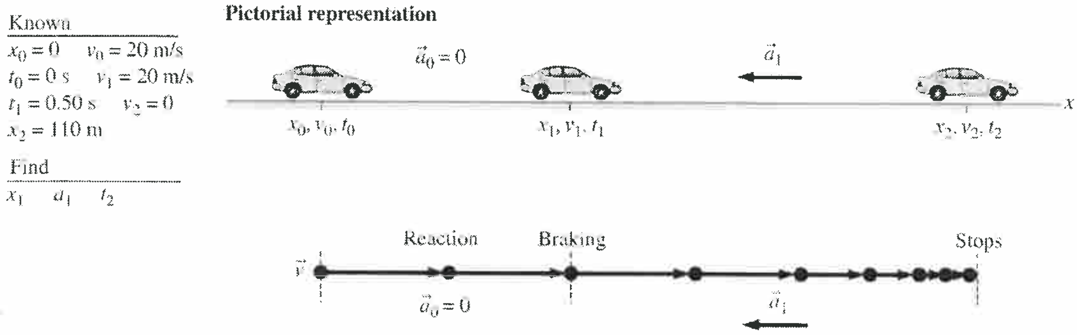
Mechanics (PHY1505)
Assignment 01 Semester 2 2018
Keys to Solutions

2.43. Model: The car is a particle and constant-acceleration kinematic equations hold.
Visualize:



Solve: **(a)** This is a two-part problem. During the reaction time,

$$x_1 = x_0 + v_0(t_1 - t_0) + 1/2a_0(t_1 - t_0)^2$$
$$= 0 \text{ m} + (20 \text{ m/s})(0.50 \text{ s} - 0 \text{ s}) + 0 \text{ m} = 10 \text{ m}$$

After reacting, $x_2 - x_1 = 110 \text{ m} - 10 \text{ m} = 100 \text{ m}$, that is, you are 100 m away from the intersection.
(b) To stop successfully,

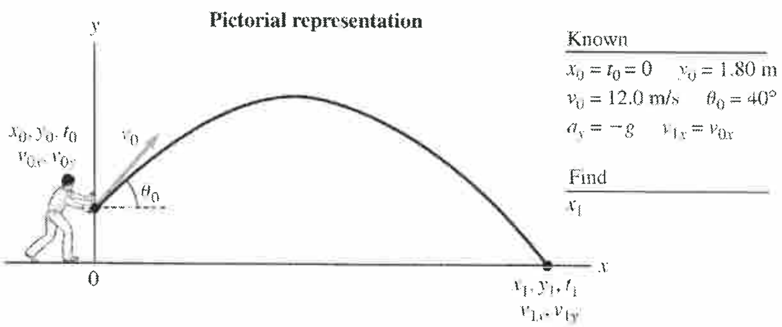
$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow (0 \text{ m/s})^2 = (20 \text{ m/s})^2 + 2a_1(100 \text{ m}) \Rightarrow a_1 = -2 \text{ m/s}^2$$

(c) The time it takes to stop once the brakes are applied can be obtained as follows:

$$v_2 = v_1 + a_1(t_2 - t_1) \Rightarrow 0 \text{ m/s} = 20 \text{ m/s} + (-2 \text{ m/s}^2)(t_2 - 0.50 \text{ s}) \Rightarrow t_2 = 11 \text{ s}$$

The total time to stop since the light turned red is 11.5 s. **[20]**

4.42. Model: Assume the particle model and motion under constant-acceleration kinematic equations in a plane.
Visualize:



Solve: (a) Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$0\text{ m} = 1.80\text{ m} + v_0 \sin 40°(t_1 - 0\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 0\text{ s})^2$$
$$= 1.80\text{ m} + (7.713\text{ m/s})t_1 - (4.9\text{ m/s}^2)t_1^2 \Rightarrow t_1 = -0.206\text{ s and } 1.780\text{ s}$$

The negative value of t_1 is unphysical for the current situation. Using $t_1 = 1.780$ s and $x_1 = x_0 + v_{0x}(t_1 - t_0)$, we get

$$x_1 = 0 + (v_0 \cos 40°\text{ m/s})(1.780\text{ s} - 0\text{ s}) = (12.0\text{ m/s})\cos 40°(1.78\text{ s}) = 16.36\text{ m} \approx 16.4\text{ m}$$

(b) We can repeat the calculation for each angle. A general result for the flight time at angle θ is

$$t_1 = \left(12\sin\theta + \sqrt{144\sin^2\theta + 35.28}\right)/9.8\text{ s}$$

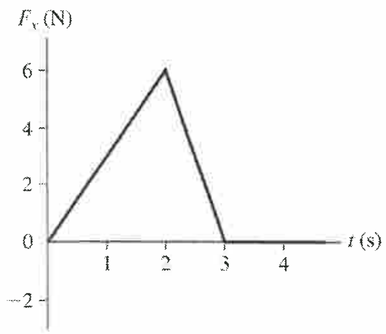
and the distance traveled is $x_1 = (12.0)\cos\theta \times t_1$. We can put the results in a table.

θ	t_1	x_1
40.0°	1.780 s	16.36 m
42.5°	1.853 s	16.39 m
45.0°	1.923 s	16.31 m
47.5°	1.990 s	16.13 m

Maximum distance is achieved at $\theta \approx 42.5°$.

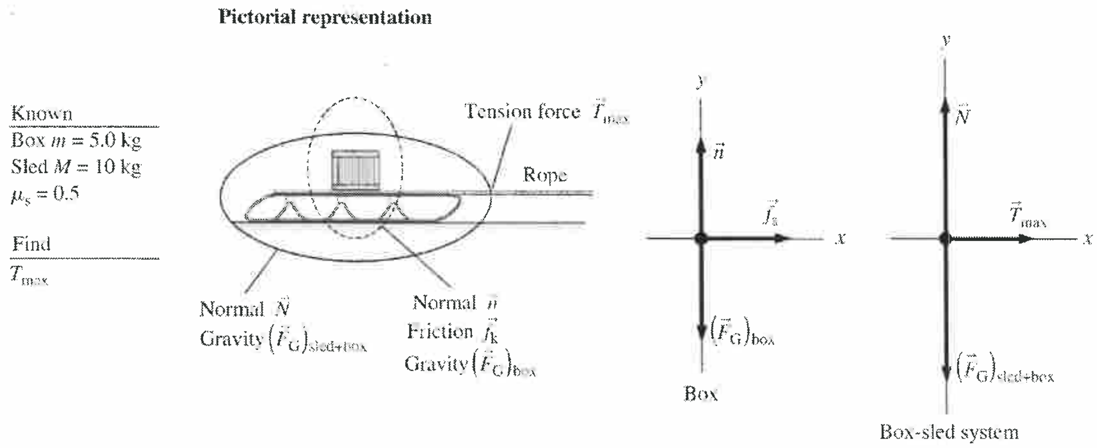
Assess: The well-known "fact" that maximum distance is achieved at 45° is true only when the projectile is launched and lands at the *same* height. That isn't true here. The extra $0.03\text{ m} = 3\text{ cm}$ obtained by increasing the angle from 40.0° to 42.5° could easily mean the difference between first and second place in a world-class meet. **[20]**

5.30. Visualize:



Solve: According to Newton's second law $F = ma$, the force at any time is found simply by multiplying the value of the acceleration by the mass of the object. Thus, for example, the point at (2 s, 3 m/s²) become (2 s, 2 m/s² × 2.0 kg) = (2 s, 6 N). **[15]**

6.50. Model: Model the small box as a particle and use the model of static friction. The acceleration of the small box must be the same as the acceleration of the large box in order for it not to slip.
Visualize: First use Newton's second law in both directions on the small box. The force that is responsible for the small box's acceleration is the static friction force. We use this to determine a_{max}. Then we use Newton's second law on the the two-box system.



Solve:
(a)

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$
$$\Sigma F_x = f_s = ma_x$$
$$(f_s)_{\text{max}} = \mu_s n = \mu_s mg = ma_{\text{max}}$$
$$a_{\text{max}} = \mu_s g$$

Now consider the two-box system.

$$\Sigma F_x = T_{\text{max}} = (M + m)a_{\text{max}}$$

Put these together to arrive at

$$T_{\text{max}} = (M + m)\mu_s g$$

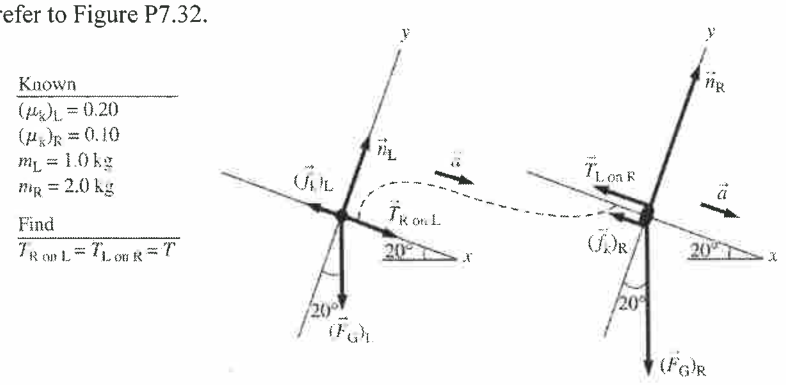
(b) Insert the known values for M and m, and look up μ_s for wood on wood in the table.

$$T_{\text{max}} = (10 \text{ kg} + 5 \text{ kg})(0.5)(9.8 \text{ m/s}^2) = 73.5 \text{ N} \approx 74 \text{ N}$$

Assess: Check the reasonableness of our answer by examining the dependence of T_{max} on μ_s: if the small box were glued to the large box $(\mu_s \to \infty)$ then one could pull on the rope with any tension desired; if the friction between the two boxes were zero then one could not pull at all without causing the small box to slip. We expect a similar dependence on g.

[15]

7.32. Model: The two blocks form a system of interacting objects. We shall treat them as particles.
Visualize: Please refer to Figure P7.32.



Left-hand block (L) Right-hand block (R)

Solve: It is possible that the left-hand block (Block L) is accelerating down the slope faster than the right-hand block (Block R), causing the string to be slack (zero tension). If that were the case, we would get a zero or negative answer for the tension in the string. Newton's first law applied in the y-direction on Block L yields

$$(\Sigma F_L)_y = 0 = n_L - (F_G)_L \cos(20°) \;\Rightarrow\; n_L = m_L g\cos(20°)$$

Therefore

$$(f_k)_L = (\mu_k)_L m_L g\cos(20°) = (0.20)(1.0\text{ kg})(9.80\text{ m/s}^2)\cos(20°) = 1.84\text{ N}$$

A similar analysis of the forces in the y-direction on Block R gives $(f_k)_R = 1.84$ N as well. Using Newton's second law in the x-direction for Block L gives

$$(\Sigma F_L)_x = m_L a = T_{R\text{ on }L} - (f_k)_L + (F_G)_L \sin(20°) \;\Rightarrow\; m_L a = T_{R\text{ on }L} - 1.84\text{ N} + m_L g\sin(20°)$$

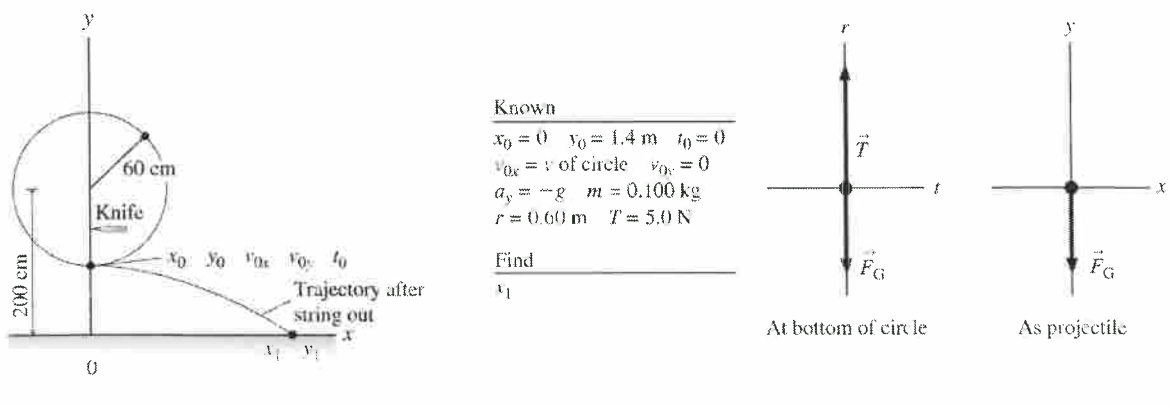
For Block R,

$$(\Sigma F_R)_x = m_R a = (F_G)_R \sin(20°) - 1.84\text{ N} - T_{L\text{ on }R} \;\Rightarrow\; m_R a = m_R g\sin(20°) - 1.84\text{ N} - T_{L\text{ on }R}$$

Solving these two equations in the two unknowns a and $T_{L\text{ on }R} = T_{R\text{ on }L} \equiv T$, we obtain $a = 2.12$ m/s^2 and $T = 0.61$ N.
Assess: The tension in the string is positive, and is about 1/3 of the kinetic friction force on each of the blocks, which is reasonable. **[15]**

8.50. Model: Model the ball as a particle swinging in a vertical circle, then as a projectile.
Visualize:

Pictorial representation



Solve: Initially, the ball is moving in a circle. Once the string is cut, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile. The free-body diagram for circular motion is shown at the bottom of the circle. Since $T > F_G$, there is a net force toward the center of the circle that causes the centripetal acceleration. The r-equation of Newton's second law is

$$(F_{\text{net}})_r = T - F_G = T - mg = \frac{mv^2}{r}$$

$$\Rightarrow v_{\text{bottom}} = \sqrt{\frac{r}{m}(T - mg)} = \sqrt{\frac{0.60 \text{ m}}{0.100 \text{ kg}}[5.0 \text{ N} - (0.10 \text{ kg})(9.8 \text{ m/s}^2)]} = 4.91 \text{ m/s}$$

As a projectile the ball starts at $y_0 = 1.4$ m with $\vec{v}_0 = 4.91\hat{i}$ m/s. The equation for the y-motion is

$$y_1 = 0 \text{ m} = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 = y_0 - \tfrac{1}{2}gt_1^2$$

This is easily solved to find that the ball hits the ground at time

$$t_1 = \sqrt{\frac{2y_0}{g}} = 0.535 \text{ s}$$

During this time interval it travels a horizontal distance

$$x_1 = x_0 + v_{0x}t_1 = (4.91 \text{ m/s})(0.535 \text{ s}) = 2.63 \text{ m}$$

So the ball hits the floor 2.6 m to the right of the point where the string was cut. **[15]**