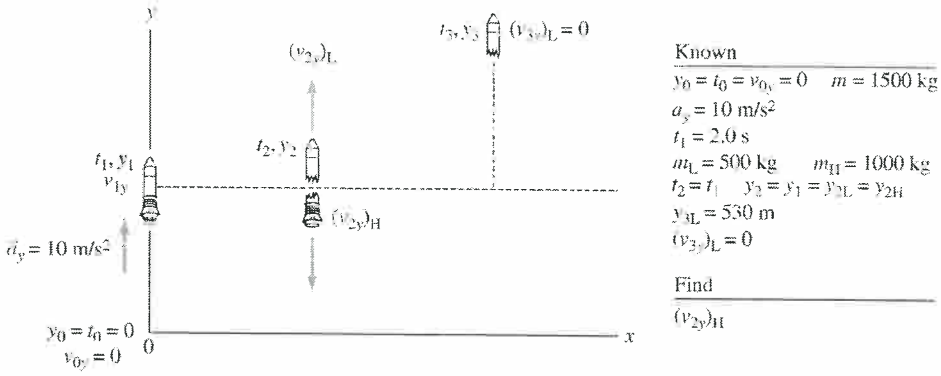
Mechanics PHY1505
Assignment 02
Solutions

9. We will model the two fragments of the rocket after the explosion as particles. We assume the explosion separates the two parts in a vertical manner. This is a three-part problem. In the first part, we will use kinematic equations to find the vertical position where the rocket breaks into two pieces. In the second part, we will apply conservation of momentum to the system (that is, the two fragments) in the explosion. In the third part, we will again use kinematic equations to find the velocity of the heavier fragment just after the explosion.

Visualize:

Pictorial representation



Solve: The rocket accelerates for 2.0 s from rest, so

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = 0 \text{ m/s} + (10 \text{ m/s}^2)(2.0 \text{ s} - 0 \text{ s}) = 20 \text{ m/s}$$

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(10 \text{ m/s}^2)(2.0 \text{ s})^2 = 20 \text{ m}$$

At the explosion the equation $p_{fy} = p_{iy}$ is

$$m_L(v_{2y})_L + m_H(v_{2y})_H = (m_L + m_H)v_{1y} \quad \Rightarrow \quad (500 \text{ kg})(v_{2y})_L + (1000 \text{ kg})(v_{2y})_H = (1500 \text{ kg})(20 \text{ m/s})$$

To find $(v_{2y})_H$ we must first find $(v_{2y})_L$, the velocity after the explosion of the upper section. Using kinematics,

$$(v_{3y})_L^2 = (v_{2y})_L^2 + 2(-9.8 \text{ m/s}^2)(y_{3L} - y_{2L}) \quad \Rightarrow \quad (v_{2y})_L = \sqrt{2(9.8 \text{ m/s}^2)(530 \text{ m} - 20 \text{ m})} = 99.98 \text{ m/s}$$

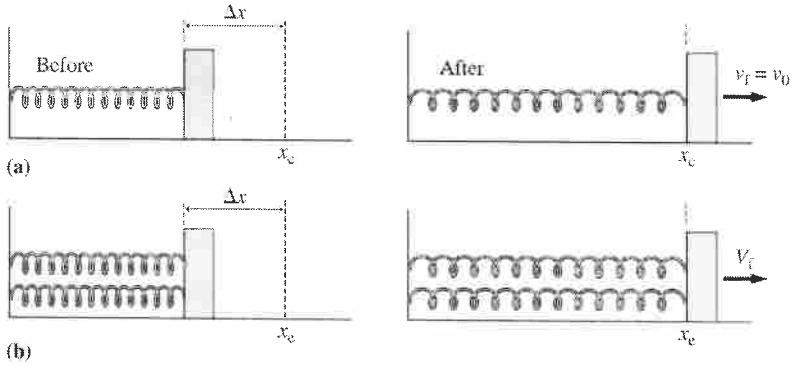
Now, going back to the momentum conservation equation we get

$$(500 \text{ kg})(99.98 \text{ m/s}) + (1000 \text{ kg})(v_{2y})_H = (1500 \text{ kg})(20 \text{ m/s}) \quad \Rightarrow \quad (v_{2y})_H = -20 \text{ m/s}$$

The negative sign indicates downward motion. **[15]**

Open Rubric

10. Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K+U_s$ is conserved.
Visualize:



Note that $x_f = x_e$ and $x_i - x_e = \Delta x$. The before-and-after pictorial representations show that we put the origin of the coordinate system at the equilibrium position of the free end of the springs.
Solve: The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the single spring is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

Using the value for v_f given in the problem, we get

$$\frac{1}{2}mv_0^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(\Delta x)^2 \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy for the two-spring case:

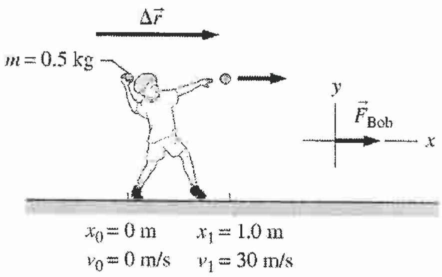
$$\frac{1}{2}mV_f^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(x_i - x_e)^2 + \frac{1}{2}k(x_i - x_e)^2 \quad \frac{1}{2}mV_f^2 = k(\Delta x)^2$$

Using the result of the single-spring case,

$$\frac{1}{2}mV_f^2 = mv_0^2 \Rightarrow V_f = \sqrt{2}v_0$$

Assess: The block separates from the spring at the equilibrium position of the spring. **[15]**

11. Model the rock as a particle, and apply the work–kinetic-energy theorem.
Visualize:



Solve: **(a)** The work done by Bob on the rock is

$$W_{Bob} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.500\text{ kg})(30\text{ m/s})^2 = 225\text{ J} = 2.3\times10^2\text{ J}$$

(b) For a constant force, $W_{\text{Bob}} = F_{\text{Bob}}\Delta x \quad \Rightarrow \quad F_{\text{Bob}} = W_{\text{Bob}}/\Delta x = 2.3\times10^2$ N.

(c) Bob's power output is $P_{\text{Bob}} = F_{\text{Bob}}v_{\text{rock}}$ and will be a maximum when the rock has maximum speed. This is just as he releases the rock with $v_{\text{rock}} = v_1 = 30$ m/s. Thus, $P_{\max} = F_{\text{Bob}}v_1 = (225\text{ J})(30\text{ m/s}) = 6750\text{ W} = 6.8\text{ kW}$. **[15]**

12. From Equation 12.16,

$$I = \int (x^2 + y^2)dm$$

A small region of area dA has mass dm, and

$$dm = \frac{M}{A}dA = \frac{M}{A}dx\,dy$$

The area of the plate is $\frac{1}{2}(0.20\text{ m})(0.30\text{ m}) = 0.030\text{ m}^2$. So

$$\frac{M}{A} = \left(\frac{0.800\text{ kg}}{0.030\text{ m}^2}\right) = 26.67\text{ kg m}^2$$

The limits for x are $0 \le x \le 30$ cm. For a particular value of x, $-\frac{1}{3}x \le y \le \frac{1}{3}x$. Note that $\pm\frac{1}{3}$ is the slope of the top and bottom edges of the triangle. Therefore,

$$I = \int_0^{30\text{ cm}} \int_{-\frac{1}{3}x}^{\frac{1}{3}x} (x^2 + y^2)(26.67\text{ kg/m}^2)dx\,dy$$

$$= (26.67\text{ kg/m}^2)\int_0^{30\text{ cm}} \left(x^2y + \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{3}x}^{\frac{1}{3}x} dx$$

$$= (26.67\text{ kg/m}^2)\int_0^{30\text{ cm}} \left(\frac{2}{3}x^3 + \frac{2x^3}{81}\right)dx$$

$$= (26.67\text{ kg/m}^2)\left(\frac{56}{81}\right)\left(\frac{1}{4}x^4\right)\Bigg|_0^{30\text{ cm}} = 0.037\text{ kg m}^2$$

[15]

13. Consider the object as a particle and take the planet to be a spherical mass.
Solve: Conservation of mechanical energy of the object gives

$$-G\frac{Mm}{R+h} = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

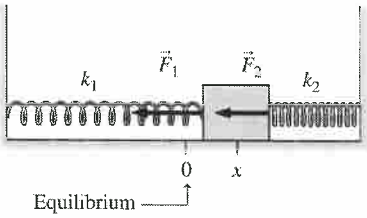
The object's mass drops out. Solving for the speed as it hits the ground,

$$v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)} = \sqrt{2GM\left(\frac{R+h-R}{R(R+h)}\right)} = \sqrt{\frac{2GMh}{R(R+h)}}$$

Assess: Compare this to $v = \sqrt{2gh} = \sqrt{\frac{2GMh}{R^2}}$, which is the result if the potential $U = mgh$ is used. **[10]**

14. The two springs obey Hooke's law.

Visualize:



Solve: There are *two* restoring forces on the block. If the block's displacement x is positive, *both* restoring forces—one pushing, the other pulling—are directed to the left and have negative values:

$$(F_{\text{net}})_x = (F_{\text{sp }1})_x + (F_{\text{sp }2})_x = -k_1x - k_2x = -(k_1 + k_2)x = -k_{\text{eff}}x$$

where $k_{\text{eff}} = k_1 + k_2$ is the effective spring constant. This means the oscillatory motion of the block under the influence of the two springs will be the *same* as if the block were attached to a single spring with spring constant k_{eff}. The frequency of the blocks, therefore, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{k_1}{4\pi^2 m} + \frac{k_2}{4\pi^2 m}} = \sqrt{f_1^2 + f_2^2}$$

[15]

15. The ideal fluid obeys Bernoulli's equation.
Visualize: Please refer to Figure P15.63. There is a streamline connecting point 1 in the wider pipe on the left with point 2 in the narrower pipe on the right. The air speeds at points 1 and 2 are v_1 and v_2 and the cross-sectional area of the pipes at these points are A_1 and A_2. Points 1 and 2 are at the same height, so $y_1 = y_2$.
Solve: The volume flow rate is $Q = A_1v_1 = A_2v_2 = 1200\times10^{-6}\ \text{m}^3/\text{s}$. Thus

$$v_2 = \frac{1200\times10^{-6}\ \text{m}^3/\text{s}}{\pi(0.0020\ \text{m})^2} = 95.49\ \text{m/s} \qquad v_1 = \frac{1200\times10^{-6}\ \text{m}^3/\text{s}}{\pi(0.010\ \text{m})^2} = 3.82\ \text{m/s}$$

Now we can use Bernoulli's equation to connect points 1 and 2:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1) = \tfrac{1}{2}(1.28\ \text{kg/m}^3)[(95.49\ \text{m/s})^2 - (3.82\ \text{m/s})^2] + 0\ \text{Pa} = 5.83\ \text{kPa}$$

Because the pressure above the mercury surface in the right tube is p_2 and in the left tube is p_1, the difference in the pressures p_1 and p_2 is $\rho_{\text{Hg}}gh$. That is,

$$p_1 - p_2 = 5.83\ \text{kPa} = \rho_{\text{Hg}}gh \ \Rightarrow\ h = \frac{5.83\times10^3\ \text{Pa}}{(13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 4.4\ \text{cm}$$

[15]