Bonus Questions for MAT2611

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Bonus Questions? What is it?

A series of *Video Lecture*s shall be uploaded to the **MAT2611-18-S1** web site. The purpose of these *Video Lectures* would be to aid understanding material of the course at your own pace in your own pre-defined sets of piecemeals.

The *Video Lecture*s would go unnoticed unless it is added with some reward system. This is augmented by the **Bonus Questions**. Each *Video Lecture* shall come paired with a **Bonus Questions** section which would test you on the material of the *Video Lecture*. The posting and submission dates, as well as the weight in the calculation of the *Year Mark*, of the **Bonus Questions** is given in the table below. Each **Bonus Question**, with the exception of the last one, has a period of fourteen days from the date of posting for submission.

The last one, i.e., **Bonus Questions 8**, has a submission time of eleven days and has lesser weight. This shall not be linked to any particular *Video Lecture*, but instead would be connected to all of them. This set of questions would depend on the full material of the course and would give you a judgement of the level of examination.

Now regarding the computation of the *Year Mark*. If A_1, A_2, A_3 be the percentage marks obtained for the first, second and third assignments, respectively, and B be the percentage mark obtained from all the questions in this document then the *Year Mark* Y shall be:

$$Y = \frac{3A_1 + 2A_2 + 3A_3 + 12B}{20}.$$

Hence the **Bonus Questions** contribute to 60% of the *Year Mark* and the assignments given do the rest.

Note: The number B shall be attained as follows:

Bonus Questions No.	Posting Date	Submission Date	Percentage Weight
Bonus Questions 1	18-Feb-2018	26-Feb-2018	8%
Bonus Questions 2	05-Mar-2018	19-Mar-2018	8%
Bonus Questions 3	12-Mar-2018	26-Mar-2018	8%
Bonus Questions 4	19-Mar-2018	02-Apr-2018	8%
Bonus Questions 5	26-Mar-2018	09-Apr-2018	8%
Bonus Questions 6	02-Apr-2018	16-Apr-2018	8%
Bonus Questions 7	09-Feb-2018	23-Apr-2018	8%
Bonus Questions 8	16-Feb-2018	27-Apr-2018	4%
Total			60%

For instance, if the marks obtained by a candidate in the eight sections of Bonus Questions have percentages $11, 44, 77, 78, 99, 33, 50, 47$ then for this candidate the calculation of B is:

$$B = \frac{8 \times (11 + 44 + 77 + 78 + 99 + 33 + 50) + 4 \times 47}{60} = \frac{3324}{60} = 55.4\%.$$

Hence, his contribution from Bonus Questions to year mark is $0.6 \times 55.4 = 33.24$.

If, further, his percentage marks for the three assignments are $40, 60, 90$, then his year mark would be

$$Y = \frac{3 \times 40 + 2 \times 60 + 3 \times 90 + 12 \times 55.4}{20} = \frac{1174.8}{20} = 58.74\%$$

I hope that the calculation of the year mark should be clear and your tensions regarding when to expect the **Bonus Questions** and their deadlines should be clear. The *Video Lectures* do augment the syllabus, although **not** in the same order as the text book. However, the sections of the textbook which correspond to a *Video Lecture* shall be stated in the respective section.

Please feel free to send your queries, suggestions, opinions. . . , directly to me at my email address ghoshpp@unisa.ac.za. Please **do not** send your emails through the *myUnisa* website or the Microsoft Office 365 service, since there are complaints of students finding me unreachable from any of the above two services; use an independent email client instead.

2. Video Lecture 2

This video lecture conveys the definition of a real vector space and provides some examples. Material relevant to this lecture shall be obtained in [**Ant, 2005**, Chapter 4.2, Chapter 8.2].

The following questions are to be answered and answers submitted to ghoshpp.unisa.ac.za on or before **March 19, 2018**.

Please use any email client, except either of the Microsoft Office 365 or *myUnisa* services — it seems that mails from these sites are not properly forwarded to the Unisa email server.

No submission to the questions in this section shall be entertained after this date.

PROBLEM 3. Which of the following are linear transformations and which are pretenders? Give reasons for your answers.

(a) $\mathbb{R} \xrightarrow{f} \mathbb{R}$ where $f(x) = 3x$.
(b) $\mathbb{R}^2 \xrightarrow{f} \mathbb{R}$ where $f(x, y) = x^2 + y^2$.
(c) $\mathbb{R}^2 \xrightarrow{f} \mathbb{R}^3$ where $f(x, y) = (3x + 4y, 7y - 4x, 2x)$.
(d) $\mathbb{R} \xrightarrow{f} \mathbb{R}$ where $f(x) = 3x + 7$.
(e) $[0, 1] \xrightarrow{f} \mathbb{R}$ where $f(x) = 2x$.
(f) $V \times W \xrightarrow{p_1} V$, where $p_1(v, w) = v$ with vector spaves V and W and the set $V \times W$ made into a vector space with vector addition defined by $(v, w) + (v', w') = (v + v', w + w')$ and scalar multiplication defined by $\lambda(v, w) = (\lambda v, \lambda w)$.

[$5 \times 6 = 30$ marks]

PROBLEM 4. Let V and W be vector spaces and $\hom(V, W)$ be the set of all linear transformations from V to W, i.e.

$$\hom(V, W) = \left\{ f : \text{the function } V \xrightarrow{f} W \text{ is a linear transformation} \right\}.$$

Define for $f, g \in \hom(V, W)$, $a \in V$ and $\lambda \in \mathbb{R}$:

$$(f + g)(v) = f(v) + g(v), \tag{1}$$

$$(\lambda f)(v) = \lambda f(v), \tag{2}$$

and

$$\hom(V, W) \xrightarrow{E_a} W \ , \text{ where } E_a(f) = f(a). \tag{3}$$

(a) Show that $\hom(V, W)$ with the vector addition defined in (1) and scalar multiplication defined in (2) is a vector space.

(b) Show also that the function $\hom(V, W) \xrightarrow{E_a} W$ from the vector space $\hom(V, W)$ to the vector space W is a linear transformation.

Taking the special case of $V = \mathbb{R} = W$, show that the function $\hom(\mathbb{R}, \mathbb{R}) \times \mathbb{R} \xrightarrow{E} \mathbb{R}$ defined by $E(f, a) = f(a)$ is however **not** a linear transformation. Note that, using the result in Problem 3((f)) $\hom(\mathbb{R}, \mathbb{R}) \times \mathbb{R}$ above is indeed a vector space.

[7 + 8 + 5 = 20 marks]

References

[Ant, 2005] (2005). *Elementary Linear Algebra with Applications*. John Wiley & Sons, 9th edition.

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