = 0$ and if $\mathbf{F}$ is continuous in $t$, then*

$$\mathbf{X}_p(t) = \int_0^t \phi(t-s)\mathbf{F}(s)\, ds,$$

*is a particular solution of $\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{F}$. Use the result above to find a general solution of*

$$\dot{\mathbf{X}} = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} t\, e^{2t} \\ -e^{2t} \end{pmatrix}.$$

**Question 3.** *Use Corollary 4.39 with $t_0 = 0$ of the study guide to solve the inhomogeneous initial value problem*

$$\dot{\mathbf{X}} = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ e^{-3t} \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

**Question 4.** *Write the companion system for the equations given below.*

$$a)\ y^{(3)} - y \sin t = f(t), \qquad b)\ \ddot{y} - \dot{} = 0.$$

**Question 5.** *Use the power series method to find a series solution for*

$$\dot{\mathbf{X}} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{X}, \quad \mathbf{X} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

*Write the final answer in term of the appropriate exponential function.*

**Question 6.** *Classify the critical points of the plane autonomous system corresponding to the second order non-linear differential equation:*

$$\ddot{x} + \nu(x^2 - 1)\dot{x} + x = 0.$$

### *8.4.2 Assignment for Semester 2*

**Only For Semester Two Students**

**Assignment Number 01**

**Unique Assignment Number: 788820**

**Compulsory Assignment for Exam Admission**

**Last Date of Submission: August 15, 2018**

Answer ***ALL*** the questions.
Some of the Questions shall be marked.

**Question 1.** *Cnsider the following system of differential equations:*

$$\mathbf{P}_{11}(D)[x] + \mathbf{P}_{12}(D)[x] = f_1(t),$$
$$\mathbf{P}_{21}(D)[x] + \mathbf{P}_{22}(D)[x] = f_2(t).$$

*(i) How do we determine the correct number of arbitrary constants in a general solution of the above system.*

*(ii) Explain briefly the difference between the operator method and the method of triangularization when used for solving the above system.*

**Question 2.** *Determine whether the system*

$$\dot{x} + x - \dot{y} - y = e^{\frac{t}{5}},$$
$$2\dot{x} - x - 2\dot{y} + y = 5e^{-t}$$

*has no solution or infinitely solution. If it has no solution, explain why, else solve $y(t)$ in term of $x(t)$, where $x(t)$ is an arbitrary function of $t$.*

**Question 3.** *Solve the system:*

$$(D^2 + 1)[x_1] - 2D[x_2] = 2t$$
$$(2D - 1)[x_1] - (2 - D)[x_2] = 5$$

*by using the elimination method (operator method).*

**Question 4.** *Consider the system of linear differential equations:*

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}.$$

*(i) Suppose $\lambda$ is an eigen value of $\mathbf{A}$ of multiplicity $k > 1$. Define a root vector corresponding to the eigen value $\lambda$.*

*(ii)* *Show that each eigen value $\lambda$ of $\mathbf{A}$ of multiplicity $k$ has $k$ linearly independent root vectors.*

*(iii)* *Suppose that $\lambda$ is an eigen value of $\mathbf{A}$ of multiplicity $k$ and $\mathbf{U}_{\lambda,1}$, $\mathbf{U}_{\lambda,2}$, ..., $\mathbf{U}_{\lambda,k}$ are root vectors corresponding to $\lambda$ of orders $1$, $2$, ..., $k$, respectively. Further, let:*

$$\mathbf{X}_i(t) = \mathrm{e}^{\lambda t}\big[\mathbf{U}_{\lambda,i} + t\mathbf{U}_{\lambda,i-1} + \frac{t^2}{2!}\mathbf{U}_{\lambda,i-2} + \dots \frac{t^{i-1}}{(i-1)!}\mathbf{U}_{\lambda,1}\big], \text{ for } i = 1, 2, \dots k.$$

*Show that $\Big\{\mathbf{X}_1(t), \mathbf{X}_2(t), \dots, \mathbf{X}_k(t)\Big\}$ is a set of linearly independent solutions of the system of linear differential equations and every solution of the system is a linear combination of the solutions in this set.*

**Question 5.** *Solve the system:*

$$\dot{\mathbf{X}} = \begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix} \mathbf{X}$$

*completely.*

*Find the unique solution that passes through the point $(4, 5, -9)$ when $t = 0$.*

**Only For Semester Two Students**

**Assignment Number 02**

**Unique Assignment Number: 870261**

**Last Date of Submission: September 14, 2018**

Answer ***ALL*** the questions.
Some of the Questions shall be marked.

**Question 1.** *(a) If $\mathbf{F}(t)$ is continuous and $\Phi$ is a fundamental matrix of $\dot{\mathbf{X}} = \mathbf{AX}$, then show that a particular solution of the non-homogeneous system $\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{F}(t)$ is given by* ***(8)***

$$\mathbf{X}_p(t) = \mathbf{\Phi}(t) \int_{t_0}^{t} \mathbf{\Phi}^{-1}(s)\, \mathbf{F}(s)\, ds.$$

*(b) Use the result in 1(a) with $t_0 = 0$ to find a* general *real solution of* ***(12)***

$$\dot{\mathbf{X}} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{X} + 2 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{2t}.$$

**Question 2.** *(i) Show that for any square matrix $\mathbf{A} = \left(a_{ij}\right)_{i,j=1}^{n}$, if:*

$$||\mathbf{A}|| = \sqrt{\sum_{i,j=1}^{n} a_{ij}^2},$$

*then: $||A|| = 0 \Leftrightarrow \mathbf{A} = \mathbf{0}$, $||\lambda \mathbf{A}|| = |\lambda| ||\mathbf{A}||$ and $||\mathbf{A} + \mathbf{B}|| \leq ||\mathbf{A}|| + ||\mathbf{B}||$.*

*(ii) Suppose $[a, b] \overset{f}{\underset{g}{\rightrightarrows}} \mathbb{R}$ are real valued continuous functions defined on the interval $[a, b]$ with $f$ differentiable and $\dot{f}(x) \leq f(x)g(x)$ for all $x \in (a, b)$.*
*Show that for each $x \in [a, b]$:*

$$f(x) \leq f(a)\mathrm{e}^{h(x)}, \qquad \text{where } h(x) = \int_a^x g(t)\mathrm{d}t.$$

*(iii) Using the statement of* ((ii))*, or otherwise, show that for real valued continuous functions $[a, b] \overset{f}{\underset{g}{\rightrightarrows}} \mathbb{R}$ of which $g$ is always non-negative and:*

$$f(x) \leq K + \int_a^x f(t)g(t)\mathrm{d}t, \qquad \text{for all } x \in [a, b],$$

*for some real number $K$, one always has for each $t \in [a, b]$:*

$$f(t) \leq K\mathrm{e}^{h(t)}, \qquad \text{where } h(t) = \int_a^t f(x)\mathrm{d}x.$$

*(iv) Show that for any solution* $\mathbf{X}(t)$ *of the system* $\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t)$*, for any* $s$ *and* $t$*:*

$$||\mathbf{X}(t)|| \leq ||\mathbf{X}(s)|| \; \mathrm{e}^{||\mathbf{A}|| \, ||t-s||}.$$

**Question 3.** *Given the system:*

$$\dot{\mathbf{X}}(t) = \begin{pmatrix} t & 2t & 0 \\ \sin t & \dfrac{1}{t} & 2 \\ 2\sin t & 2\cos t & \dfrac{1}{t^2} \end{pmatrix} \mathbf{X} + \begin{pmatrix} t \\ \mathrm{e}^{t} \\ \mathrm{e}^{-t} \end{pmatrix}$$

*and the fact that on the interval* $[1, 4]$ *there are two solutions* $\mathbf{P}$ *and* $\mathbf{Q}$ *with:*

$$\mathbf{P}(2) = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \quad \text{and } \mathbf{Q}(2) = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix},$$

*find an absolute bound for the error in estimating the solution* $\mathbf{P}$ *by the solution* $\mathbf{Q}$*.*

**Question 4.** *Consider the* $n^{th}$ *order differential equation with constant coefficients*[1]*:*

$$y^{(n)} + a_{n-1}y^{(n-1)} + a_{n-2}y^{(n-2)} + \cdots + a_1 y^{(1)} + a_0 y = f(t).$$

*Now answer the following questions with respect to this differential equation.*

*(i) Show that:*

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{F},$$

*where:*

$$\mathbf{X} = \begin{pmatrix} y \\ y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n-2)} \\ y^{(n-1)} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots - a_{n-1} & \end{pmatrix} \quad \text{and} \quad \mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ f(t) \end{pmatrix}.$$

**Note***: The matrix* $\mathbf{A}$ *is often called a* companion matrix*.*

*(ii) Show that if* $y$ *be a solution of the given differential equation above and* $\mathbf{Y} = \begin{pmatrix} y \\ y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n-2)} \\ y^{(n-1)} \end{pmatrix}$*,*

*then* $\mathbf{Y}$ *is a solution of the system of differential equations in* ((i))*.*

[1] Note, according to general convention, $y^{(n)}$ is used to denote the $n^{\text{th}}$ order derivative $\dfrac{\mathrm{d}^n}{\mathrm{d}t^n}y$, where $y$ is a function of $t$. In this notation, $y^{(0)} = y$, $y^{(1)} = \dot{y}$, $y^{(2)} = \ddot{y}$, ...

*(iii)* *Given:*

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

*find a differential equation whose companion matrix is* $\mathbf{A}$.

*(iv)* *Find the general solution of the differential equation in* ((iii)).

**Question 5.**

*(I)* *When is a function* $\mathbb{R} \xrightarrow{f} \mathbb{R}$ *said to be analytic at a point* $t_0$*?*

*(II)* *Find a solution of the system of first order differential equations by using the power series method*

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{with} \begin{pmatrix} x \\ y \\ z \end{pmatrix}(0) = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

**Question 6.**

*(a)* *State clearly (with all the assumptions and conditions) the classification scheme of critical points of* $\mathbb{R}^n \xrightarrow{f} \mathbb{R}$.

*(b)* *Classify the critical points of the plane autonomous system corresponding to the second order non-linear differential equation:*

$$\ddot{x} + \nu(x^2 - 1)\dot{x} + x = 0.$$

# 9 OTHER ASSESSMENT METHODS

There are no other assessment methods for this module.

# 10 EXAMINATIONS

## 10.1 Examination Admission

You are automatically admitted to the exam on the submission of Assignment $01$ by the due date, i.e., by $15$ March 2018. Please note that lecturers are not responsible for exam admission, and **ALL** enquiries about examination admission should be directed by e-mail to exams@unisa.ac.za.

## 10.2 Examination Period

During the year, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.

For general information and requirements as far as examinations are concerned, see the brochure my Studies @ Unisa.

## 10.3 Examination Paper

The examination shall consist of a $3$ hour paper.

The questions in the assignments should be a guideline towards the examinable material from the course.

**Note: All the theorems in the Study Guide are examinable.**

Note that the my Studies @ Unisa brochure contains general examination guidelines and examination preparation guidelines.

# 11 FREQUENTLY ASKED QUESTIONS

For any other study information see the brochure *my Studies @ Unisa*.

# 12 CONCLUSION

Read your tutorial letter carefully, follow the study guide reference and outcomes and do as many exercises as possible.